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Performance-based remaining life assessment of reinforced concrete bridge girders

M.B. Anoop[*1], K. Balaji Rao[1] and B.K. Raghuprasad[2]

[1]CSIR-Structural engineering research centre, CSIR Campus, Chennai 600113, India
[2]Department of civil engineering, Indian institute of science, Bangalore 560012, India

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Abstract. Performance-based remaining life assessment of reinforced concrete bridge girders, subject to chloride-induced corrosion of reinforcement, is addressed in this paper. Towards this, a methodology that takes into consideration the human judgmental aspects in expert decision making regarding condition state assessment is proposed. The condition of the bridge girder is specified by the assignment of a condition state from a set of predefined condition states, considering both serviceability- and ultimate- limit states, and, the performance of the bridge girder is described using performability measure. A non-homogeneous Markov chain is used for modelling the stochastic evolution of condition state of the bridge girder with time. The thinking process of the expert in condition state assessment is modelled within a probabilistic framework using Brunswikian theory and probabilistic mental models. The remaining life is determined as the time over which the performance of the girder is above the required performance level. The usefulness of the methodology is illustrated through the remaining life assessment of a reinforced concrete T-beam bridge girder.

Keywords: reinforced concrete; chloride-induced corrosion; remaining life; performability; non-homogeneous Markov chain; Brunswikian theory

1. Introduction

A rational estimation of remaining life of reinforced concrete (RC) structural elements subject to corrosion of reinforcement is required for making engineering decisions regarding the inspection/maintenance activities of these elements. Internationally, efforts are being made to develop methodologies for remaining life assessment of reinforced concrete structural elements (Balaji Rao *et al.* 2002, Val and Stewart 2003, Anoop *et al.* 2002, 2003, 2007, 2012, Choi and Seo 2009, Cusson *et al.* 2011, Gjørv 2013;Papadakis 2013, Safehiana and Ramezanianpour 2015, Cho *et al*. 2015). The remaining life in most of these studies has been estimated with respect to the ultimate limit state (of flexure). However, as stated by Freudenthal (1954), ‘the principal aspects of the operating conditions of a load-carrying structure are the limiting conditions of service and failure’. Therefore, both serviceability- and ultimate-limit states should be considered

∗Corresponding author, Ph.D., E-mail: balajiserc1@yahoo.com

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simultaneously while estimating the remaining life of RC structural elements. A performance-based approach for remaining life assessment will provide a rational framework for incorporating safety against both the ultimate limit state (ULS) and serviceability limit state (SLS) for rational estimation of the remaining life. This requires the determination of performance of the structural element at different times. A new class of models, increasingly used for performance evaluation of degrading systems, is the performability model (Platis, 2006). A performability-based approach is proposed in this study for determining the remaining life of corrosion-affected RC bridge girders.

Another important aspect in remaining life estimation is the interpretation of the data from field inspections and making expert judgment about condition state of the structural element. Human judgment plays an important role in the condition assessment and decision making. Balaji Rao *et al.* (2004) proposed a methodology for remaining life assessment of corrosion affected reinforced concrete structural elements, integrating expert judgment regarding corrosion damage level with the structural risk. In this methodology, the thinking process of the expert, in corrosion damage assessment, is modelled within a probabilistic framework using Brunswikian theory and probabilistic mental models. In the present study, this methodology is further improved for performance-based remaining life assessment of corrosion-affected reinforced concrete bridge girders. In Balaji Rao *et al*. (2004), five corrosion damage states were considered. These represented the ranges of capacity ratio (defined as ratio of the load carrying capacity of the member at any time to the required capacity for the structural member according to relevant design standards). In the present study, the state classification has been improved by using the guidelines given by U.S. Department of Transportation (1995), and also by developing an objective method for determination of condition state of corrosion-affected RC bridge girders, considering safety against both SLS and ULS. Also, Balaji Rao *et al*. (2004) used a homogeneous Markov Chain for modelling the stochastic evolution of condition state of the bridge girder with time, while in the present study, the condition state transition probabilities are considered as a function of time by using a non-homogeneous Markov Chain. The details of the studies carried out are given in the following sections.

The paper is organised as follows: The steps involved in the proposed methodology for performance-based remaining life assessment are given in the next section. Details of development of procedure for determining the condition state of RC bridge girder are presented in Section 3. Modelling the condition state evolution using non-homogeneous Markov chain is described in Section 4. The details of computation of performability for quantifying the performance of the bridge girder are presented in Section 5. Use of Brunswikian theory for handling the human judgmental aspects in condition state assessment is discussed in Section 6. Performance-based remaining life assessment is presented in Section 7, and the use of the methodology is illustrated through an application in Section 8. The results and discussion are presented in Section 9, and the summary is given in Section 10.

2. Proposed methodology for performance evaluation

The proposed methodology for performance-based remaining life assessment of corrosion-affected RC bridge girders is an improvement over the methodology given in Balaji Rao *et al.* (2004). The improvements are as follows:

• An objective method for determination of condition state of corrosion-affected RC bridge girders, considering safety against both SLS and ULS, is developed and used in the present study

• Condition state transition probabilities are considered as a function of time by using a non-homogeneous Markov Chain (NHMC) for modelling the stochastic evolution of condition state of the bridge girder with time. The elements of the transition probability matrix of the NHMC at different times are determined using Monte Carlo simulation (MCS) approach.
• Performability measure, which combines both the reliability and performance measures (Sanders and Meyer 1991), is used for describing the performance of the bridge girder.

The proposed methodology involves the following five steps.

Step 1: Determination of the condition state of the RC bridge girder at a given time
Step 2: Modelling condition state evolution of the bridge girder using Markov Chain
Step 3: Computation of performability of the RC bridge girder
Step 4: Handling the human judgmental aspects in condition state assessment based on field inspection
Step 5: Performance-based remaining life assessment

The details related to each of these steps are given in the following sections.

3 Determination of condition state of the girder–deterministic procedure

The condition of the bridge girder is specified by the assignment of a condition state. The methods of condition state assessment are generally based on results from visual inspections, and are subjective in nature (Aktan *et al.* 1996). Aktan *et al.* (1996) pointed out the need for developing an objective method for condition assessment. The chloride-induced corrosion of reinforcement affects the safety against both the ULS (due to the reduction in cross-sectional area of steel reinforcement) and SLS (due to corrosion-induced cracking of cover concrete) of the RC bridge girder. In this section, an attempt is made to develop an objective method for condition state assessment of corrosion-affected RC bridge girders considering safety against both SLS and ULS.

In the present study, the guidelines given by U.S. Department of Transportation (1995) are used for defining the condition state for the girder. It may be noted that these guidelines are applicable for the condition rating of the superstructure system, however the same is used in this study for condition rating of the individual girder. This is justifiable, since ‘the primary member rating represents the condition and functional capacity of the main members of the bridge span as a system’ (NYDoT 1997). As specified in NYDoT (1997), if the condition of a primary member is the controlling condition in a load path non-redundant system, the rating would be used for the primary member system

3.1 Modelling the reduction in area of reinforcement due to chloride-induced corrosion

For most of the cases, diffusion can be assumed to be the dominant mechanism for ingress of chlorides into concrete (Vořechovská *et al*. 2009, Shafei *et al*. 2013, Aguirre and de Gutiérrez 2013), and Fick’s second law of diffusion can be used to model the chloride penetration into concrete (Crank 1975). The time for corrosion initiation (t_i) can be determined from Fick's second law of diffusion as (Crank 1975)

$$t_i = \frac{d^2}{4D}\left[erf^{-1}\left(\frac{c_s - c_{cr}}{c_s}\right)\right]^{-2} \tag{1}$$

where d is the clear cover to reinforcement, D is the diffusion coefficient for chlorides in concrete, c_s is the surface chloride concentration and c_{cr} is the critical chloride concentration.

From a brief review of the different models for determining the remaining area of reinforcing bar subjected to corrosion (Raupach *et al.*, 2006; Markeset and Myrdal, 2008) , it is found that the model proposed by Rodriguez et al. (1996) is generally used for determining the remaining diameter of reinforcing bar. Using this model, the remaining reinforcing bar diameter, $\phi(t)$ (in mm), at any time t (in years), is estimated as

$$\phi(t) = \phi(0) - 0.0116\, \alpha\, I_{corr}(t - t_i) \tag{2}$$

where $\phi(0)$ is the bar diameter before corrosion initiation (in mm), I_{corr} is the corrosion current density (in μA/cm^2), 0.0116 is a factor which converts μA/cm^2 to mm/year, t_i is the time for corrosion initiation (in years), and α is a factor for including the effect of localized pitting.

3.2 Safety against SLS (SSLS)

In the present study, the SSLS is defined based on the degree of cracking (characterised by crack width) in the girder due to chloride induced corrosion of reinforcement. Based on a review (Anoop 2009, Ahsana *et al*. 2015) of the different models proposed by various researchers relating the level of corrosion to the formation of cracks, the following models are chosen for the crack initiation and crack width propagation due to corrosion of reinforcement in concrete. These models are based on experimental investigations on reinforced concrete beams kept under loading (Vidal *et al*. (2004)) and hence take into consideration the effect of loading on the corrosion-induced cracking. Thus, these models will be more applicable to practical conditions. The maximum crack width due to corrosion of reinforcement ($w_{\max}$, in mm) is given by (from Vidal *et al*. (2004))

$$w_{max} = 0.101\left(\Delta A_s(t) - \Delta A_{so}\right) \tag{3}$$

where $\Delta A_s(t)$ is the loss in area of reinforcement in mm^2 at time t (in years) due to corrosion, determined using $\phi(t)$ obtained from Eq. (2). ΔA_{SO} is the loss in area of reinforcement (in mm^2) resulting in crack initiation, given by

$$\Delta A_{so} = A_s\left[1 - \left[1 - \frac{\alpha}{\phi(0)}\left(7.53 + 9.32\frac{d}{\phi(0)}\right)\times 10^{-3}\right]^2\right] \tag{4}$$

where A_s is the initial area of steel cross-section in mm^2. Using Eqs. (3) and (4), the maximum crack width due to chloride-induced corrosion of reinforcement at any time is determined based on the loss in area of reinforcement at that time. The SSLS for the bridge girder is determined based on the maximum crack width using the guidelines given in Table 1.

3.3 Safety against ULS (SULS)

In this study, it is assumed that the safety against ULS of the bridge girder at any time depends on the capacity of the girder to sustain the applied loads. The load and resistance factor rating

Table 1 States of SSLS for the bridge girder

SSLS state	Description
1	Corrosion is yet to initiate
2	Corrosion initiated, but no cracking
3	Cracking initiated, but crack width less than the allowable value of 0.3 mm specified in code of practice (BIS, 2000)
4	Crack width greater than 0.3 mm, but less than 1.0mm (no spalling)
5	Crack width ≥ 1.0mm, spalling of concrete (Vu and Stewart, 2005)

(LRFR) method proposed by AASHTO (2003) is reliability-based and provides a more realistic assessment of the safe load capacity of existing bridges (Minervino *et al*., 2004). This method is adopted in the present study for defining the SULS. According to LRFR, the load rating is generally expressed as a rating factor (RF), given by (AASHTO, 2003)

$$\mathrm{RF} = \frac{\mathrm{C} - \gamma_{\mathrm{DC}}\,\mathrm{DC} - \gamma_{\mathrm{DW}}\,\mathrm{DW} \pm \gamma_{\mathrm{P}}\,\mathrm{P}}{\gamma_{\mathrm{L}}\,\mathrm{LL}(1 + \mathrm{IM})} \tag{5}$$

where

$$\mathrm{C} = \varphi_c\,\varphi_s\,\varphi\,\mathrm{R}_n\,;\ \text{with}\ \varphi_c\,\varphi_s \geq 0.85 \tag{6}$$

where C is the capacity, R_n is the nominal resistance, DC is the dead load effect due to structural components and attachments, DW is the dead load effect due to wearing surface and utilities, P is the permanent loads other than dead loads, LL is the live load effect, IM is the dynamic load allowance, γ_{DC} is the LRFD (load and resistant factor design) load factor for structural components and attachments, γ_{DW} is the LRFD load factor for wearing surface and utilities, γ_P is the LRFD load factor for permanent loads other than dead loads and γ_L is the evaluation live load factor. φ_c is the condition factor for taking into account the increased uncertainty in the resistance of deteriorated elements. φ_s is the system factor for taking into consideration the level of redundancy of the entire super structure, φ is the LRFD resistance factor.

The procedure for LRFR is comprised of three distinct procedures: 1) design load rating, 2) legal load rating, and 3) permit load rating.

Design load rating: Under design load rating, bridges are screened either at design (inventory) level reliability (reliability index, β = 3.5) or at operating level reliability (β = 2.5). While for design a relatively high value of β is chosen as the cost of compliance is marginal, for evaluation, a lower acceptable reliability is more appropriate (Minervino *et al*., 2004). For performance-based remaining life assessment, the safety of the bridge girder against the operating level at any specified time is more important. Therefore, in the present study, the design load rating is carried out only for the operating level (OL) reliability for determining the safety state of the girder.

Legal load rating: Legal load rating is required only when the RF for design load rating at the operating level (RF_{OL}) is less than one. If the bridge does not satisfy the legal load rating (RF_{LL} < 1), the safe posting load can be determined as (AASHTO 2003)

$$\text{Safe Posting Load} = \mathrm{W}(\mathrm{RF}_{\mathrm{LL}} - 0.3)/0.7 \tag{7}$$

Table 2 States of SULS for the bridge girder

SULS state	Definition	Description
1	$RF_{OL} \geq 1.0$	Bridge girder satisfies the design load rating at operating level
2	$RF_{OL} < 1.0$ & $RF_{LL} \geq 1.0$	Bridge girder does not satisfy the design load rating, but satisfies the legal load rating
3	$0.6 \leq RF_{LL} < 1.0$	Bridge girder does not satisfy the legal load rating; reduced loads can be permitted
4	$0.3 \leq RF_{LL} < 0.6$	Bridge girder does not satisfy the legal load rating; highly reduced loads can be permitted
5	$RF_{LL} < 0.3$	Bridge to be closed to traffic

(Note: RF_{OL}: rating factor for design load rating at operating level; RF_{LL}: rating factor for legal load rating)

Table 3 General appraisal rating guidelines (based on Ryan *et al*. 2006)

Condition State	Description
CS9	Excellent condition
CS8	Very good condition: no problems noted
CS7	Good condition: some minor problems
CS6	Satisfactory condition: structural elements show some minor deterioration
CS5	Fair condition: all primary structural elements are sound but may have minor section loss, cracking or spalling
CS4	Poor condition: advanced section loss, deterioration, spalling or scour
CS3	Serious condition: loss of section, deterioration or spalling has seriously affected primary structural components. Local failures are possible. Shear cracks in concrete may be present.
CS2	Critical condition: advanced deterioration of primary structural elements. Shear cracks in concrete may be present. Unless closely monitored, it may be necessary to close the bridge until corrective action is taken.
CS1	“Imminent” failure condition: major deterioration or section loss present in critical structural components. Bridge is closed to traffic but corrective action may put back in light service.
CS0	Failed condition: out of service. Beyond corrective action.

where W is the weight of the legal load rating vehicle. When RF_{LL} falls below 0.3, then the bridge should be considered for closure.

Permit load rating: This level of evaluation is required for the issuance of overweight permits based on request, and is not considered in the present study.

For a bridge girder subjected to chloride-induced corrosion of reinforcement, the nominal resistance will reduce with time after corrosion initiation. Once the reduced diameter of the reinforcing bars is determined using Eq. (2), the nominal resistance of the bridge girder, R_n, and the rating factors are computed. The SULS for the bridge girder at any time is determined based on the value of rating factor at that time using the conditions given in Table 2.

Table 4 Definitions of ULS and SLS conditions for condition state assessment

Criteria	ULS condition	SLS condition
Desirable criteria	Satisfy design load check, $(RF_{OL} \geq 1.0)$	No cracking due to corrosion
Minimum criteria	Satisfy legal load check, $(RF_{LL} \geq 1.0)$	Crack width $\leq$ 0.3 mm
Minimum tolerable limit	$RF_{LL} = 0.6$	Crack width = 1.0 mm
Intolerable	$0.30 \leq RF_{LL} \leq 0.6$	Crack width > 1.0 mm
Close the bridge	$RF_{LL} < 0.30$	-

Table 5 Rule base for determining the condition state

	Safety against ULS → / Safety against SLS ↓	SULS State				
		1	2	3	4	5
SSLS State	1	CS9	CS7	CS5	CS3	CS1
	2	CS8	CS6	CS5	CS2	CS1
	3	CS7	CS6	CS4	CS2	CS1
	4	CS5	CS4	CS4	CS2	CS0
	5	CS3	CS3	CS3	CS2	CS0

3.4 Determination of condition states

After determining the safety against the ultimate limit state (SULS) and the safety against serviceability limit state (SSLS) at any time, the condition state (CS) at that time of the girder can be determined. The general appraisal rating guidelines (Table 3) based on the Bridge Inspector's Reference Manual of FHWA (Ryan *et al*. 2006) along with the definitions of ULS and SLS conditions (Table 4) are used for this purpose. Using the information given in Tables 1-4, a rule base is formulated relating the SULS and SSLS to the condition state of the girder (see Table 5). Another consideration while developing the rule base is the number of associations (defined as the number of combinations of safety and serviceability states for a given condition state) for the different condition states. There are least uncertainties associated with assigning the highest- (corresponding to no- or least- damage) and the lowest- (corresponding to the maximum damage) condition states, while for the intermediate condition states, the uncertainties are the maximum. Accordingly, the number of associations for the highest- (1 each for CS9 and CS8) and the lowest- (2 for CS0) condition states are kept to minimum while those for the intermediate condition states are kept in an increasing and then decreasing manner (2 each for CS7 and CS6, 3 each for CS5 and CS4, 4 each for CS3 and CS2 and 3 for CS1) while formulating the rule base. In the present study, it is assumed that the SULS and SSLS obtained from the analysis represents what is observed from the field. The rulebase given in Table 5 is formulated by the authors based on review of relevant literature and intuition. Table 5 can be used to classify/assess the (corrosion) condition state of the bridge girder. The feature of this table, as can be noted, is combining the information presented in Tables 1-4 towards objective condition state assessment, which is one of the aims of the present study (the need for which has been felt in Aktan *et al*. (1996)).

After developing an analytical model for determining the condition state of the girder at any time, the next step is to model the evolution of condition state with time. This issue is addressed in the next section.

4. Stochastic modelling of the condition state evolution

Due to the uncertainties in material properties of steel and concrete and the variations in exposure conditions, the condition state of the RC bridge girder at any given time is a random variable. In a corrosion-affected RC bridge girder, the area of reinforcement reduces with time which affects the condition state of the girder. The condition state evolution of the bridge girder with time should be treated as a stochastic process. Markov Chains (MC) are found to be a useful tool for stochastic modelling of condition state evolution of RC bridge girders (Balaji Rao *et al*. 2004, Balaji Rao *et al*. 2004a, Lay and Schieβl 2003, Marcous 2006) and also for inspection/maintenance scheduling of civil infrastructure systems (Zhang and Gao 2010, Gao and Zhang 2013). In the present study, MC modelling is used for the condition state evolution of the bridge girder with time. The index space of this stochastic process is the time, which can be considered as discrete, $\{T_1, T_2, \ldots\}$. The state space of the stochastic process represents the condition state of the girder, which is also discrete, S = {CS9, CS8, …, CS0}. Hereafter, the condition states are represented as $S = \{CS_9, CS_8, \ldots, CS_0\}$. The probabilistic evolution of condition state of the bridge girder is given by the transition probability matrix (TPM). Since the cross-sectional area of reinforcement reduces with time due to chloride-induced corrosion, there is transition only from higher condition states to the lower condition states. Therefore, the TPM will be an upper triangular matrix.

Balaji Rao *et al.* (2004) used a homogeneous MC for corrosion damage evolution with time. From the results of experimental investigations reported in literature (Rodriguez *et al*. 1996), it is noted that the loss of area of reinforcement with time is nonlinear. Balaji Rao and Anoop (2014) carried out stochastic analysis of RC beams with corroding reinforcement and showed that the time-variant ultimate moment of resistance of the RC beam is nonstationary. Therefore, the condition state transition probabilities need to be considered as a function of time, and the use of a non-homogeneous Markov Chain (NHMC) model would be more rational (Balaji Rao *et al.* 2004a). Hence, in this study, a NHMC model is proposed for modelling the condition state evolution with time. Monte Carlo simulation (MCS) approach is used for determining the elements of the TPM. To account for variations in workmanship and exposure conditions, d, D, c_s, c_{cr}, I_{corr} and α are treated as random variables. The step-by-step procedure for determining the elements of the transition probability matrix (**P**) for the condition state evolution between two successive time instants, T_k and T_{k+1}, is given below.

1. Select values of mean and COV for D, c_s, c_{cr}, I_{corr}, α and cover thickness (d)

2. Determine mean and standard deviation of time to corrosion initiation (Eq. 1) using first order approximation. Based on the results of the simulation studies on corrosion initiation in reinforced concrete members, Balaji Rao *et al*. (2001) reported that t_i follows a lognormal distribution, and, the same has been considered in the present study.

3. Carry out MCS at two successive time instants, T_k and T_{k+1} considering t_i, I_{corr} and α as random variables

For each time instant and for each simulation

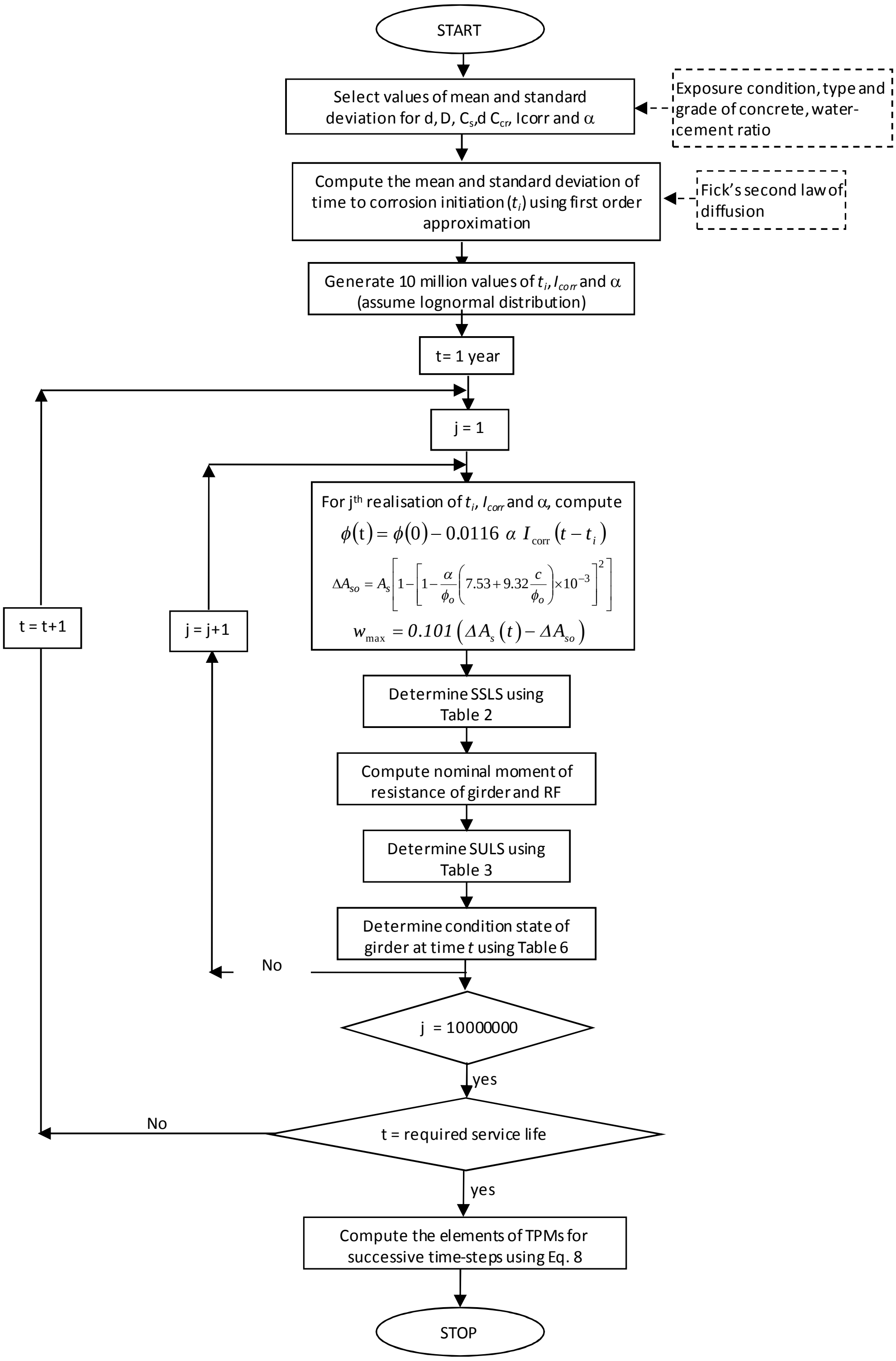

Fig. 1 Flowchart illustrating the procedure for determining the elements of TPMs for condition state evolution

i. Determine the remaining diameter of the reinforcing bar (Eq. 2) and the loss in area of reinforcement
ii. Compute the width of the crack formed due to corrosion (using Eq. 3) and identify the safety against SLS (SSLS) using Table 1
iii. Carry out LRFR and identify the safety against ULS (SULS) using Table 2
Determine the condition state (CS) for the girder using the rule base given in Table 5
4. Compute the elements of the TPM as

$$\begin{aligned}\mathbf{P}(i,\, j) = p_{ij} &= \frac{\Pr\left(CS = CS_m \mid T = T_{k+1} \cap CS = CS_l \mid T = T_k\right)}{\Pr\left(CS = CS_l \mid T = T_k\right)};\, 1 \le i \le 10;\, i \le j \le 10;\, l = 10 - i;\, m = 10 - j \\ &= \frac{\text{number of simulation cycles for which } CS = CS_m \text{ at time } T_{k+1} \text{ and } CS = CS_l \text{ at time } T = T_k}{\text{number of simulation cycles for which } CS = CS_l \text{ at time } T = T_k}\end{aligned} \tag{8}$$

A flowchart illustrating the procedure for determining the TPM is given in Fig. 1. Monte Carlo simulation method with ten million cycles is used for determining the elements of TPM at different time-steps.

The probabilistic description of the condition state after k-time steps is given by

$$\mathbf{P}(T_1, T_k) = \mathbf{P}(T_1, T_2) \times \mathbf{P}(T_2, T_3) \times \ldots \times \mathbf{P}(T_{k-1}, T_k) \tag{9}$$

The unconditional probability vector of the condition states, $\left(P^U(T_k)\right)_{1\times 10}$, after k-time steps is determined as

$$\left(P^U(T_k)\right)_{1\times 10} = \left(P(T_1)\right)_{1\times 10} \times \left[\mathbf{P}(T_1, T_k)\right]_{10\times 10} \tag{10}$$

where $\left(P(T_1)\right)_{1\times 10}$ is the vector representing the probabilities of initial condition states of the girder. After determining the unconditional probability vector of the condition states, the next step is to determine the performability of the girder, and the same is explained in the next section.

5. Performability

For performance-based remaining life assessment, the condition state of the RC bridge girder at any time should be related to the performance requirements. Development of methods for determination of system performance is an area of active research and has applications in various disciplines. A new class of models that are increasingly used in the estimation of system performance is the performability model (Platis 2006). Performability models have been introduced by Beaudry (1978), who defined combined measures of performance and reliability, and Meyer (from Platis *et al.* 1998), who proposed a general framework for performability analysis. Performability analysis is found to be useful for evaluating the performance of degrading systems (where the system is available, but not fully operational), and gives a more detailed evaluation of the system operational performance (Platis *et al.* 1998). Thus, performability analysis will be a useful method for evaluating the performance of corrosion-affected bridge girders, whose performance level degrades with time.

The performability measure combines both the reliability and performance measures (Sanders and Meyer 1991). One of the advantages of performability measure over reliability is that it is computationally easier to compute the variance of the performability measure compared to that of

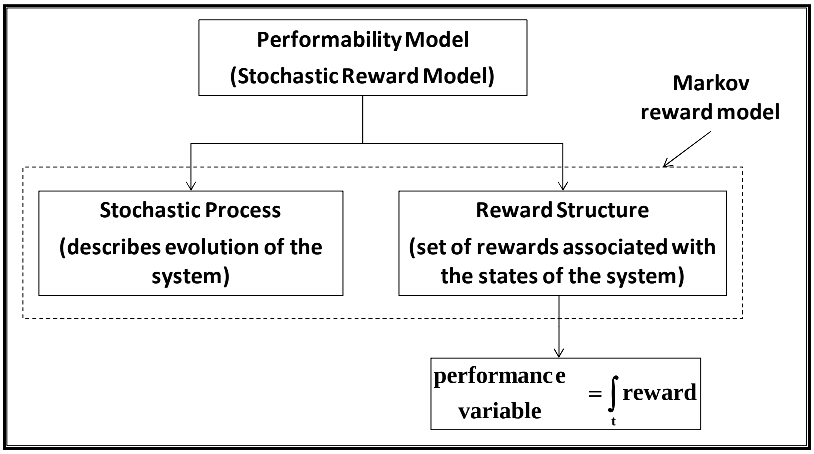

Fig. 2 Structure of a performability model (based on Sanders and Meyer 1991)

the reliability measure (Ang and Tang 1984). Thus, performability analysis will be useful for 'highlighting the uncertainties beyond expected values and probabilities' (Aven 2008). Performability models, also called stochastic reward models (Sanders and Meyer 1991), consist of a stochastic process and a reward structure (see Fig. 2). The stochastic process describes the evolution of the system. Markov chains are most commonly used in performability analysis for describing the stochastic evolution of the system (Bolch *et al.* 1998). The reward structure relates possible behaviours of the process to a specified performance variable. The reward structure is typically a set of one or more functions defined over the states of the system or on the transitions between the states in the process (Sanders and Meyer 1991).

After specifying the reward structure, the performance variable can be defined in terms of the reward structure. The performance variables can be (Sanders and Meyer 1991)

• *Instant-of-time variable*: the variable typically represents the states of the modelled system at some time 't' (corresponding reward is known as the instantaneous reward)

• *Interval-of-time variable*: the variable typically represents the accumulated benefit derived from operating the system for some interval of time (corresponding reward is known as the accumulated reward), and,

• *Time-averaged interval-of-time variable*: the variable represents the (time-averaged) rate at which reward is accumulated during an interval of time (corresponding reward is known as the time-averaged accumulated reward).

The last two performance variables are relatively insensitive to the state of the system at a particular instant as compared to the first one, especially at large times (Smith *et al.* 1988). Hence, the instantaneous reward will be more useful for making decisions on inspection, repair or replacement which should be based upon the state of the system at a given time. The accumulated reward and time-averaged accumulated reward will be useful for comparing the performances of different alternatives (for instance, for comparing the performances of different design alternatives) over a period of time for selecting the best alternative.

5.1 Computation of performability

Let the vector *r* represent the reward structure (i.e., r_{CS_i} is the reward assigned to the bridge girder if the bridge girder is in condition state CS_i). Let $CS(T_k)$ be the vector representing the condition state of the girder at time T_k. Then

$$Z(T_k) = r_{CS(T_k)} \tag{11}$$

refers to the instantaneous reward of the girder at time T_k (Bolch *et al*. 1998). The expected instantaneous reward and the variance of the instantaneous reward are given by

$$E[Z(T_k)] = \sum_{i \in S} r_i \, P_i^U (T_K) \tag{12}$$

$$Var[Z(T_k)] = \sum_{i \in S} r_i^{\,2} \, P_i^U (T_K) - \left(\sum_{i \in S} r_i \, P_i^U (T_K) \right)^2 \tag{13}$$

where $P^U(T_k)$ unconditional probability vector of the condition states at time T_k (Bolch *et al*. 1998). The probability that the instantaneous reward does not exceed a specified performance level, *z*, at time T_k is given by (Bolch *et al*. 1998)

$$p_z(T_k) = \text{Prob}[Z(T_k) \le z] = \sum_{i \in S,\; r_{i \le z}} P_i^U (T_K) \tag{14}$$

The accumulated reward, $W(T_k)$, the expected accumulated reward, $E[W(T_k)]$, and the expected time-averaged accumulated reward, $E[Y(T_k)]$, over the period (T_1, T_k) are given by

$$W(T_k) = \sum_{j=2}^{k} Z(T_j) \tag{15}$$

$$E[W(T_k)] = \sum_{j=2}^{k} \sum_{i \in S} r_i \, P_i^U (T_j) \tag{16}$$

$$E[Y(T_k)] = \frac{1}{(T_k - T_1)} \sum_{j=2}^{k} \sum_{i \in S} r_i \, P_i^U (T_j) \tag{17}$$

Consider a binary reward structure *r* defined such that a value of $r = 1$ is assigned to the 'up' states of the system and $r = 0$ is assigned to the 'down' states of the system. In this case, $E[Z(T_k)]$ gives the reliability of the system at time T_k, while $E[W(T_k)]$ gives the expected 'uptime' for the system over the period (T_1, T_k). It the reward structure *r* is considered such that r_i represents the capacity of the system in state '*i*', then $E[Z(T_k)]$ gives the expected instantaneous capacity of the system at time T_k and $p_z(T_k)$ gives the probability that the capacity of the system does not exceed '*z*' at time T_k. If r_i represents the loss occurring due to the system being in state '*i*', then $E[Z(T_k)]$ gives the expected instantaneous loss at time T_k and $E[W(T_k)]$ gives the expected cumulative loss over the period (T_1, T_k).

Table 6 Reward structures considered

Condition State	Reward Structure I: Reward rate	Reward Structure II: Penalty rate	Reward Structure III: Reward rate
CS9	9	0 (No immediacy of maintenance action)	1.0
CS8	8		
CS7	7		
CS6	6		
CS5	5	0.5 (maintenance within one year)	0.7
CS4	4		
CS3	3		
CS2	2	1.0 (immediate maintenance)	0.2
CS1	1		0.0
CS0	0		

5.2 Specification of reward rates-proposed structure

The set of rewards associated with the different states of the system is called the reward structure. It relates possible behaviours of the process to a specified performance variable. Some of the commonly used reward structures are presented in Bolch *et al.* (1998).

To study the effect of considering different types of reward rates, three reward structures are proposed in this study (Table 6).

Reward Structure I: The reward rate are taken as the condition rank of the girder when it is in a given condition state. In this case, $E[Z(T_k)]$ gives the expected value of condition rank for the girder at time T_k, and $p_z(T_k)$ gives the probability that the condition rank of the girder does not exceed 'z' at time T_k.

Reward Structure II: The reward rates are assigned based on the urgency of maintenance when the girder is in different condition states. The maintenance urgency guidelines given in Ryan *et al.* (2006) is used for this purpose. Since the reward rate changes from 0 for no immediacy of maintenance action to 1.0 for immediate maintenance action, it is more appropriate to term it as a penalty rate rather than reward rate. This reward structure will be useful for comparing the maintenance urgency for different bridge girders for planning of maintenance and allocation of resources.

Reward Structure III: The reward rates are assigned based on the non-dimensionalized weight of the legal load rating vehicle that can be permitted when the girder is in different condition states. The rewards are determined by first identifying the minimum safety states corresponding to the different condition states, and then determining the average rating factor for these safety states. After determining the average rating factor, the weight of the vehicle that can be permitted is determined using Eq. (7).

So far, the focus has been on the determination of condition state and performance evaluation. A rational methodology for remaining life assessment should be able to incorporate the results of condition assessment based on information from field inspections. A method for rationally incorporating the expert judgment regarding condition state in the MC model for remaining life assessment is explained in the next section.

6. Brunswikian theory - an approach to handle of human judgmental aspects regarding condition assessment

Field inspections are usually carried out at frequent intervals to assess the condition state of the girder. The information from field inspections is passed on to an expert or a group of experts for making judgment regarding the condition state of the girder. Expert judgment is an essential part of condition state assessment, and there will be uncertainties associated with the human mental process in making judgment. The human mental process can best be described in a probabilistic basis and Brunswikian theory provides a rational framework for handling the uncertainties associated with the human mental process in judgment.

Balaji Rao *et al.* (2004) proposed the use Brunswikian theory for taking into consideration the uncertainties associated with human mental process in making judgment regarding corrosion damage state, and used a multi-level Brunswikian lens model for corrosion damage assessment of reinforced concrete structural elements. The same model is adopted in the present study for handling of human judgmental aspects regarding condition state assessment of bridge girders. The salient details of Brunswikian theory for corrosion damage assessment as presented by Balaji Rao *et al.* (2004) is explained in this section.

6.1 Brunswikian theory

Brunswik (1952) pointed out that one’s knowledge of a distal ‘initial focal variable’ is mediated by more proximal ‘cues’ (or information) that one has about it. According to Brunswik, while the individuals are generally competent, the levels of capability differ (Wolf 2000). This is in line with the thinking in risk perception and risk communication (Reid 1999), wherein the people involved in making judgment are considered to be rational, rather than classifying them as experts and non-

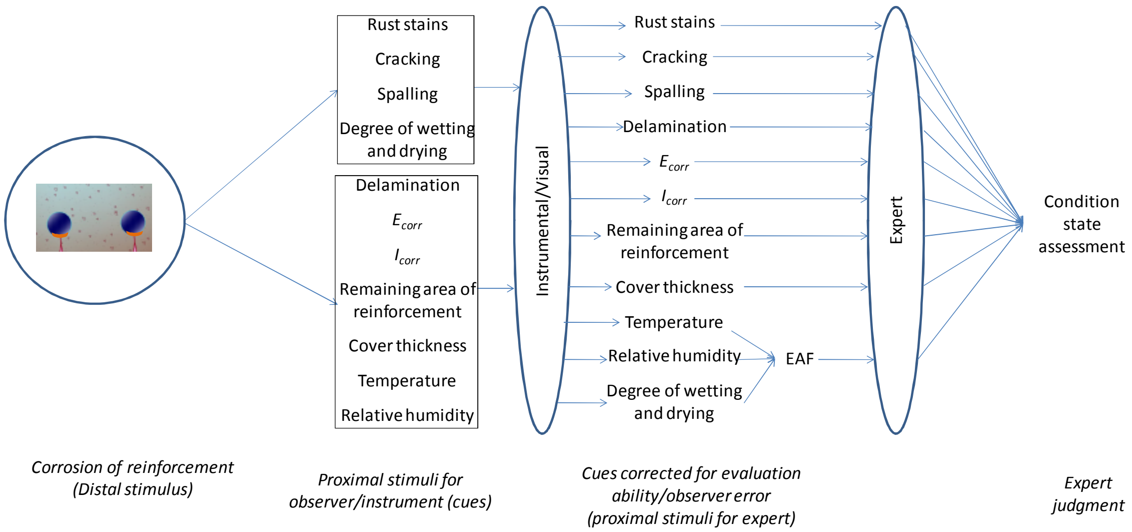

Fig. 3 Condition state assessment using Brunswikian lens model - Schematic diagram
(The cues temperature, relative humidiy and degree of wetting and drying are used for determining the environmental aggressiveness factor (EAF) characterising the exposure environment of the reinforced concrete element, see Anoop *et al*. (2002, 2003, 2007)

multiple pieces of information. Brehmer and Hagafors (1986) expanded the Brunwikian lens experts. The lens model proposed by Brunswik (1952) conceptually represents the situation wherein one individual has to make a judgment about the true state of the distal variable using model to a multilevel lens model to study the use of experts in complex judgment making. Such a multilevel lens model is used in the present study to model the condition state assessment of RC bridge girders.

6.2 Condition state assessment using Brunswikian theory

The distal stimulus of the multilevel lens model, used in condition state assessment of RC bridge girders, is the corrosion of reinforcement, which gives rise to the perceived/indicative proximal stimuli to the observer/ instrument in the form of changes in appearance and corrosion current/potential. The information on proximal stimuli (such as rust stains, amount of cracking and spalling, corrosion current density) are recorded by the observer/instrument (cues). The information recorded by the observer/instrument (cues) are corrected for the evaluation ability/human error (in the case of human observer) and for the detection capability and correctness of detection (in the case of instrument). The corrected data is the proximal stimuli for the expert who makes a judgment regarding the condition state. This is shown schematically in Fig. 3.

The same set of cues is supplied to a number of experts. Each expert is asked to identify the condition state(s) in which he believes the structural element is in, and to attach confidence level(s) for his judgment from a confidence scale. The confidence scale consists of seven levels I-VII representing 0%, 1%-20%, 21%-40%, 41%-60%, 61%-80%, 81%-99% and 100% confidence, respectively. The confidence judgment p_i is defined as the mean value of the confidence level. For the confidence levels I-VII, the values of p_i are 0%, 10%, 30%, 50%, 70%, 90% and 100%, respectively. Consistent with probabilistic mental thinking, the experts would judge the probable condition states of corrosion affected RC bridge girders, along with respective confidence levels. The final decision should take into account in some form the condition state information provided by different experts. Instead of classifying judges as experts or non-experts, it is better to consider them rational to different degrees (Reid 1999). This requires that the achievement of experts to be quantified.

The achievement index (r_a) of each expert is determined using the generalised linear model based on a number of baseline cases. Previous data on condition state assessment of distressed RC structural elements or data from laboratory experiments are used for this purpose. Similarly, the values of over- or under-confidence of each expert for different confidence levels are also determined using the probabilistic mental model (PMM) theory (Gigerenzer *et al.* 1991).

Suppose an expert has identified condition state(s) and assigned confidence level(s) to these condition state(s) based on the given data. Then the state probabilities (probability that the structure is in condition state k) at the time of inspection can be determined as

$$P_k = p_k \Big/ \sum_{s \in S} p_s \tag{18}$$

where p_k is the confidence judgment for the confidence level assigned to the condition state k ($k \in S$). For the condition states not identified by the expert, the value of p_k is taken as 0%. The condition state probabilities based on the judgment of the expert 'j' is

$$\{P\}_j = \{P_{CS_9}, P_{CS_8}, \ldots, P_{CS_0}\} \tag{19}$$

If there are n experts who make judgments independently using the same set of cues, then the state vector for the condition state combining the judgments of all the experts is obtained as

$$\{P_c\} = \sum_{j=1}^{n} w_j \{P\}_j \tag{20}$$

where $\{P\}_j$ is the condition state vector based on the judgment of the j^{th} expert, and w_j is the weight attributed to the judgment of the j^{th} expert. The weights reflect the accuracy of the expert in making the judgment. Since the achievement index is a measure of the correctness of the judgements made by the expert, more weightage can be given to the expert with higher achievement index. In the present study, the weight attributed to the judgment an expert (w_j) is determined by normalising the achievement index as given below.

$$w_j = \left(r_{a_j}\right)^m \Big/ \sum_{i=1}^{n} \left(r_{a_i}\right)^m; \qquad \mathrm{m} \geq 0 \tag{21}$$

where r_{a_j} is the achievement of the j^{th} expert and m is a value reflecting the degree of importance attached with the achievement of the expert. When m = 0, all the experts have been attributed the same weight. As the value of m increases, the degree of importance attached with the achievement of the expert increases. The achievement index can be determined using the linear regression model for each expert.

The over- or under-confidence limits associated with an expert for the different confidence levels are determined based on the judgments made on a number of baseline cases. The over- or under-confidence limit for a given confidence level, 'i', for a given expert 'j', can be determined as

$$\textit{over}\text{ - or under - confidence, ouc}_{\mathrm{i}} = n_i\left(p_i - f_i\right)/N \tag{22}$$

where N is total number of decisions made by the expert 'j', n_i is the number of times the confidence judgment p_i was used by the expert 'j', and f_i is the relative frequency of correct answers for all decisions for which confidence p_i was assigned by the expert 'j'. The over- or under-confidence takes into account the relative bias of the expert through the term n_i. The over- or under-confidence associated with the judgment of the j^{th} expert is given by

$$EC_j = \sum_{s \in S} ouc_s \tag{23}$$

The value of EC_j gives a rational way of determining the confidence to be attached with the judgment of an expert, which is only possible using the PMM by modelling human mental process on a probabilistic basis. The over- or under- confidence associated with the condition state obtained by combining the judgments of all the experts is given by

$$EC_c = \sum_{j=1}^{n} w_j EC_j \tag{24}$$

The value of EC_c will be useful as a measure of the confidence that can be put on the final

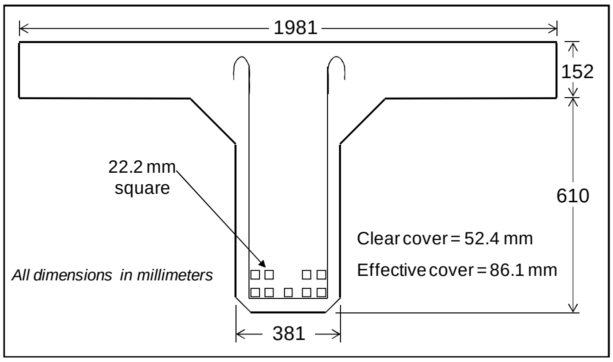

Fig. 4 Cross-sectional details of the T-beam bridge girder (AASHTO 2003)

condition state by processing the judgments of all the experts. A lower value of EC_c denotes tighter bounds on the condition state assessed and vice versa.

The condition state vector $\{P_c\}$, obtained based on the judgment of the experts, can be used along with the NHMC for modelling the condition state evolution. Suppose the inspection is carried out at time T_{ins}. Then $\{P_c\}$ represents the probabilities of condition states at time T_{ins}. The unconditional probability vector of the condition states after k-time steps from T_{ins} is obtained using Eq. (10) as

$$\left(P^U\left(T_{ins+k}\right)\right)_{1\times10} = \left\{P_c\right\} \times \left[\mathbf{P}\left(T_{ins}, T_{ins+k}\right)\right]_{10\times10} \tag{25}$$

The evaluation of the condition state is useful for predicting the future performance of the structure, and hence in the remaining life assessment.

7. Performance-based remaining life assessment

The remaining life is estimated by comparing the performance of the girder at different times with the required/acceptable performance. Since remaining life should be based on the state of the system at different times, the instantaneous reward rate ($Z(T_k)$) will be more useful for remaining life assessment. While $E[Z(T_k)]$ gives the expected instantaneous reward for the system at time T_k, it does not address the question of likelihood of satisfying a given level of performance (Smith *et al*. 1991). This question can be addressed using $p_z(T_k)$, or by its complement (1- $p_z(T_k)$), which gives the probability that the performance of the girder is above the performance level z.

8. Application

The reinforced concrete T-beam bridge girder (Fig. 4), given as an illustrative example in the LRFR Manual (AASHTO 2003), is considered in the present study. The girder has a span of 7.9m and is assumed to be located in a *severe* environment (as per the definitions of exposure conditions in IS 456-2000 (BIS 2000)). The compressive strength of concrete (f_c') is 20.7 MPa and the yield strength of steel (f_y) is 227.6 MPa. The values of mean and coefficient of variation (COV) of the

Table 7 Statistical properties of the random variables considered

variable	mean	COV*
d (mm)	52.4	0.05
D (cm^2/s)	5 x 10^{-8}	0.20
c_s (% by weight of concrete)	0.25	0.20
c_{cr} (% by weight of concrete)	0.125	0.20
i_{corr}	3.5	0.30
α	5.25	0.30

(Note: * - the COV values are assumed)

Table 8 Values of load effects and load factors to be used in load rating (from AASHTO 2003)

Parameter		Moment (KN-m)	Load factor
DC		114.88	1.25
DW		37.84	1.25
LL	Design load	400.2	1.35
LL	Legal load Type 3	238.98	1.65
LL	Legal load Type 3S2	229.43	1.65
LL	Legal load Type 3-3	196.60	1.65

random variables considered are given in Table 7. The mean value of cover thickness is taken as the nominal cover thickness specified at the design stage, and the mean values of D, c_s, c_{cr}, I_{corr} and α are taken based on the values reported in literature for similar type and grade of concrete in similar exposure conditions (Anoop and Balaji Rao 2015). All the random variables are assumed to be statistically uncorrelated with each other. The mean and standard deviation of time-to-corrosion initiation is determined using first order approximation, and it is assumed that t_i, d, I_{corr} and α follows lognormal distribution.

The RC beam considered is reinforced with 22.2 mm×22.2 mm size square bars. But the models used for determining the loss of reinforcement area corresponding to crack initiation and the remaining diameter of reinforcing bar require the diameter of the rebar. The diameter of a circular rebar with the same cross-sectional area as that of the square rebar is 25 mm, and this value is used as the initial diameter of the rebar. The number of layers of reinforcement and the numbers of bars in each layer are not modified.

For determination of SULS, the ultimate limit state of flexure is only considered in this study. The results of the dead load analysis and live load analysis for the bridge girder considered are taken from the LRFR Manual (AASHTO 2003) and are given in Table 8. The procedure shown in Fig. 1 is used for determining the elements of TPMs for condition state evolution.

9. Results and discussion

The variations in the mean and COV of the SSLS and the SULS with time are shown in Fig. 5. From these figures, it is noted that the ensemble average of SULS at a given time (<SULS>) is

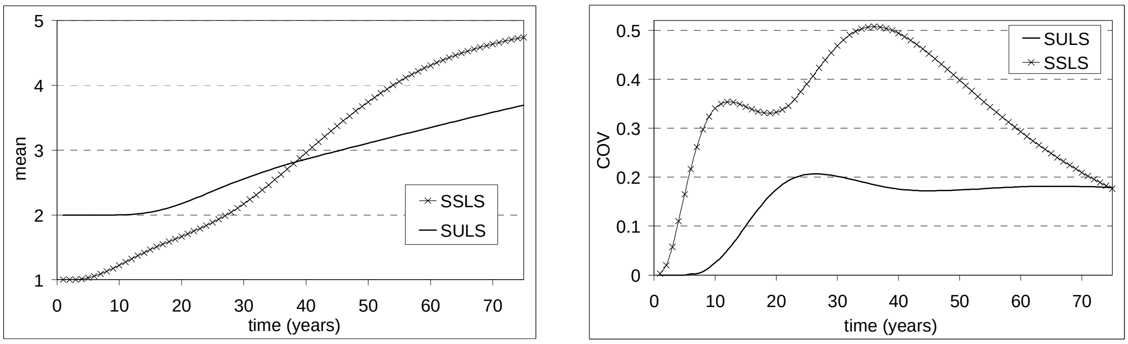

Fig. 5 Variations in mean and COV of SSLS and SULS with time

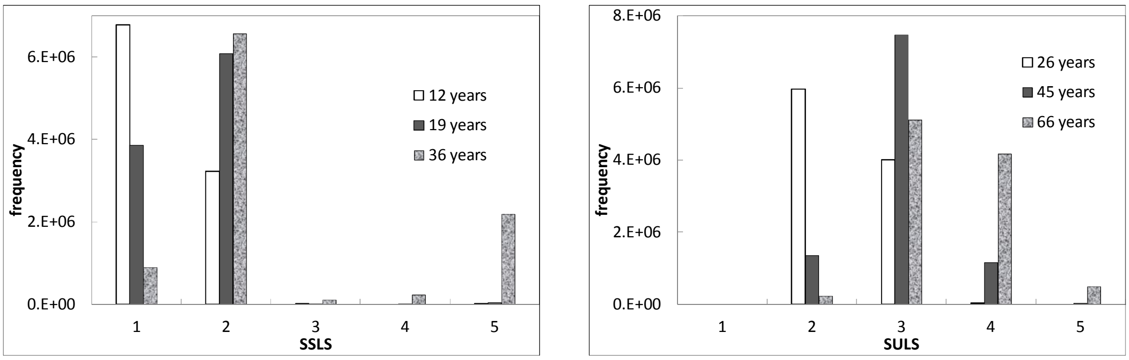

Fig. 6 Frequency distributions of SSLS and SULS at different times

higher than the ensemble average of SSLS at a given time (<SSLS>) in the initial stages. This is because even when there is no corrosion, the value of rating factor for design load at operating level (RF_{OL}) is less than one (for the bridge girder considered), and the corresponding safety state is 2. The values of COV of SULS and SSLS at a given time are initially zero indicating that there is no uncertainty regarding the SSLS and SULS (and, in turn, regarding the condition state) when there is no corrosion damage. It is also noted that the maximum value of COV of SSLS is much higher (about 2.5 times) than that of SULS. This suggests that corrosion-induced cracking behaviour of RC beams is a highly random phenomenon similar to as observed for flexural cracking behaviour (Prakash Desayi and Balaji Rao 1987).

It is noted from Fig. 5 that with the increase in time, the COV of SSLS also increases (till about 12^{th} year), then decreases (till about 19^{th} year), and then increases (till about 36 years) and decreases again. The frequency distributions of SSLS at 12^{th}, 19^{th} and 36^{th} year are shown in Fig. 6. It is noted from Fig. 6 that at 12^{th} and 19^{th} year, almost all the realisations of the girder are in SSLS 1 (no corrosion) and SSLS 2 (corrosion initiated, but no cracking). While at 12^{th} year, corrosion has not initiated in majority of the realisations, by19^{th} year, corrosion has initiated in majority of the realisations. The increase in standard deviation of SSLS from 12^{th} year to 19^{th} year is less (14.9%) compared to the increase in mean of SSLS over the same period (23.5%). This causes a reduction in the value of COV of SSLS. With further increase in time, cracking initiates in some of the realisations and these realisations move to higher damage states (SSLS 3 and above), as noted from the frequency distribution of SSLS at 36 years (Fig. 6). Hence, the value of

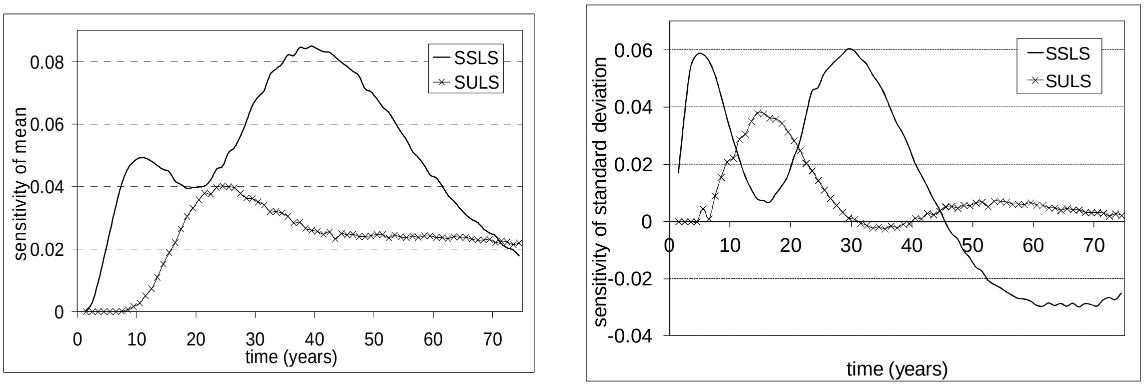

Fig. 7 Variation in sensitivities of mean and standard deviation of SSLS and SULS with time

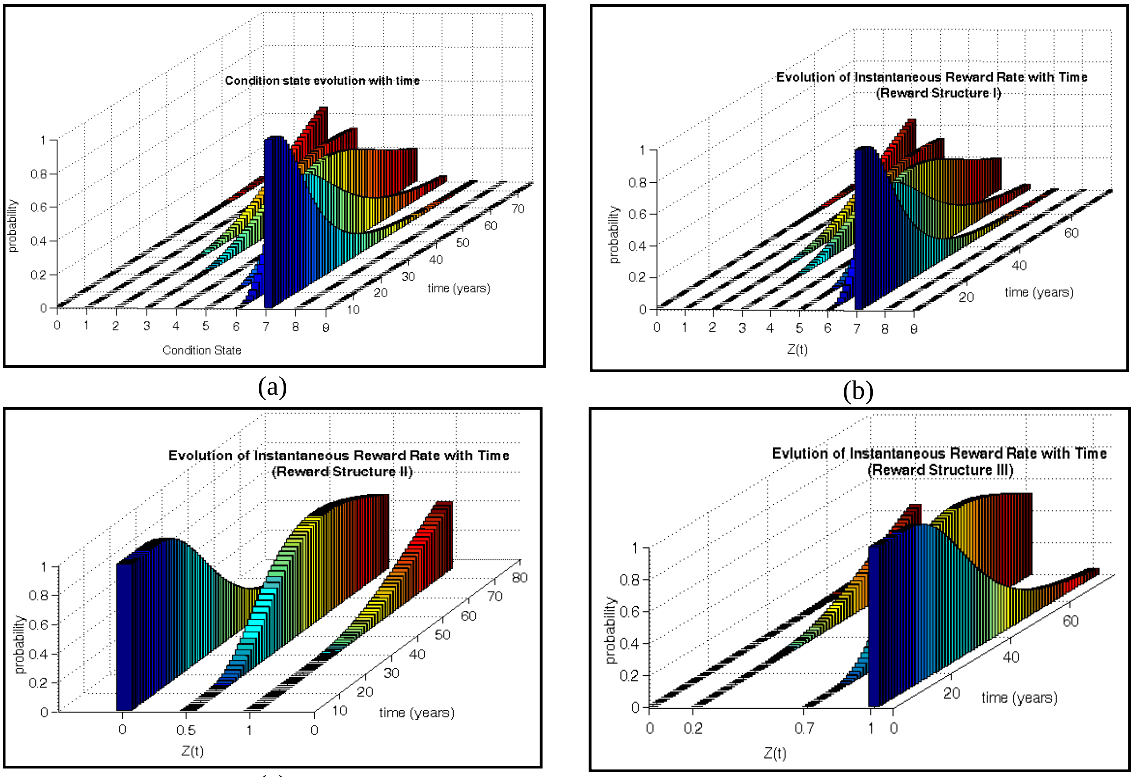

Fig. 8 (a) Evolution of condition states with time, (b)-(d) Evolution of instantaneous reward rate for the reward structures considered

COV of SSLS again starts increasing. With further passage of time, most of the realisations would move to maximum damage states (as indicated by the high values of <SSLS> in Fig. 5). Hence the value of COV of SSLS decreases, indicating a reduction in uncertainty about the possible damage state(s).

While the trend of variation in value of COV of SULS with time is similar to that of SSLS, the variation in values of COV are not very significant after reaching the maximum value (at the end of about 26 years). The frequency distributions of SULS at different times are shown in Fig. 6. It is noted from Fig. 6 that most of the realisations does not move to the maximum damage state (SULS 5) even at the end of 66 years. This is also evident from the low values of <SULS> at 66 years (about 3.5) and at the end of 75 years (about 3.7), when compared to the values of <SSLS> (about 4.5 at the end of 66 years and about 4.7 at the end of 75 years).

The variations in the sensitivities (determined as the change in the value over a time period of one year) of mean and standard deviation of SSLS and SULS with time are shown in Fig. 7. It is noted that the sensitivities vary nonlinearly with time. This is expected since the loss in area of reinforcement due to corrosion with time is nonlinear during the propagation period (Rodriguez *et al*. 1996). This observation suggests that there is a need to use NHMC for modelling the stochastic evolution of condition state of the RC girder with time. The highly nonlinear nature of sensitivities also suggests the need for a shorter time-step for computation of TPM, and hence 1-year interval is used in the NHMC. It is noted from Fig. 7 that the sensitivities of standard deviations of SSLS and SULS with time have almost opposing natures (i.e., when one increases, the other decreases and vice versa). Also, after about 45 years, the values of sensitivity of standard deviation of SSLS become negative, while those of SULS are positive. This suggests that as the age increases, the condition states are governed by the values of SULS.

The TPMs for condition state evolution at different times are determined using the procedure given in section 4, and the unconditional probabilities of condition states at different times are computed using Eq. (10). The evolution of condition states with time is shown in Fig. 8(a) and the

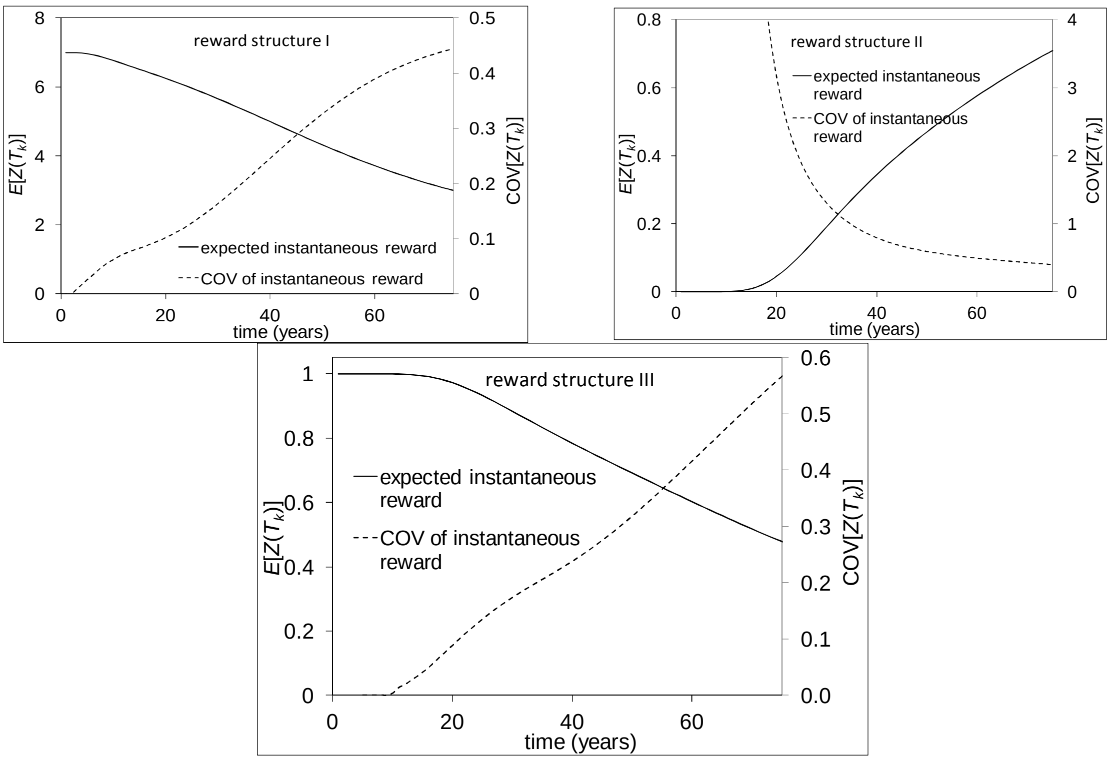

Fig. 9 Evolution of expected instantaneous reward rate and COV of instantaneous reward rate with time

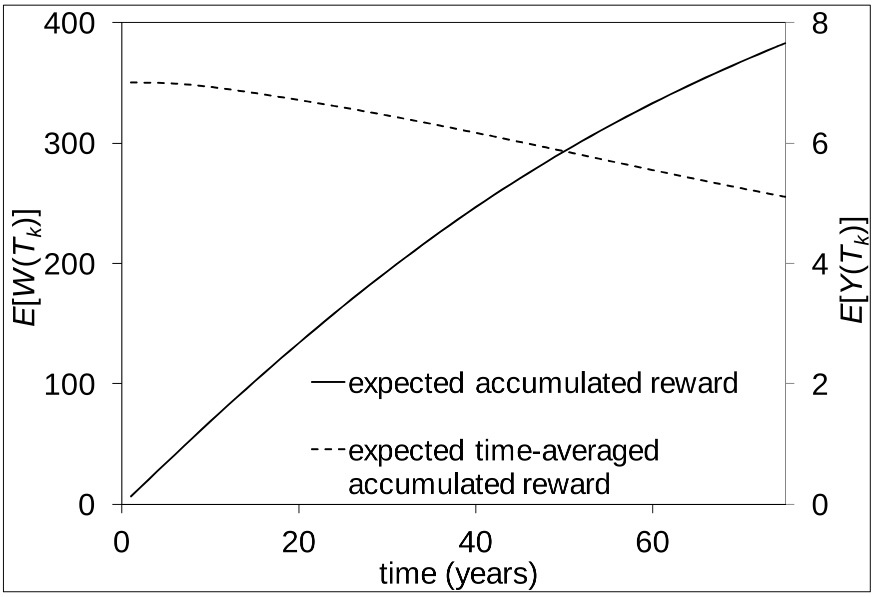

Fig. 10 Evolution of expected accumulated reward ($E[W(T_k)]$) and expected time-averaged accumulated reward ($E[Y(T_k)]$) with time considering reward structure I

Table 9 Data from inspection for the example problem

From Visual Inspection:	
Rust stains	Highly noticeable rust stains
Cracking	Several longitudinal cracks; width of crack < 0.3 mm
Spalling	No spalling
From Field Measurements:	
I_{corr} (3LP)	3.5 μA/cm^2
E_{corr}	-460 mV
Cover depth	50 mm
Remaining diameter of reinforcement	22.0 mm

evolution of instantaneous reward rate with time for the three reward structures considered are shown in Figs. 8(b)-(d). It is noted that the evolution of instantaneous reward rate with time for reward structure I. This is expected, since for reward structure I, the condition state of the structure, expressed as the condition rank, is taken as the reward rate (see Table 6). The performance measures are also computed using Eqs. (12), (13), (16) and (17). The evolution of expected instantaneous reward rate and COV of instantaneous reward rate for the three reward structures considered are shown in Fig. 9. The evolution of $E[Z(T_k)]$ and $E[Y(T_k)]$ with time for reward structure I are shown in Fig. 10. It is noted from Fig. 10 that $E[Y(T_k)]$ is less sensitive to the state of the girder at a particular instant T_k, when compared to $E[Z(T_k)]$, since $E[Y(T_k)]$ is time-averaged over $(0, T_k)$.

For reward structures I and III, it is noted that the coefficient of variation of $Z(T_k)$ increases with time, indicating the increase in uncertainty in condition state of the bridge girder with time.

But for reward structure II, the coefficient of variation of $Z(T_k)$ decreases with time. This is expected as time progresses, the condition of the bridge girder deteriorates and the need for maintenance increases. Thus, the uncertainty about the urgency of maintenance decreases.

It is assumed that a field inspection is carried out at 21 years. The information (cues) obtained from the inspection is given in Table 9. It is assumed that these information have already been corrected for the evaluation ability/human error (in the case of human observer) and for the detection capability and correctness of detection (in the case of instrument). This information (cues) is passed on to five experts, who have been asked to make judgments regarding the condition state and to assign confidence levels for their judgments from the confidence scale of I-VII.

9.1 Evaluation of the expert opinion

The expert opinions have been evaluated based on their performance in making judgment

Table 10 Results of condition state assessment made by Expert 1 (characterised in terms of ouc_i)

Confidence Level (i)	p_i	n_i	f_i	ouc_i
I	0.0	0	0	0
II	0.1	30	0	0.88
III	0.3	19	0.105	2.52
IV	0.5	16	0.438	0.68
V	0.7	22	0.682	0.27
VI	0.9	24	0.958	-0.95
VII	1.0	53	1.0	0

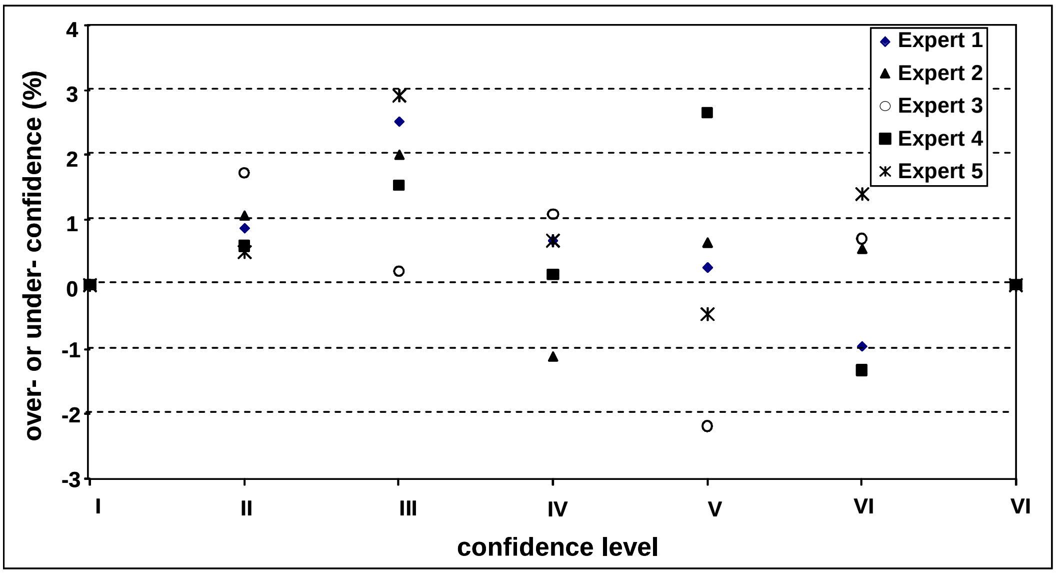

Fig. 11 The over- or under-confidence (ouc_i) corresponding to different confidence levels for the different experts

Table 11 Achievement index and weights for the experts

Expert (j)	r_{a_j}	w_j
1	0.93	0.215
2	0.90	0.208
3	0.85	0.197
4	0.81	0.188
5	0.83	0.192

regarding condition state for RC structural elements for which actual condition states are known.

One hundred baseline cases are assumed for this purpose. Five experts are considered in this study. All these five experts are considered to be equally informed and are equally capable in terms of qualification to assess the condition state. The information (cues) on the 100 structural elements (baseline cases) were passed on to these experts, who have been asked to make judgments regarding the condition state and to assign confidence levels for their judgments from the confidence scale of I-VII.

The over- or under-confidence (ouc_i) for the confidence judgment p_i for the different experts are determined. Detailed calculations for determining ouc_i have been carried out, and the results obtained for expert 1 are given in Table 10. The over- or under-confidence corresponding to the different confidence levels for the experts considered are shown in Fig. 11. The values of ouc_i reflect the probabilistic nature of mental process in judgmental decision making.

The achievement of the expert (defined by the achievement index, r_a), reflecting the correlation between the judgment and the environment, can be determined using the linear regression model for each expert. In the present study, it is assumed that such an analysis has been carried out and the values of r_a have been obtained for all the five experts. It is assumed that the experts have similar tendencies in viewing the environment and making judgments (viz. all having positive correlations). The values of weight attributed to the judgment of each expert are determined using Eq. (21). The values of achievement index and the weights for the five experts considered are given in Table 11.

9.2 Condition state assessment

The judgments regarding the condition state and corresponding confidence levels for these judgments given by all the five experts are presented in Table 12. Using these values, the condition state probabilities are determined (using Eq. (18)) and the state vector for the condition state combining the judgments of all the experts is obtained (using Eq. (20)). These probabilities are given in Table 13. The value of over- or under-confidence associated with the judgment of the each expert (EC_j) is computed using Eq. (23), and the over- or under-confidence associated with the condition state obtained by combining the judgments of all the experts (EC_c) is determined using Eq. (24) as 0.82% (the positive sign denoting over-confidence). The vale of EC_j is a measure of the confidence to be attached with the judgment of the j^{th} expert, while the value of EC_c is a measure of the confidence on the final condition state obtained by processing the judgments of all the experts considered. These measures will be useful in addressing the issues of dependability and trust essential for perception and communication of risk (Reid 1999).

Table 12 Experts' assessment of condition state and associated confidence level for the example problem

Condition State	confidence level				
	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5
CS7	V	VI	V	IV	VI
CS6	III	IV	II	IV	II

Table 13 Condition state probabilities based on experts' assessment

Condition State	Condition state probabilities					
	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Combined
CS7	0.7	0.64	0.87	0.5	0.9	0.723
CS6	0.3	0.36	0.12	0.5	0.1	0.277

(Note: condition state probabilities for the remaining condition states are zero)

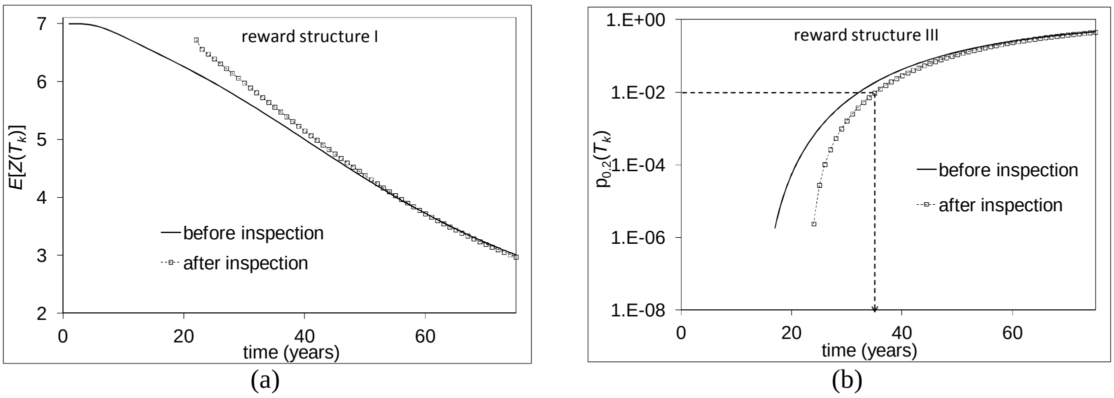

Fig. 12 Evolution of performance measures before and after considering inspection results: (a) $E[Z(T_k)]$ with reward structure I, (b) $p_{0.2}(T_k)$ with reward structure III

9.3 Performability analysis including inspection results

Using the condition state vector obtained using the judgments of the experts, the condition state vectors at different time instances after inspection are determined. The variation in the performance measures after including the inspection results are shown typically in Fig. 12(a)-(b). From these figures, it is noted that the effect of incorporating inspection results decreases with time. This is expected, since, when the damage is very high, the uncertainties about the condition states reduce.

9.4 Performance-based remaining life assessment

The reward structure III is considered in the present study for performance-based remaining life assessment. The remaining life is assumed as the time over which the quality of service (defined as the non-dimensionalized weight of the legal load rating vehicle that can be permitted on the bridge) offered by the bridge girder is greater than 0.2 with a probability of 0.99, i.e., the time over which $p_{0.2}(T_k)$ is less than 0.01. The variation of $p_{0.2}(T_k)$ with time is shown in Fig. 12(b).

From Fig. 12(b), it is noted that $p_{0.2}(T_k)$ becomes more than 0.01at about 35 years. Thus the remaining life of the bridge girder after the inspection at 21 years is 14 years. It is also noted from Figs. 12(a)-(b) that $p_{0.2}(T_k)$ is more sensitive to the inspection results as compared to $E[Z(T_k)]$.

10. Summary

A methodology for performance-based remaining life assessment of corrosion-affected reinforced concrete bridge girders is proposed. The methodology takes into consideration: 1) the safety and serviceability of the structural element, and, 2) uncertainties associated with expert judgment on condition state based on information from field inspections. The main features of the methdology are:

• use of Brunswikian theory for taking into consideration the uncertainties associated with human mental process in making judgment regarding condition state: the people involved in making judgment are considered to be rational, rather than classifying them as experts and non-experts, which is in line with the thinking in risk perception and risk communication (Reid 1999).

• use of performability measure for describing the performance of the bridge girder: the computation of performability is useful for making financial decisions by answering questions like 'How serviceable is the structure', and help in bridging the gap between where health monitoring stops and where owner's decisions begins, thus in completing the value chain as indicated by Wong and Yao (2001).

• use of NHMC for modelling the stochastic evolution of condition states with time

The usefulness of the methodology is illustrated through the remaining life assessment of a reinforced concrete T-beam bridge girder. The limited studies presented suggest that the methodology shows promise for remaining life assessment of corrosion-affected RC bridge girders.

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