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Numerical model to simulate shear behaviour of RC joints and columns

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Abstract. In this paper, a new numerical model for the evaluation of the seismic behaviour of existing reinforced concrete (RC) structure with considering the effect of variation of applied axial load on columns is presented. Focus is given on developing accurate and practical models for simulating nonlinearities in joint core as well as column under varying axial load. For exterior as well as interior joints, according to experimental and finite element results, principal tensile stresses versus shear deformation relations in joint core are recommended. Moreover, to consider the effects of stirrup on the principal tensile stress-shear deformation relation, an incremental procedure is proposed based on the Mohr theory. According to the proposed numerical model, complex nonlinear behaviour of joint core is simulated by two diagonal axial springs so that the effect of variation of axial load is also considered. The properties of these springs are determined based on the proposed principal tensile stress-shear deformation relation in the joint core. Moreover, the shear and flexural nonlinear behaviour of RC beams and columns is also simulated by rotational springs. A simplified methodology is developed to consider the effects of axial load variation on shear and flexural nonlinear behaviour of RC columns. To demonstrate the capability of the proposed numerical model at structural level, two RC frames with various failure modes are investigated. The results confirm the ability of the model in predicting the nonlinear behaviour of the frame, which can provide an alternative method for engineers in practice.

Keywords: beam-column joints; numerical model; shear effect; nonlinear analysis; principal tensile stresses

1. Introduction

Traditionally, RC beam-column joints were considered rigid with the basic assumption that the shear failure of the joint core can be neglected in nonlinear analyses. However, when shear demand increases during seismic actions, RC joints are subjected to high shear force in the joint core. A large portion of these shear force is endured by the transverse reinforcements. According to this fact, without transverse reinforcements, significant diagonal shear cracking may occur in the joint core that will lead to reduction in the joint strength. Therefore, in absent of transverse reinforcement in the core, the joint and later on the structure will be vulnerable against seismic

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actions which has been proved by many previous studies (Calvi *et al*. (2002a), Calvi *et al*. (2002b), Sasmal *et al*. (2010), Niroomandi *et al*. (2010), Park and Mosalam (2013), Costa *et al*. (2013), Asha and Sundararajan (2014), Niroomandi *et al*. (2014), Shayanfar *et al*. (2016), Parate and Kumar (2016)). Calvi *et al*. (2002a) and Calvi *et al*. (2002b) studied the effects of shear failure in joint core on the seismic behaviour of existing RC frame systems designed only for gravity loads. According these works, typical deficiencies in RC structures (i.e. joints with plain round bars with 180°-hooks, absence of stirrup etc.) might cause severe resistance reduction leading to brittle behaviour mechanism. To predict seismic complete behaviour of these RC structure, the modelling of the joint core is required. Numerous models have been proposed to predict the nonlinear behaviour of joints in the literature. Alath and Kunnath (1995) simulated the joint behavior with a zero length rotational spring model and rigid links to introduce the joint zone geometry. Biddah and Ghobarah (1999) modeled the joint inelastic behavior with discrete rotational springs to describe the effects joint shear and bond slip deformations. Youssef and Ghobarah (2001) proposed a joint model using two diagonal springs and 12 translational springs to model the joint shear distortion and the effects of bar slippage and concrete crushing, respectively. Pampanin *et al*. (2003) came up with a joint model using a rotational spring with zero length and rigid elements to describe the joint zone geometry. The properties of this spring were computed according to the principal tensile stress- shear deformation in the joint core. Lowes and Altoontash (2003) proposed joint models using 12 zero-length spring and a rotational spring to simulate the response of joint. Sharma *et al*. (2011) provided a model including two shear springs and a rotational spring to simulate inelastic behaviour in the joint core.

It is generally known that the nonlinear behaviour of the RC joints as well as RC column is a function of the applied axial load on column (Clyde *et al*. (2002), Pantelides *et al*. (2002), Lynn (2001), Sezen (2002)). However, at structural level, the joint shear capacity as well as column flexural and shear capacity are significantly affected by variation of level of applied axial load on column (Li *et al*. (1991), Pampanin *et al*. (2002), Ousalem *et al* (2004), Akguzel and Pampanin (2012)). Therefore, for a comprehensive nonlinear analysis, this effect should be considered in the calculation of the properties of RC joints and columns.

Sharma *et al*. (2013) tested a four storey RC frame. In order to model this frame, analytical methods were applied to frame model in nonlinear analysis. The seismic performance of frame was predicted relatively good in the case of the considering of the effect of joint failures. Their numerical results confirm the impact of considering the various failure mode in the analysis. Therefore, in order to derive the inelastic response of RC frames, simulating and predicting the expected failure in each member is required. While the models available in literature for simulating the shear effect in the joint core as well as beam and column are not simple enough for using in existing commercial programs. Moreover, the mentioned models generally require large number of springs or special plastic hinge or program to analyze the global behavior of the RC structural. Therefore, providing a practical model to simulate effects of shear behaviour for beam and column as well as more complex shear behavior at the core of the joint is required. Moreover, the effects of varying axial load on column as well as joint should be considered for understanding the complete and realistic response of the structure.

In this study, for simulating nonlinearities in joint core as well as beam and column, an accurate and practical models is developed so that the shear and flexural nonlinear behaviour of beams and columns are simulated by rotational springs and the nonlinear behaviour of the core of joints is simulated by two diagonal axial springs. Moreover, a procedure is proposed for considering the effect of variation of applied axial load on joints as well as column. The properties of joint springs are computed based on the principal tensile stress-shear deformation relation in joint core. Therefore, for exterior as well as interior joints, according to experimental and finite element results, principal tensile stresses versus shear deformation relations in joint core are recommended. Moreover, for seismically detailed beam-column joints, the effects of the transverse reinforcements on principal tensile stress-shear deformation curve is calculated using an incremental procedure.

2. Nonlinear model of RC frame

In order to derive the nonlinear behaviour of RC structures using the lumped plasticity concept, the calculation of plastic hinge properties is required. Considering only the flexural nonlinear behaviour of the structural components with popular assumption that shear failure of members can be simplified and neglected in analysis, may lead to quite misleading results in terms of the seismic performance of the structure. Therefore flexural and shear inelastic behaviour of the structural components should be considered. Especially, for the RC structures without transverse reinforcement in the joint zone under seismic action, the joints behaviour has a significant role in the performance evaluation of RC structure. To simulate these behaviour, the numerical model with two diagonal axial springs in joint core and rotational springs in beam and column is proposed as shown Fig. 1. Here, h_b and h_c define beam and column section depth, respectively. L_b and L_c are half of the bay width and the storey height, respectively. As can be seen in figure, flexural and shear inelastic behaviour of beam and column are modeled by the rotational springs. In practical application, it is useful to analyze the structural response using commercial software. Therefore, the determination of the accurate properties of these springs is required.

3. Flexural and shear effect in beam and columns

3.1 Flexural behaviour

This section describes the computation of the properties of the rotational springs according to proposed model (see Fig. 1). In order to compute the properties of these springs, the determination of the moment- curvature of section is required which can be derived based on the principle of strain compatibility and equilibrium and material constitutive relations for concrete and steel. In a RC member, the confinement effect by transverse reinforcements should be considered in the stress–strain properties of concrete. In the present study, the stress–strain model proposed by Mander *et al*. (1988) is used for computing the properties unconfined and confined concrete that are considered for concrete of cover and core of section, respectively. Also, the elastic-plastic model is assumed for steel bars. When moment-curvature relation of section was formulated, then, moment-rotation curve can be determined by Eqs. (1)-(2) as

$$\theta_i = \frac{\varphi_i L}{2} \quad \text{for } \varepsilon_s \le \varepsilon_y \tag{1}$$

$$\theta_i = \frac{\varphi_y L}{2} + (\varphi_i - \varphi_y) L_p \quad \text{for } \varepsilon_s \ge \varepsilon_y \tag{2}$$

where φ_i and φ_y denote the curvature of reinforced concrete section and the yield curvature; ε_s and ε_y represent strain in reinforcing bar and reinforcement yield strain; L and L_p are the distance of critical section to the point of contra-flexure and the plastic hinge length, respectively. In this study, to compute the plastic hinge length, the equation recommended by Pauley and Priestley (1992) has been followed as

$$L_p = 0.08L + 0.022 d_b f_y \tag{3}$$

where d_b and f_y define diameter of main reinforcing bars, in mm and yield strength of reinforcement bars, in MPa.

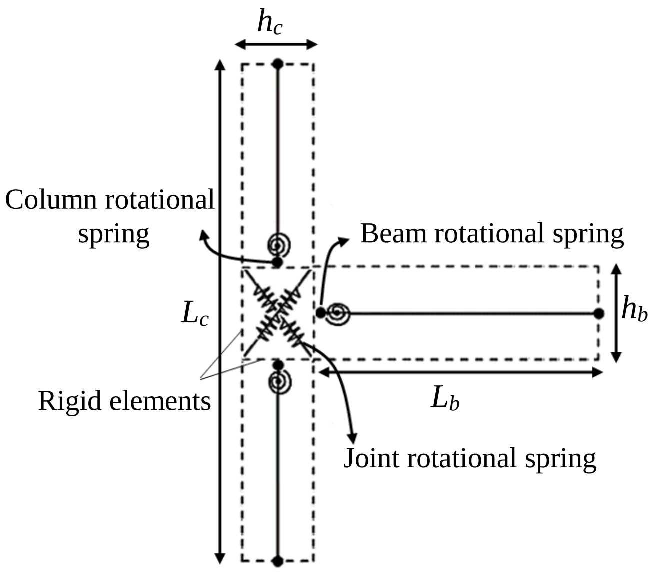

Fig. 1 Proposed model for exterior RC beam-column joints

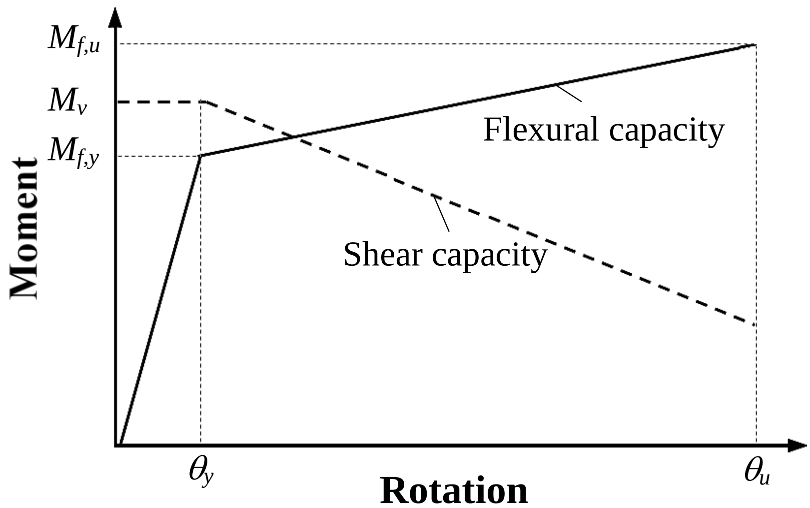

Fig. 2 Shear model for interaction between shear and rotation

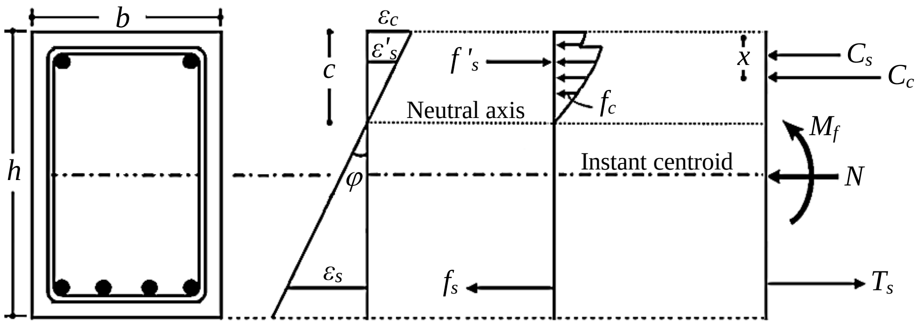

Fig. 3 Sectional analysis of a RC member section

The flexural stiffness of the RC members is decreased under the seismic actions. To simulate this phenomenon in the nonlinear analysis, the equivalent flexural stiffness, E_cI_{eff}, can be used. It is described by Eq. (4) as

$$E_c I_{eff} = \frac{M_y}{\varphi_y} = \frac{M_y L}{2\theta_y} \tag{4}$$

where M_y and E_c define the yield moment and the concrete elastic modulus, respectively.

It is clear that the properties of moment-rotation for a RC column is significantly sensitive to the applied axial load. Accordingly, at structural level, variation of applied axial load on RC column under seismic action causes numerous complexity in calculation of behavioural properties of RC column. The relation between the axial load, N, and the column shear force generated by lateral loads, V_c, can be repressed as

$$N = N_g + KV_c = N_g + KV_n = N_g + K\frac{M_f}{L} \tag{5}$$

in which

$$A = \frac{K}{L} \tag{6}$$

The axial load versus lateral force relationships can be obtained from preliminary static pushover analyses on structure. In Eq. (5), N_g and K define the column axial load due to gravity load and the axial load factor, respectively. V_n and M_f define nominal shear capacity and flexural moment of a RC column. With no considering the effect of variation of axial load (K = 0), the natural axis depth, c, can be computed based on the force equilibrium at any level of axial load as

$$C_c + C_{si} - T_{si} - N = 0 \tag{7}$$

where C_c is the compressive force of the concrete; C_{si} and T_{si} define the compressive and tensile forces of longitudinal reinforcements (see Fig. 3). Finally, using obtained natural axis depth, the flexural moment can be calculated as follows

$$M_f = C_c\left(\frac{h_c}{2} - x\right) + C_{si}\left(\frac{h_c}{2} - d_i\right) + T_{si}\left(d_i - \frac{h_c}{2}\right) \tag{8}$$

On the other hand, considering the effect of variation of axial load ($K \neq 0$), according to Eq. (5), the column axial load as a function of axial load factor and section properties can be written as follows

$$N = N_g + AM_f = N_g + C_c(A\frac{h_c}{2} - Ax) + C_{si}(A\frac{h_c}{2} - Ad_i) + T_{si}(Ad_i - A\frac{h_c}{2}) \tag{9}$$

Substituting Eq. (9) in Eq. (7), the force equilibrium can be written as

$$C_c(A\frac{h_c}{2} - Ax - 1) + C_{si}(A\frac{h_c}{2} - Ad_i - 1) + T_{si}(A\frac{h_c}{2} - Ad_i + 1) + N_g = 0 \tag{10}$$

where x and d_i define the distance from the location of the concrete compressive force to the extreme compression fiber of section and the effective depth for the longitudinal reinforcement. As a result, using Eq. (10), the natural axis depth and consequently, flexural moment-curvature of a RC column can be calculated so that the effect of variation of axial load is considered.

3.2 Shear behaviour

For RC members, the shear capacity reduces while the inelastic flexural displacement increases. Numerous studies have been conducted by researchers for considering this effect in predicting of nonlinear behaviour of RC members (Aschheim and Moehle (1992), Priestley *et al*. (1994), Sezen and Moehle (2004), Sung *et al*. (2005), Park *et al*. (2006), Park *et al*. (2012)). Sung *et al*. (2005) proposed a new shear model for RC column according to study conducted by Aschheim and Moehle (1992). This model implements the shear effects to the flexural moment–rotation curve as shown in Fig. 2. $M_{f,y}$ and $M_{f,u}$ are yield and ultimate moment corresponding to yield and ultimate rotation in RC member, respectively. As can be observed in Fig. 2, the model needs to define the equivalent shear moment, M_v, as a function of flexural rotation in RC member which can be computed as

$$M_v = V_n \times L \tag{11}$$

where the nominal shear capacity, V_n can be calculated as

$$V_n = 0.53(R + \frac{N}{140A_g})\sqrt{f'_c}A_e + V_s \qquad N > 0 \tag{12}$$

for RC columns

$$V_n = 0.53(R + \frac{N}{35A_g})\sqrt{f'_c}A_e + V_s \qquad N < 0 \tag{13}$$

$$V_n = 5.2211R\frac{\rho_l^{1/3}}{d^{1/4}}f'^{1/3}_c(0.75 + \frac{1.4}{L/d})A_e + V_s \qquad \text{for RC beams} \tag{14}$$

in which

$$R = \frac{\theta_u - \theta_i}{\theta_u - \theta_y} \qquad 0 \leq R \leq 1 \tag{15}$$

$$V_s = \frac{A_v f_{yv} d}{s} \cot(\theta') \tag{16}$$

where f'_c is concrete compressive strength; A_v, s, f_{yv} and d are the total transverse reinforcement area, the centre to centre spacing of transverse reinforcement, yield stress of transverse reinforcement and effective height of cross-section, respectively; ρ_l is flexural reinforcement ratio. Moreover, in this study, A_e in Eqs. (12) to (14) and θ' in Eq. (16) are assumed equal to 80% of the total cross-section area (A_g) and 30° (Priestley *et al*. (1994)), respectively. In additional, as can be seen in Eq. (14), for a RC beam, the concrete shear capacity is computed based on study conducted by Okamura and Higai (1980). In order to consider the effects of axial load variation on shear behaviour of RC columns a simplified methodology has been provided. For this purpose, substituting Eq. (5) into Eq. (12), the nominal shear capacity can be written as

$$V_n = 0.53\left(\frac{\theta_u - \theta}{\theta_u - \theta_y} + \frac{N_g + KV_n}{140A_g}\right)\sqrt{f'_c}A_e + V_s \tag{17}$$

Rearranging the above equation gives

$$V_n = \frac{0.53(140RA_g + N_g)\sqrt{f'_c} + 175V_s}{140 - 0.53K\sqrt{f'_c}} \tag{18}$$

Therefore, for $N > 0$, the equivalent shear moments as a function of axial load factor, can be calculated by Eq. (19) as

$$M_v = \frac{0.53\sqrt{f'_c}(140RA_g + N_g) + 175V_s}{175 - 0.53K\sqrt{f'_c}}L \tag{19}$$

On the other hand, for $N < 0$, the equivalent shear moments corresponding flexural rotation as a function of axial load factor, can be written as

$$M_v = \frac{0.53\sqrt{f'_c}(35RA_g + N_g) + 43.75V_s}{43.75 - 0.53K\sqrt{f'_c}}L \tag{20}$$

Therefore, providing these equations, the shear model proposed by Sung *et al*. (2005) has been modified to consider the effect of the variation of the applied axial load on RC column.

4. Shear effect in exterior beam-column joint

This section describes assessment of shear behaviour in core of exterior joint which is considered rigid in most nonlinear analysis. The proposed joint model was illustrated in Fig. 1. As can be seen, the nonlinear behaviour of the core of joints is simulated by two diagonal axial springs. The properties of these springs are calculated based on the principal tensile stress -shear deformation in joint core. In this study, for RC non-ductile joints with various anchorage of beam longitudinal reinforcement in joint core, the principal tensile stress-shear deformation curves are proposed based on experimental and analytical studies conducted by (Priestley (1997), Hakuto *et*

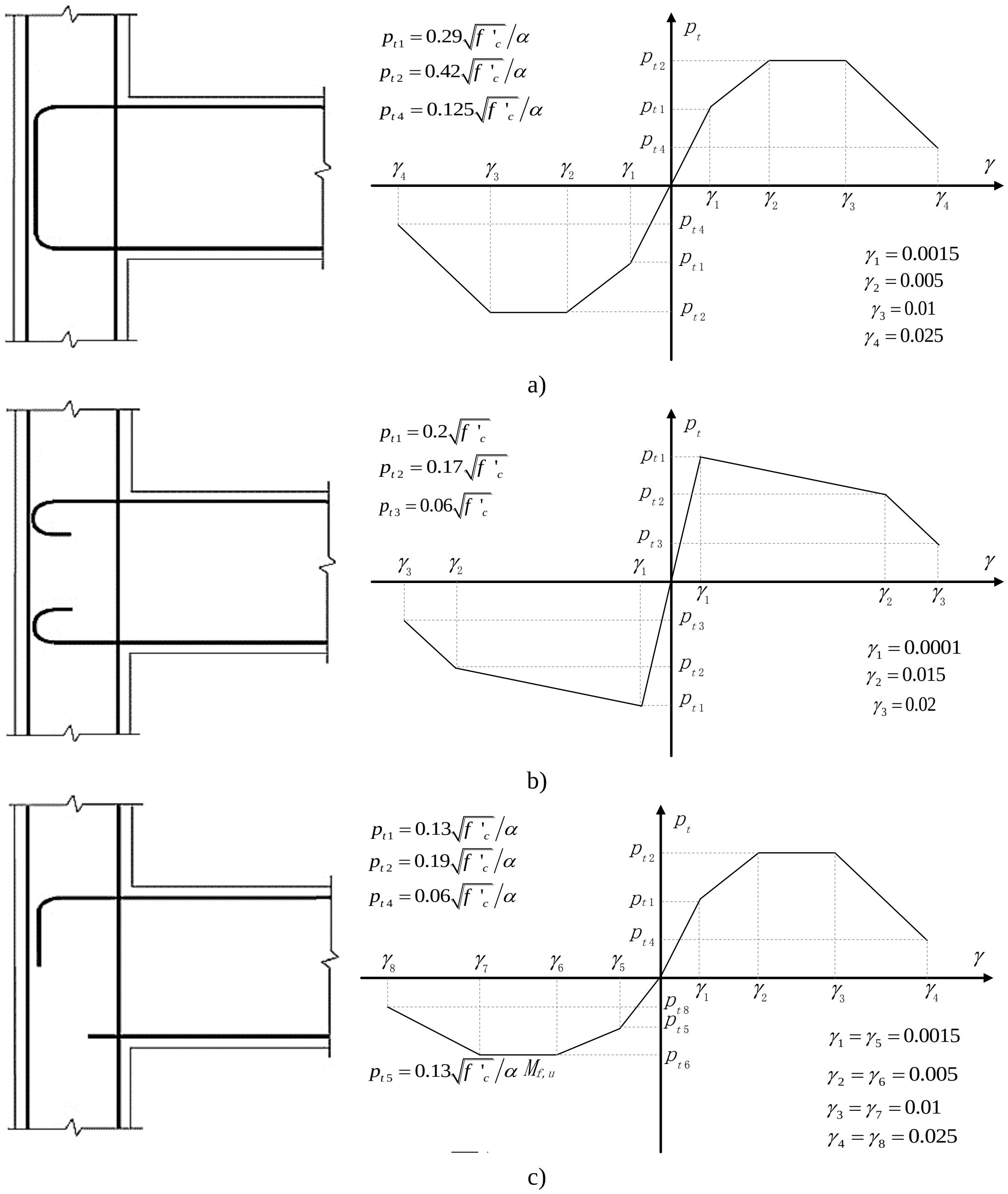

Fig. 4 The principal tensile stress-shear deformation relation for exterior joints with a) deformed bars and 90°-hooks b) plain round bars and 180°-hooks c) deformed bars and 90°-hook / straight anchorage.

al. (2000), Clyde *et al*. (2002), Pantelides *et al*. (2002), Sharma *et al*. (2011), Akguzel and Pampanin (2012), Sharma (2013)) and finite element study conducted by Genesio (2012) as shown in Fig. 3. Here, α defines joint aspect ratio; p_{t1}, and γ_1 are the principal tensile stress and shear deformation corresponding to first diagonal cracking, respectively. The other parameters are defined in the figure. As can be seen in Fig. 3, for case of joints with 90°-hook as well as straight anchorage, hardening behaviour can be observed after first diagonal cracking while, for joints with 180°-hook, the maximum principle tensile stress occurs corresponding to first diagonal cracking in joint core. It should be noted that for exterior RC joints with a short embedment length, the bond failure becomes the critical parameter. This means that the bond failure occurs prior to the shear failure in joint core. On the other hands, for exterior RC joints with a sufficient embedment length, the shear failure becomes the critical parameter. Therefore, to compute the properties of diagonal axial springs, the proposed principal tensile stress-shear deformation curve for exterior joint with 90°-hook, can be used (Pantelides *et al*. (2002)).

4.1 Principal tensile stress–shear deformation relation of the joints with stirrups

This section describes the computation of principal tensile stress-shear deformation relation for joints which transverse reinforcements are used in the joint core for preventing shear failure and for increasing the joint shear strength. In order to derive the effect of stirrups on the principal tensile stress-shear deformation curve, an incremental procedure is suggested. According to Akguzel and Pampanin (2012) and Del Vecchio *et al*. (2015), the stirrup contribution corresponding to a value of ε_s, can be obtained from the horizontal tensile force in the stirrups (see Fig. 5)

$$T_s = A_v f_s \tag{21}$$

in which

$$f_s = E_s \varepsilon_s \qquad \text{for} \quad \varepsilon_s \le \varepsilon_y \tag{22}$$

$$f_s = f_y \qquad \text{for} \quad \varepsilon_y \le \varepsilon_s \le \varepsilon_{sh} \tag{23}$$

$$f_s = f_y + (f_u - f_y)\left(\frac{\varepsilon_s - \varepsilon_{sh}}{\varepsilon_u - \varepsilon_{sh}}\right)^{0.5} \qquad \text{for} \quad \varepsilon_{sh} \le \varepsilon_s \le \varepsilon_u \tag{24}$$

where A_v and f_s define the total area and the tensile stress in the stirrups, respectively. ε_y, ε_{sh} and ε_u represent the yielding, hardening and ultimate strain in stirrups, respectively. The principal tensile stress contributed by stirrup can be calculated using the equivalent stirrups area in the panel of the joint as (see Fig. 5)

$$p_{t,s} = \frac{T_s \cos\theta_s}{b_c \dfrac{h_b}{\cos\theta_s}} = \frac{A_v f_s}{b_c h_b} \cos^2\theta_s \tag{25}$$

where θ_s defines the direction of principal stresses; bc is width of the column. On the other hand, principal tensile stress contribution due to stirrups can be computed using Mohr`s circle as

$$p_{t,s} = \sqrt{\left(\frac{f_v - f_h}{2}\right)^2 + v_s^2} - \frac{f_v + f_h}{2} \tag{26}$$

where f_h and f_v represent the horizontal and vertical stresses at the mid-depth of the panel of joint, respectively. v_s define joint shear stress contributed by stirrups which can be calculated as

$$v_s = \frac{\rho_s f_s}{\tan\theta_s} \tag{27}$$

where ρ_s is the stirrup ratio in the longitudinal direction. Putting Eq. (25) in the Eq. (26), we have

$$\frac{A_v f_s}{b_c h_b}\cos^2\theta_s = \sqrt{(\frac{f_v - f_h}{2})^2 + v_s^{\,2}} - \frac{f_v + f_h}{2} \tag{28}$$

Squaring both the sides and simplifying, we have

$$\begin{aligned} &-\left(\frac{A_v f_s}{b_c h_b}\right)^2 \cos^6\theta_s + \frac{A_v f_s}{b_c h_b}\left(\frac{A_v f_s}{b_c h_b} - f_v - \rho_s f_s\right)\cos^4\theta_s \\ &\qquad + \left(\frac{A_v f_s}{b_c h_b} f_v + \frac{A_v f_s^{\,2}}{b_c h_b}\rho_s - \rho_s f_v f_s - \rho_s\ f_s\right)\cos^2\theta_s + \left(f_v \rho_s f_s\right) = 0 \end{aligned} \tag{29}$$

Therefore, the direction of the principal stresses contributed by stirrups, θ_s, is derived by solving Eq. (29). Now, the vertical direction strain in joint panel can be computed as follows

$$\varepsilon_v = \frac{\sigma_2 - \sigma_2 \tan^2\theta_s}{E_c} + \varepsilon_s \tan^2\theta_s \tag{30}$$

in which

$$\sigma_2 = -\frac{\tan^2\theta_s + 1}{\tan\theta_s} v_s \tag{31}$$

Therefore, the joint shear deformation can be determined as

$$\gamma = \frac{2(\varepsilon_1 - \varepsilon_s)}{\tan\theta_s} \tag{32}$$

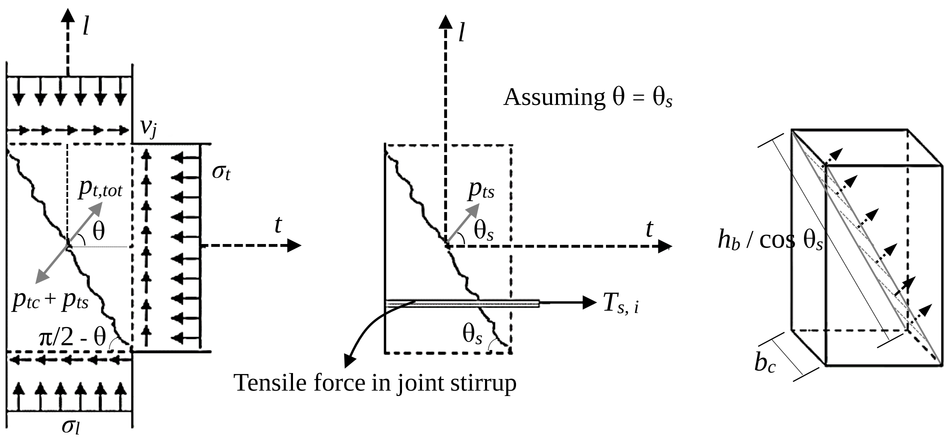

Fig. 5 Internal stresses and stirrup mechanism in the joint core

in which

$$\varepsilon_1 = \frac{\varepsilon_f - \varepsilon_v \tan^2\theta_s}{1-\tan^2\theta_s} \tag{33}$$

The total principal tensile stress contributed by concrete and transverse reinforcements can be calculated as

$$p_{t,tot} = p_{t,c} + p_{t,s}\cos(\theta-\theta_s) \tag{34}$$

According to Paulay and Priestley (1992), the angle θ can be calculated as a function of the joint core dimensions (see Fig. 4)

$$\theta = \frac{\pi}{2} - \mathrm{atan}(\frac{h_b}{h_c}) \tag{35}$$

Where h_b is the height of the beam. The concrete contribution, $p_{t,c}$, corresponding to joint shear deformation determined by Eq. (32), can be calculated from the principal tensile stress-shear deformation curve proposed for RC joints with no stirrup. Finally, the crushing of the concrete compressive strut should be checked. The principal compression stress can be computed as follows (upper limit for the principal compression stress has been proposed by Priestley (1997))

$$p_c = \sqrt{(\frac{f_v - f_h}{2})^2 + (v_{tot})^2} + \frac{f_v + f_h}{2} \le 0.5f'_c \tag{36}$$

in which

$$v_{tot} = \sqrt{p_{t,tot}{}^2 + f_v\, p_{t,tot}} \tag{37}$$

It is important to note that the principal tensile stress corresponding to first diagonal cracking in the joint core cannot be influenced by stirrup because the material property of concrete cannot be changed by stirrup. Therefore, the effects of stirrup is considered after first diagonal cracking in joint core. As a result, the principal tensile stress-shear deformation in the joint with considering the effects of stirrup can be derived using the proposed method.

4.2 Analytical calculation of joint spring properties

The characteristics of the joint's springs are dependent on principal tensile stress-shear deformation curve as that mentioned above. These curves should be transform into axial load-axial deformation to determine the properties of the diagonal axial springs corresponding to proposed model. The formulations to calculate the properties of these springs from the principal tensile stress-shear deformation curve are given below

The column shear force, V_c, can be calculated using internal equilibrium in the joint core

$$V_c = T_b - V_{jh} \tag{38}$$

where T_b and V_{jh} define the tensile force in the beam longitudinal reinforcements and the joint horizontal shear force, respectively. For an exterior beam-column joint, the beam shear force, V_b, can be expressed using the external equilibrium as follows

$$V_b = V_c \frac{\xi}{L_b} \tag{39}$$

where ξ is a function of geometric dimensions of joint core which can be calculated as

$$\xi = \frac{L_c L_b}{L_b + 0.5 h_c} \tag{40}$$

Substituting Eq. (39) into the Eq. (38), and then multiplication of the both sides of equation with ξ, we have

$$M_b = \xi T_b - \xi Vjh \tag{41}$$

in which

$$M_b = V_b L_b \tag{42}$$

Multiplication of the both sides of the Eq. (41) with $1/M_b$ and rearranging, we have

$$1 = \frac{\xi T_b - \xi V_{jh}}{M_b} \rightarrow \frac{\xi - jd}{jd} = \frac{\xi V_{jh}}{M_b} \tag{43}$$

Thus, rearranging Eq. (43), we get

$$M_b = \frac{\beta \xi}{\xi - \beta} V_{jh} \tag{44}$$

in which

$$\beta = \frac{M_b}{T_b} < h \tag{45}$$

Finally, the beam shear force corresponding to horizontal shear strength in joint core can be calculated as follows

$$V_b = \frac{\beta \xi}{(\xi - \beta) L_b} V_{jh} \tag{46}$$

As can be seen in above equation, the horizontal shear strength in joint core should be defined as input parameter. For this, the equation proposed by Sharma *et al*. (2011) which is a function of principle tensile stress in joint core can be used. Therefore, the joint shear strength can be computed as

$$V_{jh} = \frac{V_{jv}}{\alpha} \tag{47}$$

in which

$$\frac{V_{jv}}{b_c h_c} = \sigma - f_v \tag{48}$$

$$\sigma = \frac{2f_v + \alpha^2 p_t + \alpha\sqrt{\alpha^2 p_t^{\ 2} + 4p_t(f_v + p_t)}}{2} \tag{49}$$

$$\alpha = \frac{h_b}{h_c} \tag{50}$$

where σ is the vertical joint shear stress; f_v is column axial stress due to gravity load. As can be seen in above equations, the joint strength is a function of applied axial load on column, however, the effect of variation of axial load is not considered in the equation proposed by Sharma *et al*. (2011). As a result, the Sharma's equation is independent of variation of applied axial load on joint core. For considering this effect, the relation between the shear force generated in column due to nonlinear behaviour in joint core and applied axial load on column should be determined. It can be obtained substituting Eq. (39) into the Eq. (46) as

$$V_c = \frac{\beta}{(\xi - \beta)} V_{jh} \tag{51}$$

Therefore, using Eq. (51), the column shear force versus the axial load relation corresponding to each value of the principle tensile stress can be calculated as shown in Fig. 6. On the other hand, the relation between the axial load and the column shear force can be defined using Eq. (5). As can be seen in Fig. 6, the column shear force corresponding to a value of principle tensile stress with considering the effect of variation of axial load can be determined at intersection of two results and only the principal tensile stress corresponding to shear deformation in joint core should be defined. Furthermore, when column shear force is calculated with no considering axial load generated by seismic actions, the value of the calculated strength ($V_{c,g}$) can be misleading and unsafe.

It should be noted that value of internal beam moment arm, β, in Eqs. (46) and (51) is a function of beam section properties in adjacent of joint core. In order to calculate the appropriate value of internal beam moment arm, it can be determined by a simplified procedure that is provided in following. As can be seen in Eq. (44), the beam moment in adjacent of joint core is a function of value β. On the other hand, relationship between value β and beam moment can be derived performing moment-curvature analysis of beam section in adjacent of joint core. Therefore, to compute the nonlinear properties of beam- column joint, the value β can be determined with the intersection of the M_b- β curves obtained from Eq. (44) and moment-curvature analysis as shown in Fig. 7. As can be seen in figure, when the slop of the curve obtained from Eq. (44), θ_2, is lower than value M_u / β_u, θ_1, it means that beam failure occurred prior to joint failure. In this case, the joint capacity is higher than the beam capacity. Therefore, in this study, the value β has been considered equal to effective depth of the beam section. Finally, by using the above-mentioned process, the principal tensile stress can be converted into the beam and column shear force. The axial force in the diagonal springs corresponding to the proposed model can be calculated using force equilibrium in the rigid elements provided along the edges of the joint core as

$$P = \frac{V_c(L_c - h_b) - 0.5 V_b h_c}{2h_b}\sqrt{1+\alpha^2} \tag{52}$$

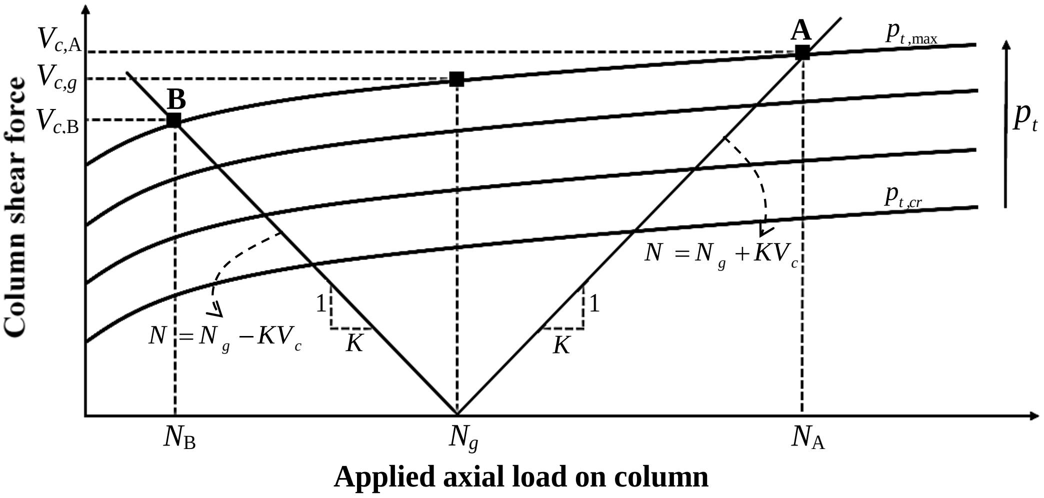

Fig. 6 Interaction between column shear force and axial load on joint core

According to the Mohr theory, the diagonal axial displacement in joint core, Δ , can be calculated by

$$\Delta = \gamma \frac{\sqrt{h_c^{\ 2} + h_b^{\ 2}}\,\sin(2\lambda)}{2} \tag{53}$$

in which

$$\lambda = \text{atan}(\frac{h_b}{h_c}) \tag{54}$$

According to Eqs. (52)-(53), the properties of the diagonal axial springs in joint core can be calculated using the principal tensile stress-shear deformation relation as input parameter.

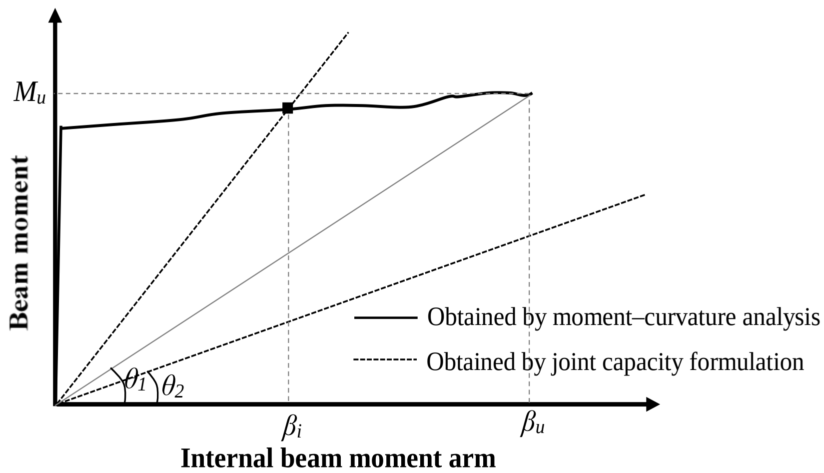

Fig. 7 Estimation of accurate value of internal beam moment arm

4.3 Verification of proposed model for exterior RC beam-column joints

In this section, to validate proposed model for simulating exterior RC beam-column joints, three joints with no stirrup in joint core, which were tested by Clyde *et al*. (2000), Pantelides *et al*. (2002), Akguzel and Pampanin (2012) and one exterior joint with stirrup in joint core which was tested Mahini and Ronagh (2011), are chosen. For each beam–column joint, two nonlinear analysis are performed with and without considering nonlinear behaviour in the core of the joint. It is noteworthy that the nonlinear analyses are performed using SAP 2000 (2008). The comparison of experimental and simulated results for joints are shows in Fig. 8. The results of nonlinear analysis prove that the joint proposed model can well simulate the nonlinear behaviour of the RC beam-column joint subjected to cyclic loading. However, in the joints without stirrup in the joint core, analysis with no joint model, simulate significantly higher ductility and strength than the one extracted from the experiment.

To assess the effect of varying axial load coefficient on the properties of axial springs in the joint core, the joint tested by Akguzel and Pampanin (2012) is selected. Fig. 9 shows load-deformation of axial springs that are computed with or without considering the coefficient of *K*. As can be seen, neglecting the actual applied axial load on column can lead to the conservative results for joint with positive value of *K* and non-conservative for joint with negative value of *K* which is unsafe.

5. Shear effect in interior beam-column joint

The process which is considered to predict the behaviour of interior joints is similar to the exterior joints. The proposed model for interior beam-column joints are shown in Fig. 10. According to the model, the behaviour of core joint is determined by two axial springs. The characteristics of these springs can be calculated by Eq. (55) to Eq. (61) as

$$P = \frac{V_c(L_c - h_b) - V_b h_c}{2h_b}\sqrt{1+\alpha^2} \tag{55}$$

in which

$$V_c = \frac{V_b(L_{b1}+L_{b2}+h_c)}{L_c} \tag{56}$$

$$V_b = \frac{V_{jh}}{\eta - \lambda} \tag{57}$$

$$\lambda = \frac{(L_{b1}+L_{b2}+h_c)}{L_c} \tag{58}$$

$$\eta = \frac{L_{b1}\beta_2 + L_{b2}\beta_1}{\beta_1\beta_2} \tag{59}$$

$$\beta_1 = \frac{M_{b,L}}{T_{b,L}} \tag{60}$$

$$\beta_2 = \frac{M_{b,R}}{T_{b,R}} \tag{61}$$

where β_1 and β_2 depend on the slope of the moment- tensile force curve which can be calculated so that was explained for the exterior joint; L_{b1} and L_{b2} are the distance of critical section to point of contra-flexure for the left and right beam, respectively; $M_{b,L}$ and $M_{b,R}$ define moment in left and right beam at the face of column. Based on studies conducted by Hakuto *et al*. (2000), Dhakala *et al*. (2005), Pantelides *et al*. (2008) and Sharma (2013), the principal tensile stress-shear deformation curve for interior joint is proposed as shown in Fig. 11. Therefore, using proposed principal tensile stress-shear deformation relation, the characteristics of axial springs can be calculated. It should be noted that for the interior joints of a regular structure can be assumed that shear force generated by seismic actions at the opposite sides of the interior joint will neutralize each other. As a consequence, there is no axial force contribution in these joints due to seismic action.

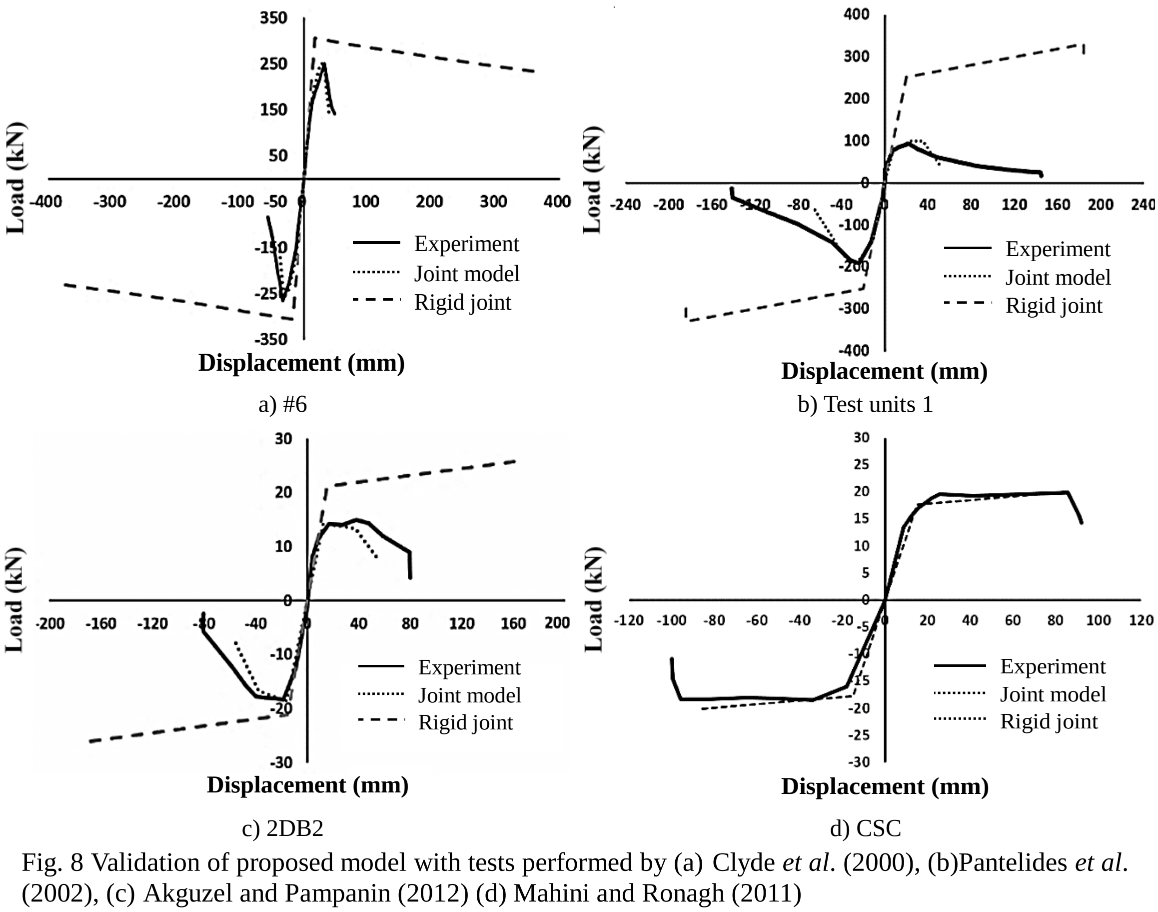

Fig. 8 Validation of proposed model with tests performed by (a) Clyde *et al*. (2000), (b)Pantelides *et al*. (2002), (c) Akguzel and Pampanin (2012) (d) Mahini and Ronagh (2011)

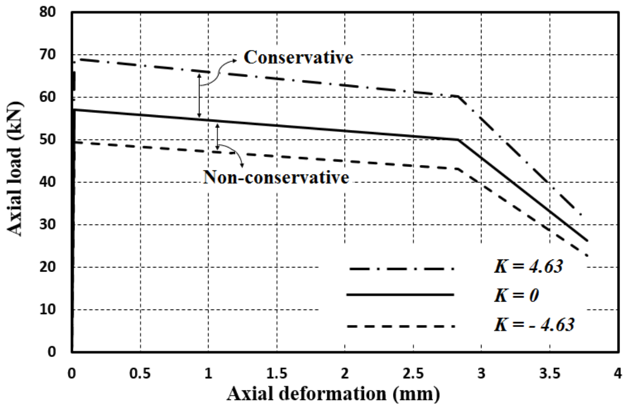

Fig. 9 Properties of axial springs with or without considering the coefficient of *K*

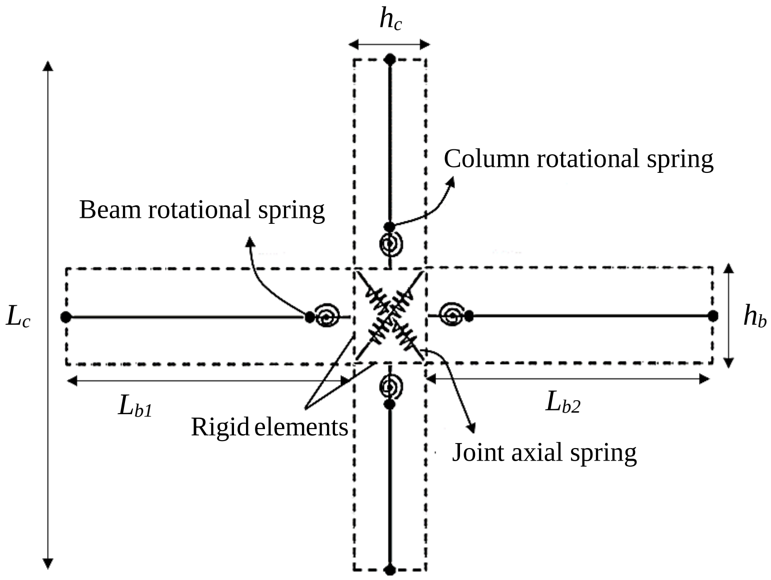

Fig. 10 The proposed model for interior joint simulation

However, it is clear that in cases of irregular or tall structure, the effect of variation of axial load should be considered. For this purpose, the provided procedure for exterior joints in current study can be used.

5.1 Verification of proposed model for interior RC beam-column joints

In this section, to validate proposed model for simulating interior RC beam-column joints, two joints, which were tested by Dhakala *et al*. (2005) are chosen. For each beam–column joint, two nonlinear analysis are performed with and without considering nonlinear behaviour in the core of the joint. It is noteworthy that the nonlinear analyses are performed using SAP 2000 (2008). The comparison between the experimental results and numerical modelling results are shown in Fig. 12. Moreover, for visualizing the significance of modelling of joint core in interior joints, the

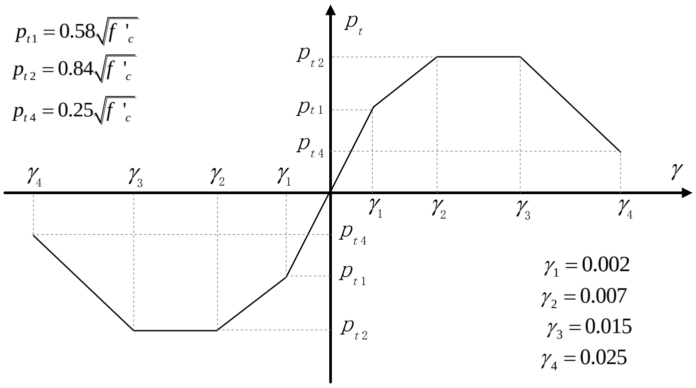

Fig. 11 Proposed principal tensile stress-shear deformation curve for interior joints

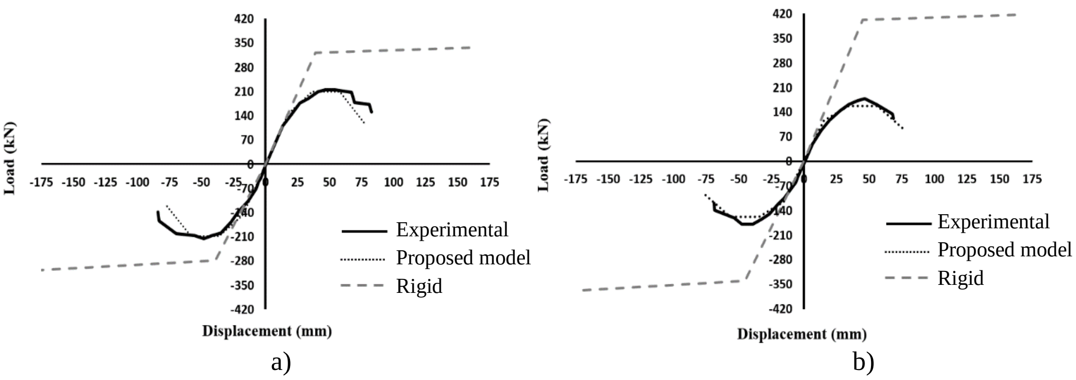

Fig. 12 Validation of model with tests performed by Dhakala *et al.* (2005) for specimens a) C1 b) C4

results of the numerical analysis when the model have not any spring in joint core is also shown in Fig. 12. As can be seen, the good agreement confirms the impact of simulating the joint core using the proposed model. Also, the misleading results can be obtained when the joint core is considered to be rigid.

6. Verification of the proposed model at structure level

In order to assess the performance of the proposed model at structural level, two frames, which were tested by Calvi *et al*. (2002b) and Duong *et al*. (2007), are chosen and the results predicted by the proposed model are compared with the one captured from the experiments. The failure mode of the frame tested by Duong *et al*. (2007), is reported as shear failure in the beams. In this case, the nonlinear analysis is performed in various cases in order to assess the importance of considering the effects of the shear in RC members. In the second frame tested by Calvi *et al.* (2002b), the failure mode is shear failure in the core of the exterior joints. Thus, in this case the ability of the proposed model to predict the behaviour of the joint and also the effect of the joint modeling in the nonlinear analysis are investigated. It is noteworthy that nonlinear analyses are performed using SAP 2000 (2008).

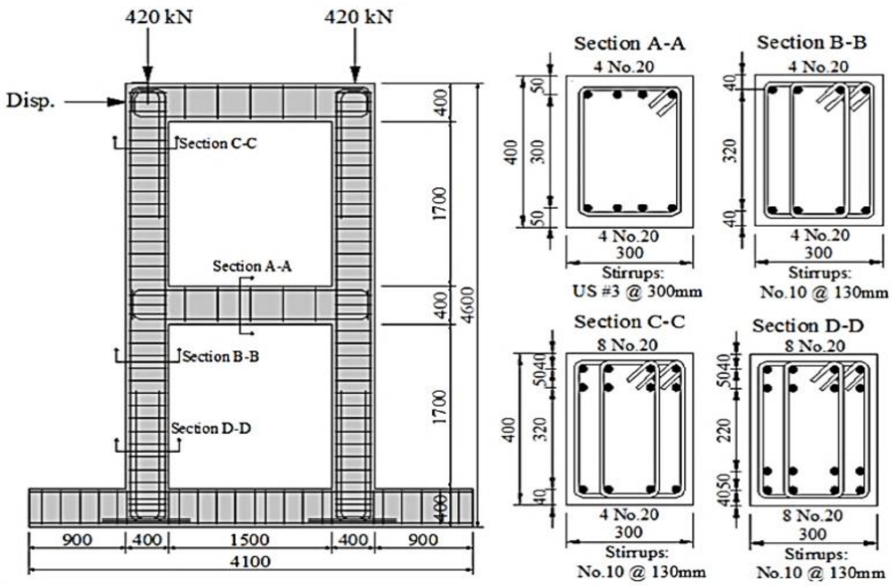

Fig. 13 Geometry and reinforcement details of frame tested by Duong *et al*. (2007)

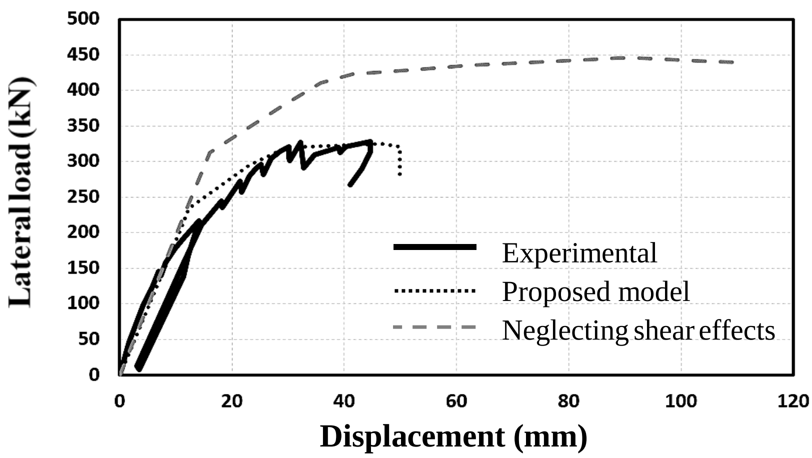

Fig. 14 Comparison of experimental (Duong *et al*. (2007)) and numerical results

6.1 Test by Duong et al. (2007)

The dimensions and reinforcement details of the test frame are shown in Fig. 13 as reproduced in Duong *et al*. (2007). As can be observed, the transverse reinforcement in the joint core was reported as Ø10 at 100mm and beam longitudinal reinforcements were anchored with 90°-hooks bent into the core. The test program was involved, a lateral displacement that applied to the second storey beam, while an axial load of 420 KN was applied to each column and held constant throughout the testing procedure. The loading sequence produced significant shear damage in the two beams of the frame and only slight increase in crack widths in the joint core. In order to accurately investigate the proposed model, the plastic hinges of the beams and columns are defined

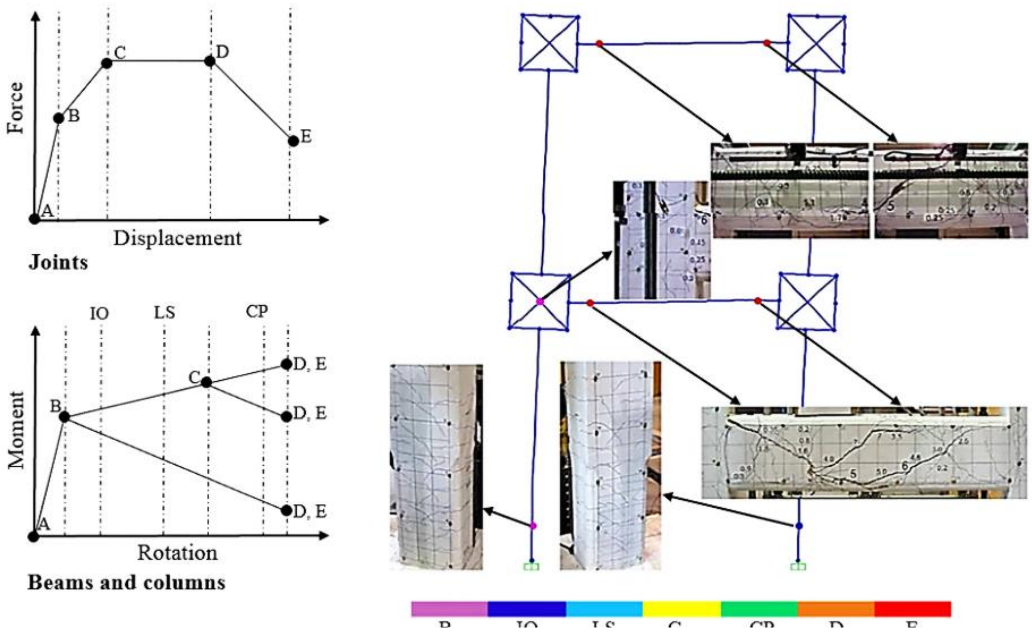

Fig. 15 Comparison of experimental (Duong *et al*. (2007)) and numerical failure patterns

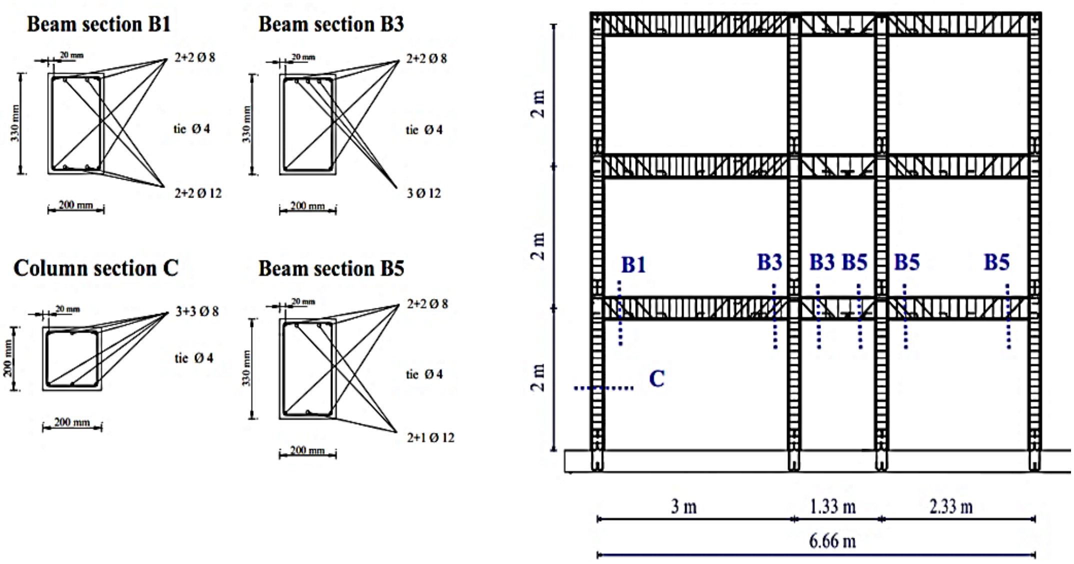

Fig. 16 Geometry and reinforcement details of frame tested by Calvi *et al*. (2002b)

(i) without considering the shear effects with the assumption that the shear failure of members are neglected in the analysis (ii) with considering the shear effects using the proposed procedure in the current study. The comparison of experimental and numerical load-displacement curves are shown in Fig. 14. Good agreement can be seen between the load-displacement curves extracted from the numerical analysis and experimental results when the shear effects are considered.

As can be observed from Fig. 14, the analysis with no considering the shear effects, leads to

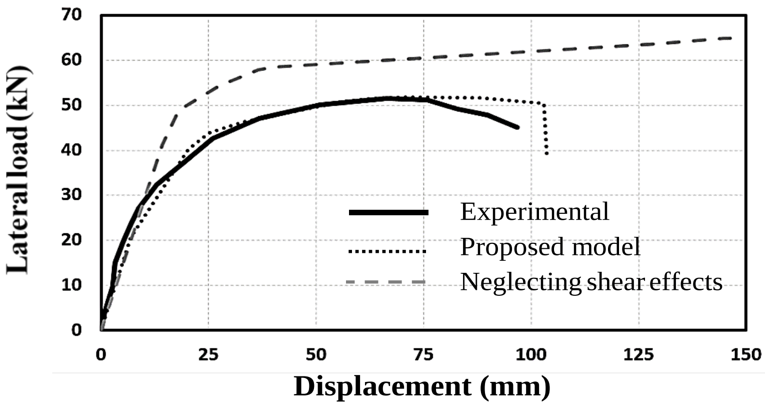

Fig. 17 Comparison of experimental (Calvi *et al*. (2002b)) and numerical results

non-conservative prediction of response of the frame. The failure modes observed in the test namely the shear hinges at the beams and the joint cracking can be successfully simulated using the proposed numerical model (see Fig. 15). Also, these results confirm the accuracy of the definition of beam plastic hinges which were calculated based on the proposed procedure in section 3.2. It should be noted that the performance level of immediate occupancy (IO), life safety (LS) and collapse prevention (CP) were determined for the beam and columns according to values of recommended in FEMA-356 (2000).

6.2 Test by Calvi et al. (2002b)

The frame geometry and the reinforcement details are shown in Fig. 16 as reproduced in Calvi *et al*. (2002b). As can be seen, the joint has no stirrup and the beam longitudinal reinforcement was anchored with end hooks in the joint core. The average compressive strength of concrete was measured as 14.06MPa. The frame system was subjected to quasi-static cyclic loading at increasing levels of top displacement. The loading history consisted of a series of three cycles at increasing level of top drift (± 0.2%; ± 0.6%; ± 1.2%) with one conclusive cycle at ± 1.6%. At each loading step, the load at roof story, second story and first story was kept fixed at 1, 0.9 and 0.45, respectively. The yield and ultimate strength of 8mm diameter reinforcement bars was measured as 385.64MPa and 451.22MPa, respectively, while the same for 12mm diameter reinforcement bars was measured as 345.87MPa and 458.63MPa, respectively. In order to accurately investigate the proposed model, two types of nonlinear analysis were performed (i) without considering the shear effects in the core of the joints with the popular assumption that the core of joints can be considered rigid (ii) with considering the shear effects in the core of the joints using the proposed procedure in the current study. The lateral load versus displacement curves extracted from the nonlinear analyses of the current study are compared to the results reported from the experiment in Fig. 17. As seen in the figure, by simulating the shear effects in the core of joints, the nonlinear analysis estimates the lateral load versus displacement relation, strength and displacement capacity with reasonable agreement. However, neglecting the effects of the shear failure in the joint, the results of the nonlinear analysis in both terms of strength and ductility are severely more than the actual values. The failure mode reported from the experiment is compared with the failure mode predicted by nonlinear analysis based the proposed model in Fig. 18. The

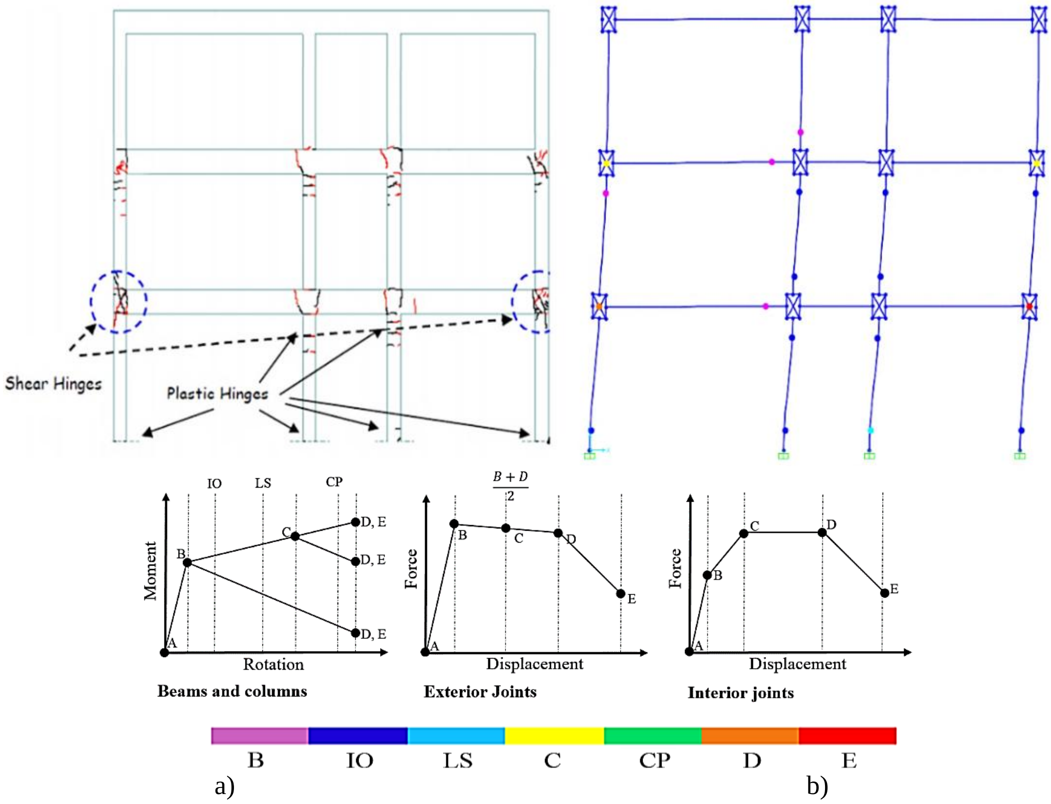

Fig. 18 Failure global patterns of the frame: (a) experimental (Calvi *et al*. (2002b)); (b) numerical

typical failures observed from the experiment including the shear failure in the core of the exterior joint as well as flexural plastic hinges in other members can be successfully replicated in the nonlinear analysis. Therefore, the proposed numerical model can correctly predict not only the lateral load versus displacement relation but also the failure patterns for the RC structures.

7. Conclusions

The aim of the current research is providing a realistic, accuracy and practical model to perform the nonlinear analyses of RC structure. The conclusions derived are given below.

• In order to consider the shear behaviour of beams and columns, the effects of shear have been applied to the moment-rotation curve of the member. Therefore, the shear and flexural behaviour of members were considered in nonlinear analysis by flexural hinges. According to the proposed model, different failure modes including flexural, shear-flexural and shear failure can be predicted with comparison between the flexural and the shear moment.

• A new model, which consists of two diagonal axial springs was proposed to simulate the nonlinear behaviour of the core joint. In order to determine the properties of axial springs for non-ductile joints, the principal tensile stress-shear deformation curves have been proposed based on experimental and finite element results. Not only the model can accurately and realistically predict the response of RC joints in terms strength and ductility but also it is compatible with the commercial softwares such as SAP2000 (2008).

• The proposed numerical model can correctly estimate not only the load-displacement curves but also the expected failure patterns for the RC frames with various failure modes.

• At structure level, considering the effect of variation of applied axial load on column is required. This effect was considered in the calculation of properties of columns as well as joints.

• According to this study, neglecting the effects of shear stresses in RC members might result into non-conservative results.

• With no considering the shear effects in the core of RC joints might lead to higher capacity, which is dangerous and misleading.

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