

Behaviour of RC Beams with non-bonded flexural reinforcement: A numerical experiment

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Abstract. The present work is concerned with a numerical investigation of the behaviour of reinforced-concrete beams with non-bonded flexural tension reinforcement. The numerically-established behaviour of such beams with and without transverse reinforcement is compared with its counterpart of similar beams with bonded reinforcement. From the comparison, it is found that the development of bond anywhere within the shear span inevitably leads to inclined cracking which is the cause of ‘shear’ failure. On the other hand, the lack of bond within the shear span of the beams is found, not only to prevent cracking within the shear span, but, also, to lead to a flexural type of failure preceded by the formation of horizontal splitting of concrete in the compressive zone. It is also found that delaying the extension of horizontal splitting through the provision of transverse reinforcement in the beam mid span can lead to flexural failure after yielding of the tension reinforcement. Yielding of the tension reinforcement before the horizontal splitting of the compressive zone may also be achieved by reducing the amount of the latter reinforcement.

Keywords: beams; concrete-steel interaction; finite-element analysis; numerical testing; reinforced concrete; non-bonded reinforcement

1. Introduction

The implementation of beam theory in Reinforced Concrete (RC) design relies on the assumption of perfect bond between concrete and steel. In fact, it is widely considered that safeguarding adequate bond between concrete and steel leads to efficient design solutions. However, the bond stresses developing for deformation compatibility purposes at the interface between concrete and flexural reinforcement causes inclined (‘shear’) cracking in reinforced concrete (RC) beam-column elements (Kong and Evans 1987). Since shear cracking results in brittle modes of failure, a large amount of research work carried out to date on RC design has been concerned with addressing the ‘shear’ problem. Solutions to this problem are invariably obtained through the development of (i) criteria for predicting inclined crack extension leading to loss of load-carrying capacity and (ii) methods for assessing the amount and arrangement of transverse

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reinforcement capable of delaying such inclined crack extension until the code (e.g., American Concrete Institute 2011, Eurocode 2 2004, Eurocode 8 2004) performance requirements for load-carrying capacity and ductility are satisfied.

It could be argued, however, that an alternative approach to safeguarding against shear failure is to prevent the formation, rather than the extension, of inclined cracking. Since the causes of such cracking are inextricably linked with the interaction between concrete and the longitudinal steel bars through bond, this could be achieved by preventing the development of bond between concrete and steel. Although the behaviour of RC structural elements without bond between concrete and flexural reinforcement has been investigated since the late-50s (Watstein and Mathey 1958, Lorentsen 1965, Zielinski and Abdulezer 1977, Cairns and Zhao 1993, Cairns 1995, Woo Kim and White 1999), only recently there has been published work discussing the applicability of the above reasoning in practical RC seismic design (Pandey and Mutsuyoshi 2004, Iemura *et al.* 2004, Kotsovou and Mouzakis 2011, Kotsovou and Mouzakis 2012a, Kotsovou and Mouzakis 2012b, Kotsovos *et al.* 2013). However, such work has been rather limited, and it has neither included an investigation of the causes of the observed behaviour, nor has it led to results revealing advantages sufficiently significant to divert the construction industry from adopting reinforcement technologies which are linked with the development of bond between concrete and steel.

The work presented herein forms part of an attempt to explore the possibility of using non-bonded reinforcement in specific cases for which there has already been evidence of improvement of the performance of structures under seismic excitation (Pandey and Mutsuyoshi 2004, Iemura *et al.* 2004, Kotsovou and Mouzakis 2012a, Kotsovou and Mouzakis 2012b, Kotsovos *et al.* 2013). The effect of the use of non-bonded reinforcement on the behaviour of RC beams under transverse loading is investigated by means of numerical experiments carried out through the use of a Finite Element (FE) model which has been reported to produce realistic predictions of structural-concrete behaviour under both static (monotonic and cyclic) and dynamic (seismic and impact) loading conditions in all cases investigated to date (ADINA 2012, Kotsovos 2015). The effect of the use of non-bonded reinforcement on the behaviour of RC beams under transverse loading is investigated by means of numerical experiments; the beam geometry and reinforcement details are those of beam OA1, tested by Bresler and Scordelis (1963), the experimentally-established behaviour of which is also compared with its numerically-predicted counterpart in order to provide additional evidence of the validity of the adopted numerical tool. The effect on structural behaviour of the amount of flexural reinforcement, concrete strength and the presence of an axial force is also investigated as such information may be used for developing a method for designing RC structural elements with non-bonded flexural reinforcement.

2. Numerical experiments

2.1 Background

For an RC beam with bonded reinforcement and a shear span (a_v)-to-depth (d) ratio smaller than about 2.5, shear failure, in the absence of transverse reinforcement, is linked to the appearance of a large inclined crack which forms independently of flexural cracking and, eventually, extends into the compressive zone causing failure (Kong and Evans 1987). This type of failure is independent of the concrete-flexural steel interaction and, therefore, the behaviour of beam elements with $a_v/d \leq 2.5$ are not part of the investigation reported herein.

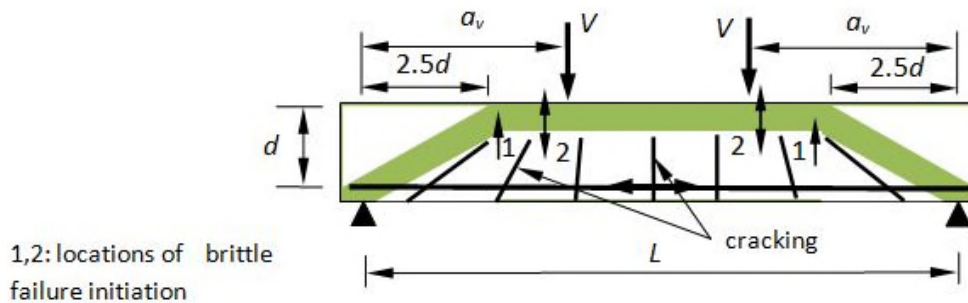


Fig. 1 CFP model for a simply-supported RC beam with longitudinal steel bonded to concrete under two-point loading

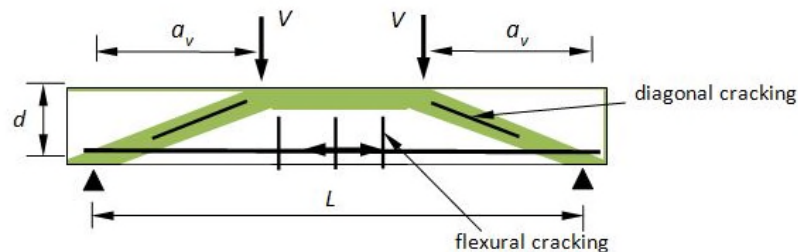


Fig. 2 Proposed model for a simply-supported RC beam with longitudinal steel not bonded to concrete under two-point loading

On the other hand, for beams with bonded reinforcement and $a_v/d > 2.5$, shear failure is linked to concrete-flexural steel interaction. The deep inclined crack that forms prior to loss of load-carrying capacity is essentially an extension of the flexural crack that forms closest to the support (Kong and Evans 1987). The formation of this inclined crack dictates the path along which the compressive force developing by bending is transmitted to the supports. According to the compressive force path (CFP) theory (Kotsovos and Pavlovic 1999, Kotsovos 2014) (which is based on Kani's seminal work (Kani 1964) and developed through extensive use of test results obtained by Leonhardt and Walther (1964)) the causes of failure are associated with the development of tensile stresses developing across the compressive force path as shown in Fig. 1 (Kotsovos and Pavlovic 1999, Kotsovos 2014). The figure also indicates that the path essentially comprises horizontal and inclined portions connected at a distance of $2.5d$ from the nearest simple support. There is, therefore, an indirect link between the causes of shear failure and concrete-flexural steel interaction, since the latter, through promoting the formation of the inclined crack, affects the shape of the path of the compressive force developing on account of bending.

It is proposed herein that, in the absence of bond between concrete and flexural steel, the inclined crack referred to in the preceding paragraph will not occur, and, hence, the path of the compressive force will be completely different. In this case, the compressive force is transferred to the support along the inclined line essentially connecting the load point closer to the support with the support (see Fig. 2). Then, the beam element is likely to suffer two types of cracking: flexural cracking and cracking in the direction of the inclined compressive force. The occurrence of the

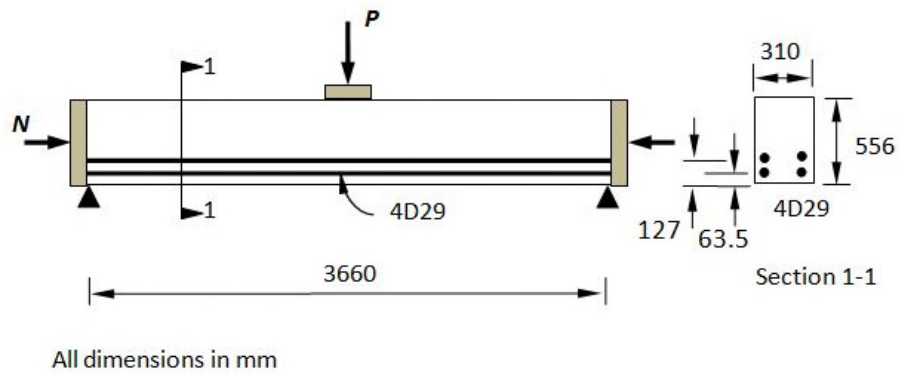


Fig. 3 Geometric characteristics and longitudinal reinforcement arrangement of beams tested

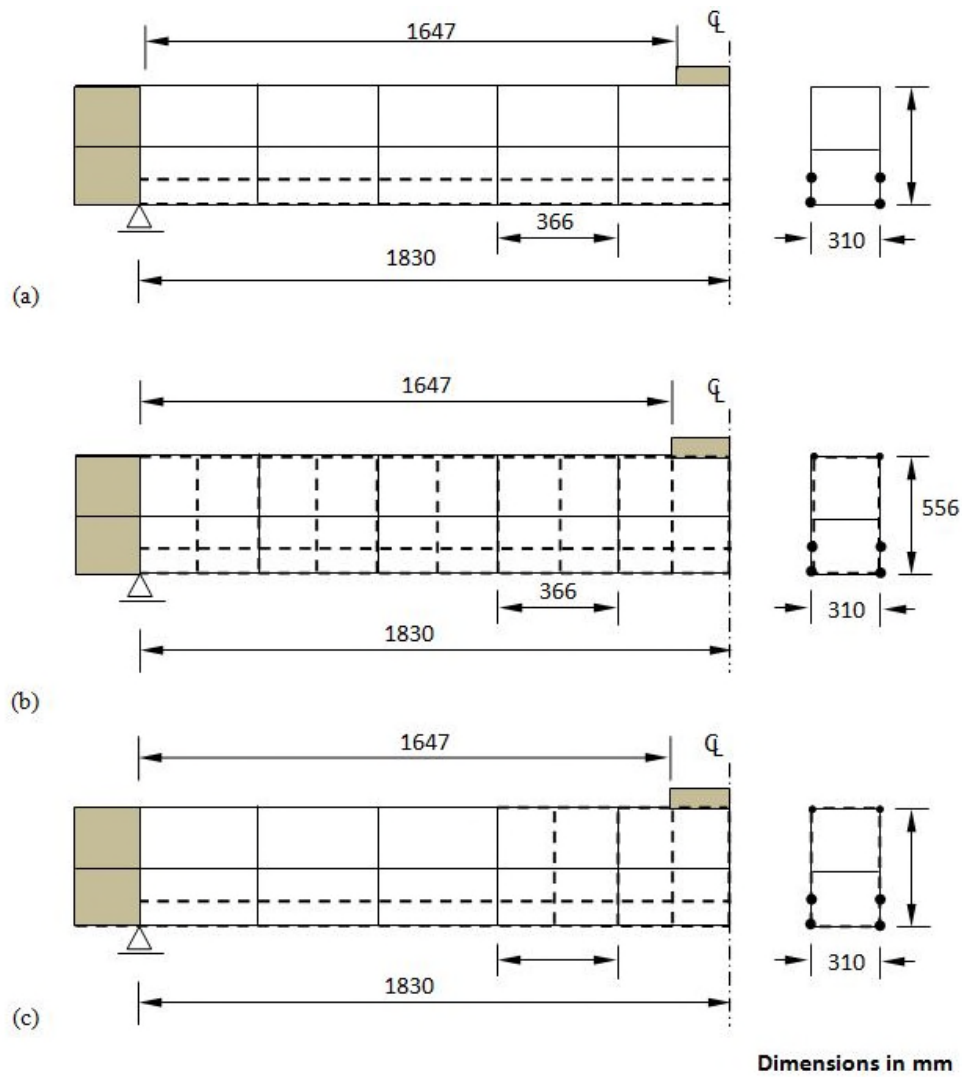


Fig. 4 Finite element characterization of beams: (a) B and NB; (b) BS; and (c) NBS

latter cracking, however, may be prevented through the provision of transverse reinforcement as proposed elsewhere (Kotsovou and Mouzakis 2012a, Kotsovou and Mouzakis 2012b) where it is demonstrated that such reinforcement is significantly less than the code specified amount of shear reinforcement for beams with bonded steel reinforcement.

2.2 RC beams investigated

On the basis of the reasoning presented in the preceding section, the RC beams selected for the numerical investigation have an $a_v/d > 2.5$. Their geometry, longitudinal reinforcement and material characteristics are those of beam OA1 tested by Bresler and Scordelis (1963) (see Fig. 3). From the figure, it can be seen that they have a 310 mm wide x 556 mm high rectangular cross section and a 3660 mm span. The tension reinforcement comprises four 29 mm diameter bars arranged in two layers at distances of 127 mm and 63.5 mm from the tensile face of the beams; thus, the beam's effective depth is equal to 461 mm. The uniaxial cylinder compressive strength of concrete is 22.5 MPa whereas the yield stress and strength of the steel bars are 555 and 660 MPa, respectively. Since steel end plates have been used to anchor the non-bonded reinforcement (elements shaded grey in Fig. 3), these plates have also been used for the beams with the bonded reinforcement. The point load has been distributed over a finite area of the upper face of the beam through the use of a steel plate, the dimensions of which are shown in Fig. 4.

Load is applied at mid span (thus, $a_v/d = 3.97$) and increases monotonically to failure. The beams without transverse reinforcement are designated as B and NB for the cases of bonded and non-bonded reinforcement, respectively, with beam B being essentially beam OA1 tested by Bresler and Scordelis (1963). Beams B and NB strengthened with transverse reinforcement so as to reach their load-carrying capacity after yielding of the tension flexural reinforcement are designated as BS and NBS; the transverse reinforcement (8 mm diameter two-legged stirrups at 100 mm spacing) is designed in accordance with current code provisions for beams with bonded reinforcement (Eurocode 2 2004) and, for such beams, it is required throughout the shear span. For the beams with non-bonded flexural reinforcement, it is placed only within the middle portion of the beam as discussed later by reference to Fig. 4. The reinforcement cage is complemented with 8 mm diameter compression longitudinal bars as indicated in Fig. 4. The yield stress and strength of the 8 mm diameter bars are assumed to be similar to those of tension longitudinal bars.

2.3 FE Model

As discussed in Section 1, the behaviour of the structural forms investigated is established from the results obtained from numerical experiments carried out through the use of a FE package which has been found to produce realistic predictions of structural-concrete behaviour in all cases investigated to date. Full details of the package may be found in numerous publications, which formed the subject of two textbooks (Kotsovos and Pavlovic 1995, Kotsovos 2015), and in the manual of the well-known package ADINA (2012) where the model has been recently incorporated.

The package is capable of performing not only static, but also dynamic analysis, the latter being effected through the unconditionally stable average acceleration method of the implicit Newmark integration scheme. Moreover, it uses three-dimensional (3D) non-linear (NL) analysis in order to allow for (a) the NL behaviour of concrete under triaxial stress conditions, which invariably develop prior to local failure (i.e. cracking), and (b) the introduction of non-homogeneity and

stress redistribution after the occurrence of cracking. Concrete is modelled by using 27-node brick Lagrangian elements, whereas 3-node isoparametric line elements of appropriate cross-sectional area possessing axial stiffness only are used to model the steel reinforcement.

The nonlinear analysis is based on the iterative procedure known as the modified Newton-Raphson which is used to calculate stresses, strains and residual forces. Every Gauss point is checked, at first, in order to determine whether loading or unloading takes place, and then in order to establish whether any cracks close or form. The development of a crack is followed by immediate loss of load-carrying capacity in the direction normal to the plane of the crack. At the same time, shear stiffness is also reduced drastically to 10% of its value before the occurrence of the crack. Three cracks can form at each integration point, with the first crack modifying the state of stress from triaxial to biaxial, the second, from biaxial to uniaxial, and the third leading to local loss of load-carrying capacity.

Depending on the results of the previous checks, changes are introduced to the stress-strain matrices of the individual FE's and, consequently, to the stiffness matrix of the structure. Based on these modified matrices, deformation, strain and stress corrections are evaluated. Convergence is accomplished once the above corrections become very small. It should be pointed out that the formation and closure of cracking is checked separately during each load step.

The implementation of the above procedure is based on the use of suitable analytical models describing the behaviour of concrete, steel and their interaction under short-term loading conditions. Although the models adopted for this purpose are fully described in Kotsovos and Pavlovic (1995), it is important to highlight some of their main features. The material model of concrete behaviour is characterised by both simplicity (fully brittle, with neither strain-rate nor load-path dependency, fully defined by a single material parameter - the uniaxial cylinder compressive strength f_c) and attention to the actual physical behaviour of concrete in a structure (unavoidable triaxiality which is described on the basis of experimental data of concrete cylinders under definable boundary conditions).

The material model used to describe the deformational behaviour of the steel reinforcement in either tension or compression follows current code recommendations (Eurocode 2 2004), so that the stress-strain curve is fully defined by using the values of the yield stress and the ultimate strength together with the values of the corresponding strains, with the yield strain taken as the ratio of the yield stress to the elastic modulus of elasticity. For the case of bonded reinforcement, the assumption of perfect bond is considered to provide an adequate description of the interaction between steel and concrete. This is compatible with the smeared-crack approach adopted for the analytical description of the cracking processes of concrete, as well as the fact that the tensile strength of concrete is smaller than that of the experimentally-established strength of the bond between the two materials.

The beams are subdivided into $10 \times 1 \times 2$ 27-node Lagrangian brick concrete elements with a $3 \times 3 \times 3$ integration rule as indicated in Fig. 4, which also shows that, as for the case of physical testing, in order to safeguard against bearing failure, the load is applied through steel plates (exhibiting linear-elastic behaviour) represented by the shaded 27-node Lagrangian brick elements monolithically connected to the upper and end faces of the beams. The longitudinal reinforcement is represented by two line elements on either side face of the beams indicated in the figure by the dotted lines and dots at the longitudinal and transverse, respectively, sections of the beams. The tension reinforcement arrangement is equivalent to that of the beam in Fig. 3 in terms of both bar location and cross-sectional characteristics. For the bonded reinforcement, the line elements are aligned between successive brick element nodes, in either longitudinal or transverse directions,

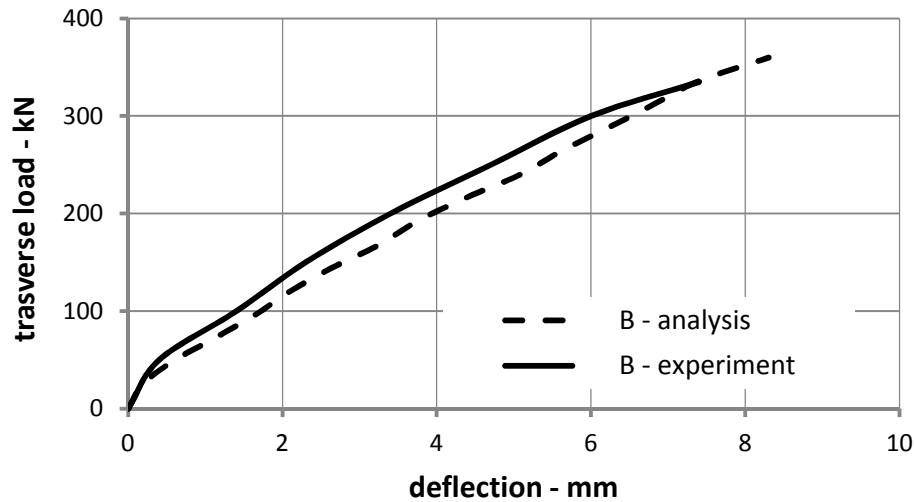


Fig. 5 Numerically-predicted and experimentally-established load-deflection curves of beam B

with the nodes common to both concrete and steel indicating the development of bond between the two materials. For the non-bonded reinforcement, the lack of bond is effected by placing single line elements along the beam span bonded to concrete at the cross sections through the supports and at mid span where the bottom nodes are common to the brick and line elements. The presence of the mid span node is essential in order to induce a common deflection to concrete and steel at mid span.

3. Results

The main results obtained from the analyses are shown in Table 1 and Figs. 5 to 11. Fig. 5 and 6 show the load-deflection curves of the beams investigated; the numerically-predicted load-deflection curve of beam B is compared with its experimentally-established counterpart in Fig. 5, whereas Fig. 6 depicts the load-deflection curves of all beams numerically investigated. Figs. 7 and 8 depict typical numerically-established crack patterns of the beams at various stages of the applied loading, while the effects of concrete strength (f_c) and percentage of tension reinforcement (ρ) of beams NBS is indicated in Figs 9 and 10, respectively. Finally, an indication of the effect of axial force on the load-deflection curves of beams NBS is provided in Fig. 11.

4. Discussion of results

Fig. 5 shows that the numerically-predicted curve for beam B correlates very closely with its experimentally-established counterpart. In fact, the maximum load sustained by the beam is numerically-predicted to be 360 kN and deviates from its experimentally-established counterpart of 335 kN by about 7%. (It is reminded that beam B is essentially beam OA1 tested by Bresler and

Table 1 Beam load-carrying capacities

Specimen	MSL- kN	NB/B	NBS/BS
B	360	-	-
NB	432	1.2	-
BS	528	-	-
NBS	552	-	1.05

MSL: Maximum sustained load

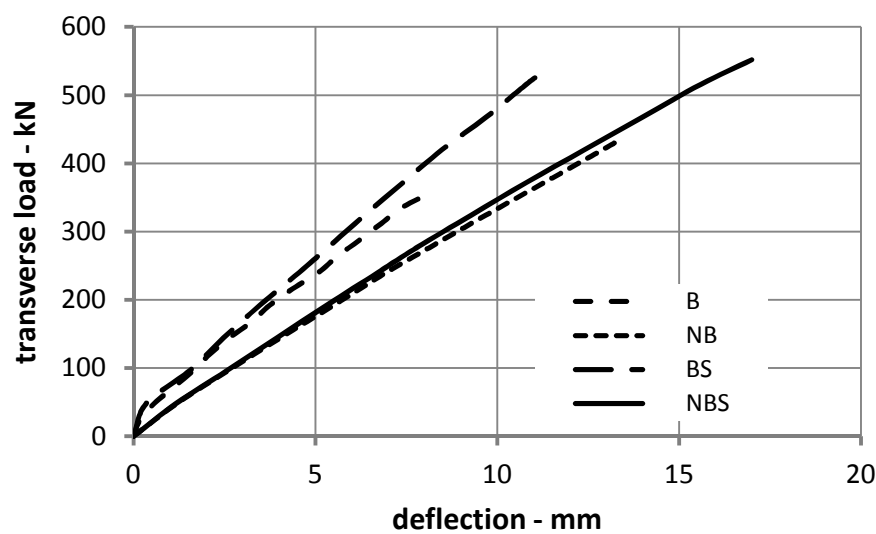


Fig. 6 Numerically-predicted load-deflection curves of beams investigated

Scordelis (1963)). Such a close correlation demonstrates yet again the validity of the FE model adopted for conducting the numerical experiments.

From Table 1, it can be seen that the beams with non-bonded flexural reinforcement (beams NB and NBS) exhibit a higher load-carrying capacity than their counterparts with bonded reinforcement (B and BS) for both types of beams, i.e. those with (beams BS and NBS) and those without (beams B and NB) transverse reinforcement. Such behaviour is similar to that established experimentally elsewhere (Cairns 1995, Woo Kim and White 1999). Moreover, for the beams with transverse reinforcement (beams BS and NBS), the transverse reinforcement of beam NBS is significantly less than that of beam BS, since, as indicated in Fig. 4, the transverse reinforcement of the former beam is confined within the region of the mid span rather than spread throughout the beam span. On the other hand, from Fig. 6, it can be seen that the stiffness of beams with non-bonded flexural reinforcement (beams NB and NBS) is significantly smaller than that of their counterparts with bonded reinforcement (beams B and BS), with the presence of transverse reinforcement leading to a small increase in stiffness for all beams. Although, regarding the serviceability limit state, the behaviour (e.g. stiffness) of beams with non-bonded reinforcement may be considered deficient, the situation may be remedied through an increase of the percentage of the flexural reinforcement.

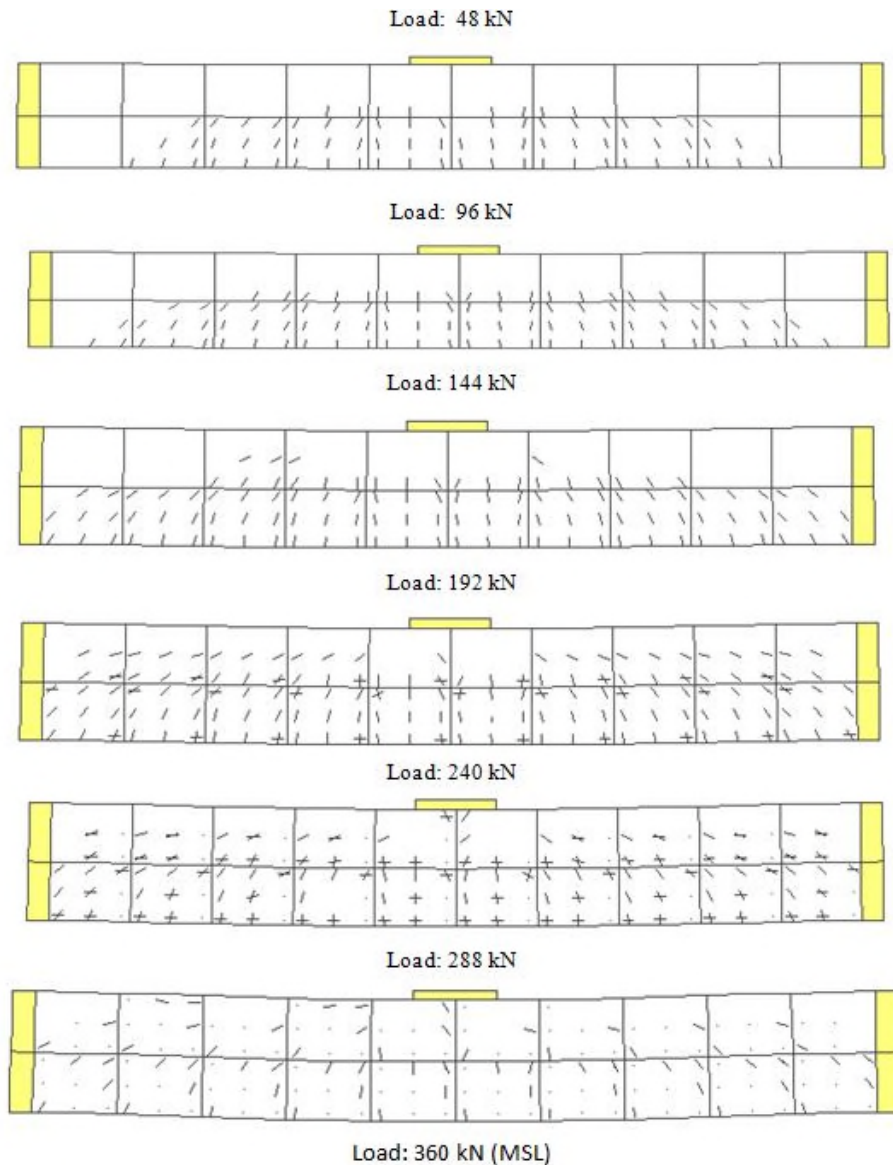


Fig. 7 Crack patterns of beam B at various load levels (short lines and dots indicate cracking intersecting the beam side face at an angle larger and smaller, respectively, than 45°)

The crack patterns for the beams without transverse reinforcement (beams B and NB) under various load levels are qualitatively similar to those of the beams with transverse reinforcement (beams BS and NBS). Therefore, the differences in the crack patterns of the beams B and NB shown in Figs. 7 and 8, respectively, are also typical for beams BS and NBS. From Fig. 7, it can be seen that, for the beams with bonded flexural reinforcement (beams B) cracking initiates at the tensile face of the beam mid span and, progressively, with increasing load, spreads towards the supports, with the individual cracks initiating at the location of the longitudinal bars and extending towards the upper face in the direction of the load point eventually leading to a brittle (shear) type

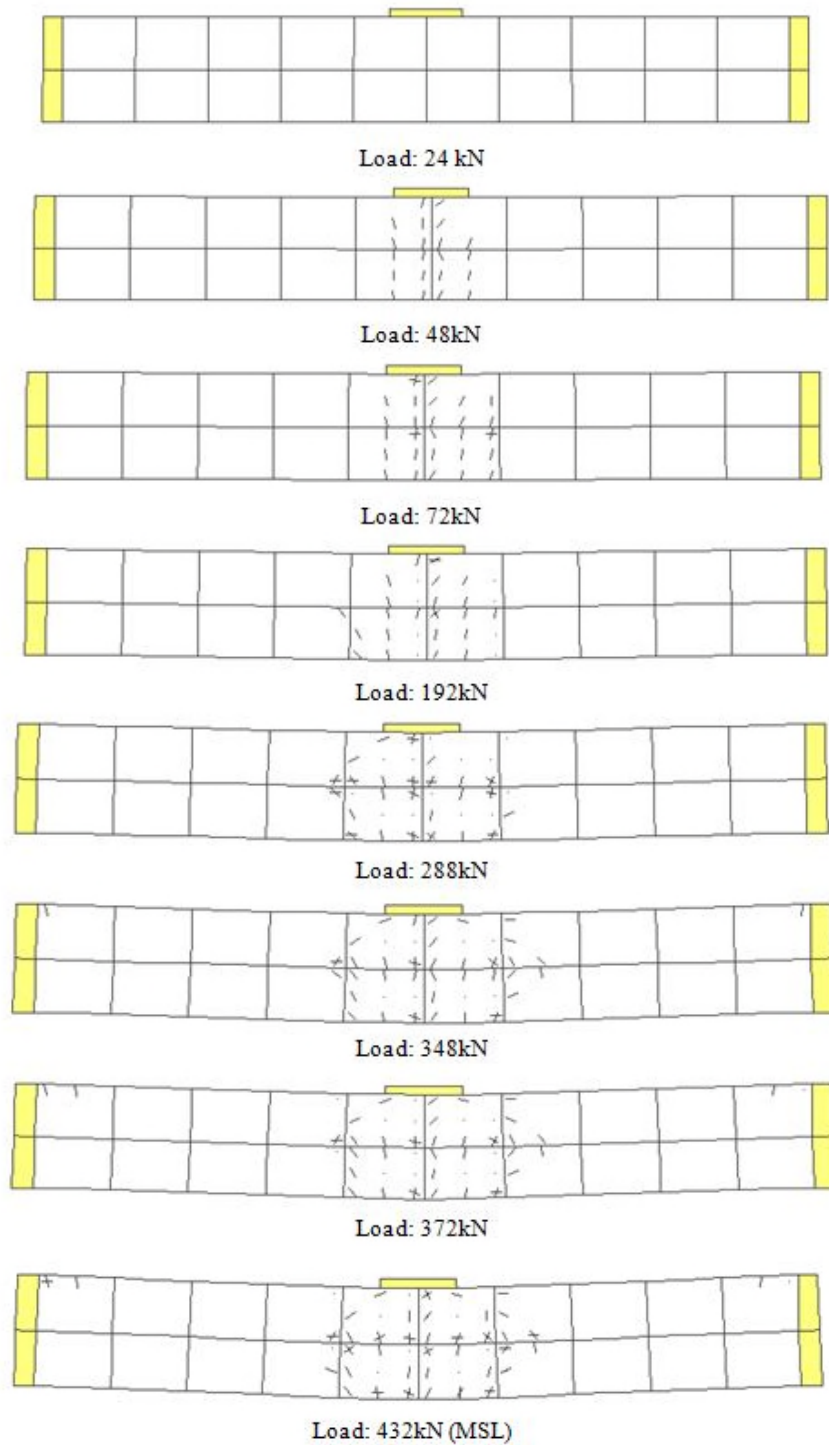


Fig. 8 Crack patterns of beam NB at various load levels (short lines and dots indicate cracking intersecting the beam side face at an angle larger and smaller, respectively, than 45°)

of failure. The provision of stirrups merely delays this cracking process and, if the stirrups are designed in compliance with current code provisions (Eurocode 2 2004) (beam BS), loss of flexural capacity precedes shear failure.

On the other hand, for the beams with non-bonded flexural reinforcement, Fig. 8 shows that cracking also initiates at the tensile face of the beam in the region of the mid span; it is initially near vertical and extends rapidly towards the upper face of the beam. Thereafter, both ends of vertical cracking extend near-diagonally, but only to a small distance from the beam mid span with increasing load; loss of load-carrying capacity eventually occurs due to the formation of near horizontal cracking on either side of the loading plate. It is interesting to note that the provision of transverse reinforcement (such as that indicated in Fig. 4(c)) only in the region of the beam mid span is sufficient for delaying the extension of near-horizontal cracking until yielding of the tension reinforcement occurs first. It is also interesting to note that, in contrast with the beams with bonded flexural reinforcement, the absence of concrete-steel interaction due to the use of non-bonded flexural reinforcement prevented the formation of inclined cracking within the beam shear spans.

Figs. 7 and 8 also show that, as the load increases, more than one crack form at particular locations: two intersecting dashes represent the occurrence of two cracks at the same location, whereas a dot indicates local loss of load-carrying capacity due to the occurrence of a third crack at the same location.

The effect of the amount of tension flexural reinforcement on the behaviour of beams NBS (i.e. the beams with non-bonded flexural reinforcement strengthened as shown in Fig. 4) is indicated in Fig. 9 which shows the load-deflection curves of the beams for the cases of flexural reinforcement ratio $\rho = 1.1\%$, 1.5% and 1.8% . The figure shows that, although all beams are characterised by a flexural mode of failure, unlike stiffness, which increases with the reinforcement ratio, ductility appears to increase with decreasing reinforcement ratio. This is because the beams with the

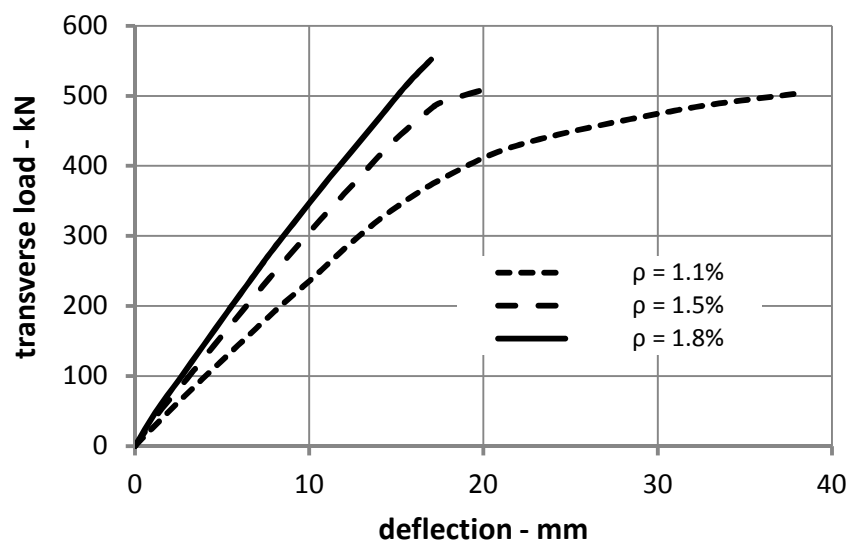


Fig. 9 Effect of percentage of tension reinforcement (ρ) on load-deflection curves of the strengthened beams with non-bonded flexural reinforcement

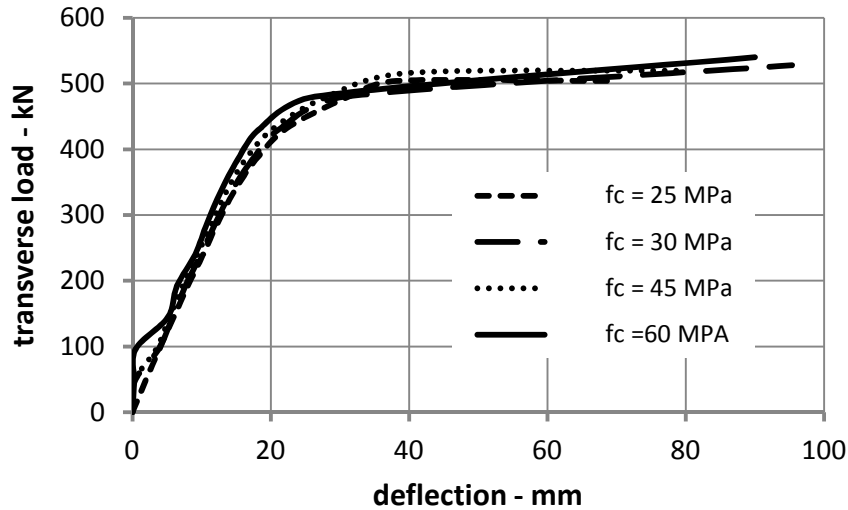


Fig. 10 Effect of concrete strength (f_c) on load-deflection curves of beams NBS with percentage of tension reinforcement $\rho = 1.1\%$

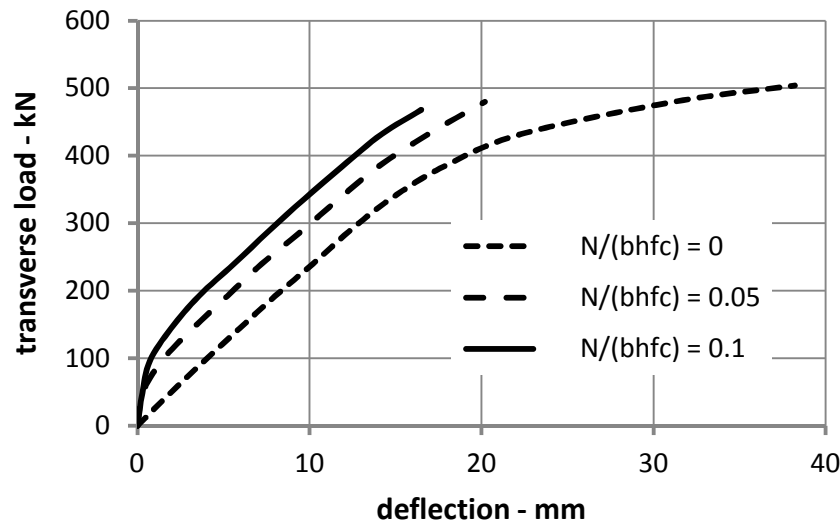


Fig. 11 Effect of axial force (N) on load-deflection curves of beams NBS with reinforcement ratio $\rho = 1.1\%$ (b and h are width and height of beam cross section and f_c is the uniaxial cylinder compressive strength)

higher reinforcement ratio reach their flexural capacity when the strain of the flexural reinforcement is only marginally larger than the value corresponding to yield. The smaller the reinforcement ratio the larger the values of both the steel strain and the rotation of the mid-span cross section thus leading to an increase in ductility. On the other hand, from the load-deflection curves shown in Fig. 10 for the case of beams NBS with a reinforcement ratio $\rho = 1.1\%$, concrete

strength appears to have only a small, if any, effect on the beam load-carrying capacity and ductility.

Finally, from Fig. 11, it appears that the presence of an axial force, even as small as 5% of the beam strength in compression, causes a significant reduction in ductility. This is because the presence of the axial force increases the depth of the compressive zone thus reducing the steel strain below the yield value, thus also preventing an increase in load-carrying capacity, even for the of the lowest reinforcement ratio of 1.1% considered in the present work. However, the presence of an axial force in RC beam-column elements with bonded flexural reinforcement increases the element shear capacity and, therefore, resorting to alternative means for reducing the likelihood of shear failure is not as important as it is in the absence of an axial force. The effect of the presence of axial force on the behaviour of linear elements with non-bonded reinforcement forms the subject of an on-going research programme.

5. Conclusions

It is confirmed that the interaction between concrete and flexural reinforcement through bond is the cause of inclined cracking developing in RC beams from the early stages of transverse loading. RC beams with non-bonded flexural reinforcement are characterised by the presence of flexural cracks only developing in the region of the maximum bending moment. These cracks penetrate deeply in the compressive zone and, at the late load stages, their ends may extend near-diagonally only to a short distance from their initial position. Failure eventually occurs due to near horizontal splitting of the compressive zone in the region of the maximum bending moment. Strengthening the region of the maximum bending moment with stirrups delays the horizontal splitting of the compressive zone until yielding of the flexural reinforcement in tension. Reducing the amount of the flexural reinforcement leads to an increase of the beam ductility, but reduces stiffness. Although the presence of an axial force does not affect the mode of failure in that it is flexural in nature, load-carrying capacity and ductility decrease with increasing axial force due to a reduction of the steel strain below the yield value.

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