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Weight minimum design of concrete beam strengthened with glass fiber reinforced polymer bar using genetic algorithm

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Abstract. This paper presents a generalized formulation for optimizing the design of concrete beam reinforced with glass fiber reinforced polymer bar. The optimization method is formulated to find the design variables leading to the minimum weight of concrete beam with constraints imposed based on ACI code provisions. A simple genetic algorithm is utilized to solve the optimization task. The weights of concrete and glass fiber reinforced polymer bar are included in the formulation of the objective function. The ultimate limit states and the serviceability limit states are included in formulation of constraints. The results of illustrated example demonstrate the efficiency of the proposed method to reduce the weight of beam as well as to satisfy the above requirement. The application of the optimization based on the most economical design concept have led to significant savings in the amount of the component materials to be used in comparison to classical design solutions.

Keywords: weight minimization; reinforced concrete; design optimization; genetic algorithm

1. Introduction

Fiber Reinforced polymer (FRP) reinforcement has recently been produced to replace steel in many applications, particularly in corrosion-prone reinforced concrete (RC) structures. In addition, FRP has some other advantages like high strength-to-weight ratio, good fatigue resistance. Therefore, FRP has been widely recommended by the researcher to be incorporated in concrete design. Glass fiber reinforced polymer(GFRP) bars, the most widely used FRP rebars, are available from a number of producers (Bank 2006, Shraideh and Aboutaha 2013). Experimental results proved that GFRP concrete beam provides significant increase in flexural capacity (El-Helou and Aboutaha 2015, Zhao *et al*. 2011). Design of concrete structure reinforced with GFRP is an important task in civil engineering application. The purpose of structural design is construction of structure to fulfill the optimality and safety criterions in respect of strength, stiffness and stability requirement (Karkauskas 2004, Norkus and Karkauskas 2004).

Because of this increasing demand, some design manual and national standards have been published in different countries for the design of concrete elements reinforced with FRP. However, most of those proposals mainly focus on determining the cross sectional area of the FRP with assumed concrete dimensions by trial and error method without considering the relative cost of concrete and FRP. This iterative process has two major disadvantages: 1) more computational effort may be exhausted on repetitive analyses and 2) the design may not be economical though the solution satisfies the performance criteria (Kwak and Kim 2008). On the other hand, optimum design procedure may lead to significant saving in cost of the structure (Islam *et al*. 2013, Rahman and Jumaat 2012).

The minimum cost design is special subject (Raue and Hahn 2005). Similarly, weight minimum structure is also important to reduce the cost of foundation of the structure. During the past two decades, a large number of research have been made in the area of the optimum design of reinforced concrete (RC) structures since 70’s using different types of method (Adamu *et al*. 1994, Coello *et al*. 1997, Islam *et al*. 2013, Islam *et al*. 2012, Raue and Hahn 2005). However, most of studies consider only steel as an internal reinforcement embedded in concrete. Limited or no study has been found in the optimization of FRP reinforced concrete beam.

In comparison with steel, FRP reinforcement usually possesses a lower modulus of elasticity that may result higher reinforcement strains, wider cracks and larger deflections. Therefore, the serviceability limit state may often govern the design of FRP reinforced concrete structure. In addition, FRP never yields and consequently the preferred mode of failure is concrete crushing which is significantly different from that of steel reinforced concrete. Since the properties of FRP is different from those of steel, the design manual of steel reinforced concrete may not be applicable to FRP reinforced concrete beam (Leung 2004).

This paper presents the development of a procedure for optimum design of concrete beam reinforced with GFRP using Genetic Algorithm. This study is aimed at optimizing the weight of GFRP reinforced concrete beam subjected to the limitations specified by ACI code. For conducting the optimum mix design procedure, genetic algorithm has been applied to search best combination of design variables for

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producing GFRP reinforced concrete beam. The analytical approach is presented using an example of certain particular set of component materials.

2. Optimization problem

"Optimal" means the most economical solution (Kasperkiewicz 1995, Rahmanian *et al*. 2014). Optimization procedures try to seek the "best" solutions for a desired objective function, $f(X)$, while satisfying the prevailing constraints. Maximization can be easily converted into minimization problem since the maximization of $f(X)$ is equivalent to the minimization of-$f(X)$ (Perera and Varona 2009). To apply optimum technology, it is essential to convert beam design problem into an optimization task, including objective function and constraint functions. In the present optimization problem, the purpose is to minimize the unit weight of GFRP reinforced concrete beam.

To introduce the subject formulation of optimization problem can be presented in the following mathematical form

Minimize

$$f(X)$$

Subject to

$$g_j(X) = 0; j = 1; 2; : : :; m$$

$$h_k(X) <=> 0; j = 1; 2; : : :; n$$

Genetic Algorithm (SGA) that has been successfully used in many optimization applications (Baušys and Pankrašovaite 2005, Camp *et al*. 2003, Koumousis and Georgiou 1994, Rahman and Jumaat 2012, Rajeev and Krishnamoorthy 1992, Šešok and Belevičius 2008, Tsai *et al*. 2011, Yan *et al*. 2011, Zhao *et al*. 2011) demonstrates the general idea and working of a typical Genetic Algorithm. Holland (1975) introduced genetic algorithms as a robust optimization technique and Goldberg (Goldberg 1989) developed it in the engineering area. As an alternative to standard optimization methods like mathematical programming method genetic algorithms are computationally powerful in their search for improvement without the derivatives of the specific problem in order to conduct a search operation. In standard GAs the specifications of a number of parts or attributes (the genotype bits) is taken and combined to create a function (the phenotype), and it is the fitness of this function that is to be optimized. Detail description of genetic algorithm is beyond the scope of this paper. However, the basic algorithms of genetic algorithms have been covered widely in the literature (Goldberg 1989, Sgambi *et al*. 2014)

3. Objective function

The selection of the objective function has a significant influence on the optimization problem. This function is utilized to demonstrate a measure of how the variables have performed in the problem domain. In the case of a minimization problem, the best individuals will have the smallest numerical value of the related objective function (Perera and Varona 2009). The optimization of FRP reinforced concrete beam design can be formulated by using unit weight as an objective. The total weight of the FRP reinforced concrete beam is affected by the weight of concrete and weight of FRP reinforcement. Therefore, the weight optimum design is a compromise between weight of the FRP reinforcement and the concrete. This compromise minimizes the total weight of concrete beam. The design variables are the diameter of GFRP bars and cross-sectional dimension of concretes that are allowed to change with purposes of minimizing concrete production process. The mathematical form of the objective function C for the design of GFRP reinforced concrete beam is

Minimize

$$W = bh\, W_c + ASW\, S$$

Where W=weight of beam per unit length (meter), b=width of beam, h=height of beam, W_c=unit weight of concrete A_s=area of FRP bar, W_s=unit weight of GFRP. In the weight optimization of FRP reinforced concrete beam, the cross-sectional dimension of concrete and amount of FRP reinforcement will be obtained.

4. Constraint and penalty function

In structural optimization problem, some technical performance and practical limitation have to be satisfied though the application of constraints functions. Therefore, the design guidelines on flexure reinforced concrete beam with FRP proposed by the American Concrete Institute (ACI 440.1R-06 2006) are formulated through constraint functions. The constraints include flexural constraints and serviceability constraints. The ACI design guideline are based on limit state principle that provides an acceptable level of safety. Certain values for the design loading and the design strength of the materials will be used to evaluate these limit state. Load factors and strength reduction factors stated in conventional RC design will be used.

Genetic algorithms cannot handle the constraints explicitly. Several techniques are proposed to handle constraint optimization problem (Carlos 2002). Among them the rejection policy and penalty function approach are the common and the most widely used method. In a penalty approach, the constraint optimization problem is converted to unconstraint optimization problem by the application of penalty function. Therefore, it is essential to transform all the constraints into the penalty function. The function of constraints is to shrink the extent of the design space to be searched in accordance with our objectives that has to achieve. If very strict constraints are applied it is impossible to find a solution and the problem has no degrees of freedom (it is over-constrained), so it is important to try to balance design freedom with ideal preferences.

5. Flexural constraint

The flexure performance characteristic of resistance parameter and loading condition along with the principle of equilibrium forms an equation relating to design variables. The concrete beam should be designed to provide necessary strength to carry the factor loads. Hence, the required moment strength of a section should be calculated by making use of load factors and safety factors as required ACI code. According ACI 440.1R-06 (2006) the factored nominal moment capacity, φM_n, of the member must be greater than the factored applied moment, M_u. These constraints provide acceptable levels of safety against ultimate limit states. This constraint is presented in the following form

$$\varphi M_n > M_u \tag{2}$$

The nominal bending moment capacity of FRP reinforced concrete beam can be calculated in a fashion similar to that of a steel-reinforced section which considers equilibrium of forces and that plane section remain plain after bending. It is also necessary to consider probable mode of failure that control the flexure strength of concrete beam. For the case of FRR reinforced beam, yield strength of the steel rebar will be replaced by the ultimate strength of FRP bar because FRP bar never yield. Therefore, most GFRP reinforced flexure member will be over-reinforced. Therefore, the preferred mode of failure should be concrete crushing that is significantly different form that of steel reinforced concrete beam.

6. Constraint against serviceability

The serviceability constraints are formulated in terms of limit deflections, cracks and stesson the steel reinforcement and concrete stress Since GFRP has lower modulus of elasticity, the deflection of FRP-reinforced beams for equivalent reinforcement ratio to steel reinforced beams will be much larger. Therefore, deflections and crack width should be controlled for the serviceability limit state specified in ACI 440.1R-06 (2006) (Jnaid and Aboutaha 2013). Deflection should be estimated for both short term and long term loads that might sustained under service loads condition and it must be less than the allowable deflections permitted by the code. An effective second moment of area based on cracked, I_{cr}, and gross, I_g second moment is used to calculate the deflections of the beam similar to steel reinforced concrete members. The stresses in FRP bars under sustained service loads should be less that creep rapture and fatigue limit.

For deflection

$$\Delta_{LT}=\Delta_{LL}+\lambda[\Delta_{DL}+0.2\Delta_{LL}] \leq l/24 \tag{3}$$

For crack width

$$w = \frac{2.2}{E_f}\beta k_b f_f \sqrt[3]{d_c A} \leq 0.71 \text{ mm} \tag{4}$$

FRP bar stress

$$f_f = \frac{M_s}{A_f d(1-k/3)} \leq 0.20 f_{fu} \tag{5}$$

Where $\Delta_{LT,}$ Δ_{DL} and Δ_{LL} are the total deflection, deflection due to dead load and deflection due to live load, respectively. λ is the multiplier of additional long term deflection9 (1.2 for more than 5 year). w is crack width that should be less than 0.71 mm, β is ratio of distance from neutral axis to extreme tension fiber to the distance from the neutral axis to the center of tensile reinforcement, M_s is moment due to sustained load, f_f is bar stress under service load, A_f is area of FRP reinforcement, d is effective depth of beam, k is the ratio of depth of neutral axis to depth of reinforcement. f_{fu} is design tensile strength of FRP reinforcement considering reduction of service environment. In addition to design constraint, some bounds of design variable are applied based on ACI code provision, aesthetical requirement, practical consideration and availability of materials

7. Application of the method

Application of optimization method requires the fixation of some constant parameter. The constant parameter includes weight of the concrete, weight of FRP bars, the compressive strength of the concrete, the ultimate design strength of FRP bars, live load and span of the beam. The weight optimization method starts with calling these constant parameters. The present study considers an example provided by ACI 440.1R-06 (2006) for flexural reinforced concrete beam with Glass Fiber reinforced polymer bar. This example consists of simply supported interior beam for a l=3.35 meter. The beam should be design for the strength limit state. The following FRP manufacture-guaranteed rebar properties should be used: 1) f_{fk}=620.6 Mpa, 2) ε_{fk}=0.014, 3) E_f=44800 Mpa.

A simple genetic algorithm was applied for optimization of FRP reinforced concrete beam design problem using SolveXL add in with excel. To apply the genetic algorithm, a number of genetic operations like generation, selection, crossover and mutation have to be performed. The generation is a population of candidate solutions as a starting point which is usually random. The population size used in this study is 100. Among the three selection operator, tournament selection method was applied in application. Crossover and mutation make the genetic algorithm more powerful. Cross over forms a new chromosome from the two parental chromosomes by reproduction operation. Single point crossover technique was selected for this operation. Mutation creates diversity among the population by changing the gene. The mutation rate used in this study is 0.05.

8. Result

It is demonstrated how the procedure mentioned in preceding section can be utilized with the help of modern digital computer to find the optimal value of beam section and FRP reinforcement. One example taken from the literature (ACI440.1R-06 2006) was solved and demonstrated in order to illustrate the use of the proposed method, to prove its capabilities, and to validate its results. After a number of trial-and-error adjustments, 100 generations were selected to meet the accuracy requirements for the example.

Table 1 Result of continuous design optimization of FRP reinforced concrete beam

Design variable	Optimum value of variables if *b* is not restricted	Optimum value of variables if *b* is restricted to 175	Tradition value of the variable
Beam width, *b* (mm)	120	176.4	178
Beam height, *h* (mm)	302	244	305
Diameter of FRP bar (mm)	20.82	30.884	19
Weight per unit length (Kg/m)	89.92	109.32	132.56
Saving in weight	29%	12%	0%

Table 2 Result of discrete design optimization of FRP reinforced concrete beam

Design variable	Optimum value of variables if *b* is not restricted	Optimum value of variables if *b* is restricted to 175	Tradition value of the variable
Beam width, *b* (mm)	120	175	178
Beam height, *h* (mm)	290	241	305
FRP bar size	7	10	6
Number of FRP bar	2	2	2
Weight per unit length (Kg/m)	86.61	107.53(111.31)	132.56
Saving in weight	29%	12%	0%

The beam is to be designed to carry a service live load of w_{LL}=5.8 KN/m(20% sustained) and superimposed service dead load of W_{SDL}=3.0 KN/m.(does not include beam self weight, which must be added to this to obtain the total dead load on the beam. The beam deflection should not exceed *l*/24, which is the limitation of long-term deflection. Due construction restriction, the depth of the member should not exceed 356 mm. In these examples, the optimal width and depth, the optimal area of GFRP bars of a rectangular RC beam is calculated to attain the least weight of the beam, while satisfying the ACI requirements.

The above problem has been solved by genetic algorithm approach mentioned in the proceeding with continuous variable. The number of bas has been kept 2(two) which is fixed. The results from design optimization process are shown Table 1 and compared with traditional design approach. The weight saving for using design optimization technique has been mentioned in table.

Since the strength of FRP reinforcing bar decreases with the diameter of the bar, it should be considered in the design process. Therefore, a linear regression model has been developed to take into consideration the effect of bar diameter on strength of the bar. Fig. 1 has shown the model with respective equation. The co-efficient of correlation of this model is .9769 which is considerably excellent. In addition, diameters of FRP bar available in market are discrete value rather than continuous. Hence, the optimization task should be performed with discrete design variables to evaluate the effect of discretization on optimization process.

A discrete algorithm is usually needed to solve the

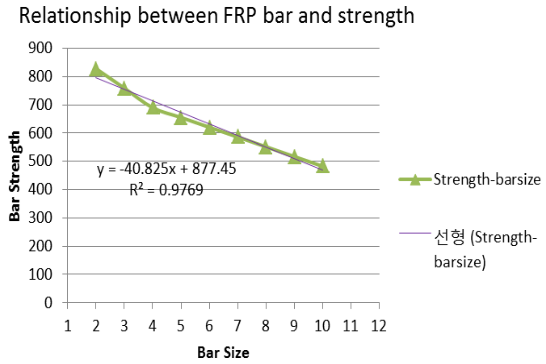

Fig. 1 Relationship between FRP bar size and its strength

discrete optimization problem and consequently, SolveXL add in software is not capable of solving discrete optimization problem. However, if the discrete value of the design variables varies uniformly or in a systematic way, the discrete design variables can be converted into integer variables by making a relationship between them. Since the bar diameter of FRP material varies uniformly it can be represented by corresponding nominal bar size which is integer.

The same problem mentioned previously has again been solved by genetic algorithm approach with discrete design variable. Moreover, the number of bar, in this case, are allowed to change. The results from discrete design optimization process are shown Table 2 and compared with traditional design approach. The weight saving using design optimization technique and the effect of discretization has been demonstrated in table.

9. Conclusions

This paper provides general approach of optimal design procedure incorporating the cost parameter to find the most economical dimension of concrete beam and area of GFRP for producing GFRP reinforced concrete beam. Simultaneously it satisfies the technical and structural performance requirement. A simple and efficient procedure utilizing digital computer has been developed to solve the optimization problem. With the application of genetic algorithm, the most important parameter like the section of beam and amount of FRP reinforcement has been optimized with respect to minimum weight of the reinforced concrete beam.

The optimum design procedure helps to reduce time by eliminating laborious trial and error attempts. It is usually difficult for a novice design engineer to predict in advance near optimum reinforced concrete section. He has to try different combination of beam parameter to achieve this optimal point except the most experienced design engineer.

- The GA procedure can successfully be applied in the design of concrete beam reinforced with GFRP.
- The application of the optimization based on the most economical design concept may lead to significant savings in the amount of the component materials to be used in comparison to classical design solutions.
- The discrete optimization problem can be solved by

mixed integer optimization algorithm provided that the discrete design parameters vary uniformly or in a systematic pattern.

• The discretization of design variable have further optimized the previous solution. Therefore, GA procedure is more suitable in discrete optimization than in continuous design optimization problem.

• Restriction of beam width have significantly influenced the performance of the design optimization process.

• The selection of genetic parameter has a considerable influence on the solution of optimum design problem.

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