

Probabilistic-based prediction of lifetime performance of RC bridges subject to maintenance interventions

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Abstract. In this paper, a probabilistic- and finite element-based approach to evaluate and predict the lifetime performance of reinforced concrete (RC) bridges undergoing various maintenance actions is proposed with the time-variant system reliability being utilized as a performance indicator. Depending on their structural state during the degradation process, the classical maintenance actions for RC bridges are firstly categorized into four types: Preventive type I, Preventive type II, Strengthening and Replacement. Preventive type I is used to delay the onset of steel corrosion, Preventive type II can suppress the corrosion process of reinforcing steel, Strengthening is the application of various maintenance materials to improve the structural performance and Replacement is performed to restore the individual components or overall structure to their original conditions. The quantitative influence of these maintenance types on structural performance is investigated and the respective analysis modules are written and inputted into the computer program. Accordingly, the time-variant system reliability can be calculated by the use of Monte Carlo simulations and the updated the program. Finally, an existing RC continuous bridge located in Shanghai, China, is used as an illustrative example and the lifetime structural performance with and without each of the maintenance types are discussed. It is felt that the proposed approach can be applied to various RC bridges with different structural configurations, construction methods and environmental conditions.

Keywords: RC bridges; lifetime performance; degradation process; maintenance actions; time-variant system reliability; finite element; Monte Carlo simulations

1. Introduction

Due to aging and environmental attacks, such as concrete carbonation or chloride penetration, the increasingly deteriorating structural performance of many in-service reinforced concrete (RC) bridges all over the world has become a major threat to their structural integrity. Thus, timely and effective maintenance interventions need to be carried out to keep the structures in a healthy condition during their service lives. When selecting a suitable maintenance strategy for a deteriorating RC bridge, one

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of the most critical issues is accurately forecasting the quantitative effects of the various maintenance actions on its structural performance. Additionally, the decision making process should be based on uncertainty, because of the inherent randomness involved in material properties, loads, construction, environmental conditions and maintenance actions. Therefore, a probabilistic-based approach, which can effectively predict the effects of various maintenance actions, is necessary for establishing efficient and economic maintenance plans for the different types of RC bridges that are deteriorating. There is a great deal of research on the lifetime performance assessment of RC structures that considers maintenance interventions (Estes and Frangopol 2001, Yang and Frangopol *et al.* 2004, Neves and Frangopol 2005, Neves and Frangopol *et al.* 2006, Neves and Frangopol *et al.* 2006, Yang and Frangopol *et al.* 2006, Yang and Frangopol *et al.* 2006, Okasha and Frangopol 2009). In (Neves and Frangopol 2005, Neves and Frangopol *et al.* 2006, Neves and Frangopol *et al.* 2006, Augusti and Ciampoli 2008), maintenance actions were classified into two groups: time-based and performance-based. Furthermore, several actions were taken into account: (1) silane treatment and the replacement of expansion joints were categorized as time-based; (2) minor concrete repairs and rebuilding were defined as performance-based; and (3) protection was classified as performance- and time-based. In (Okasha and Frangopol 2009), maintenance actions were categorized as preventive and essential, and two specific maintenance actions were investigated: painting and replacing bars. In (Yang and Frangopol *et al.* 2004, Yang and Frangopol *et al.* 2006, Yang and Frangopol *et al.* 2006), maintenance actions were classified into the same two types as (Okasha and Frangopol 2009), but the preventive actions were further divided into proactive and reactive. The objective of proactive actions was to delay any onset of damage (Kececioğlu 1995, Estes and Frangopol *et al.* 1998, Akgul 2013), while reactive actions were aimed at eliminating or reducing the effects of any deterioration. Most previous studies, to the best of the authors' knowledge, have accounted for the quantitative effects of various maintenance actions on structural performance in terms of some assumed and idealized mathematical formulae. For simple structure systems, the actual effects of the various maintenance actions may be expressed using such formulae. However, in the case of complex structure systems, it is difficult for these effects to be expressed as closed-form solutions and thus a finite element-based approach is required (Nasrellah and Manohar 2011).

A probabilistic and finite element-based approach is proposed in this paper so that the effects of various maintenance actions on structural performance with respect to deteriorating RC bridges can be precisely evaluated. Furthermore, the time-variant system reliability, which is an effective method of comprehensively reflecting the time-dependent and probabilistic structural behavior throughout a bridge's service life, is used as a performance indicator throughout the paper. Depending on their structural state during the degradation process, the classical maintenance actions for RC bridges are firstly categorized as four types: Preventive type I, Preventive type II, Strengthening and Replacement. The quantitative effects of these maintenance types on a bridge's structural performance are investigated and the respective analysis modules are written and inputted into the computer program-Concrete Bridge Durability Analysis System (CBDAS), which was developed by the author (Tian 2009). Consequently, the time-variant system reliability can be calculated by the combination of Monte Carlo simulations and the updated CBDAS. Finally, an existing RC continuous girder bridge located in Shanghai, China is used as an illustrative example. The lifetime structural performance with and without each of the maintenance types is discussed in terms of the time-variant system reliability. It is felt that the approach proposed in this paper can be applied to various RC bridges with different structural configurations, construction methods and environmental conditions.

2. Maintenance actions

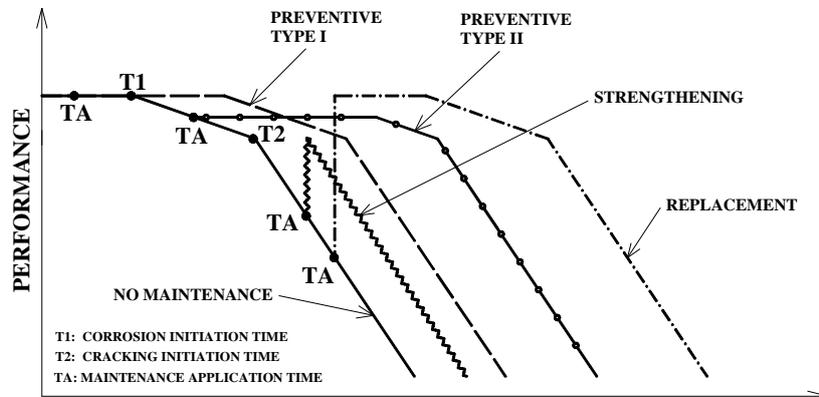


Fig. 1 The effects of the various maintenance types

In general, maintenance actions are categorized as preventive and essential (Okasha and Frangopol 2009). The purpose of the former is to delay a performance indicator's threshold value being reached by slowing down the rate of deterioration or preventing deterioration in performance (Tabsh and Nowak 1991, Nowak 2000, Frangopol and Miyake *et al.* 2002, Beaurepaire and Jensen *et al.* 2013); whereas the latter is commonly planned to improve the performance indicator when its threshold value is reached. In this study, based on the bridge's structural state during its service life, the maintenance actions usually employed on deteriorating RC bridges are classified into four types: Preventive type I, Preventive type II, Strengthening, and Replacement. Preventive type I is utilized before the onset of corrosion in the reinforcing steel and can delay the start of any corrosion. Preventive type II, which is commonly used when the reinforcing steel has corroded, can sufficiently suppress the corrosion process over a period of time. Strengthening can improve a bridge's structural performance by using various maintenance materials when it has deteriorated significantly (e.g., a threshold value is about to be reached). Replacement is utilized to restore the individual components or the overall structure to their original states by replacing them if the cost of other maintenance types is too high or if it will not be possible to maintain the structure in a safe condition. The qualitative influence of the four maintenance types is illustrated in Fig. 1. In order to accurately evaluate the quantitative influence of these maintenance types, it is necessary to investigate their effects on structural resistances (e.g., bending capacity and shear capacity) and load effects (e.g., bending moment and shear force).

2.1 Preventive type I

The objective of Preventive type I is to delay the onset of corrosion in the reinforcing steel. Specifically, silane treatment is a typical maintenance action of this type and is often used in maintaining RC structures in the United Kingdom (Neves and Frangopol *et al.* 2006). It is carried out by the application of a thin coating, which is sprayed on the concrete surface and results in a decrease of the concrete's permeability. As a result, the penetration rate of chloride ions into concrete is slowed, leading to a delay in the onset of steel corrosion. According to the above description, it is clear that this maintenance type does not lead to an improvement in structural performance but merely delays the reduction of structural resistance. Thus, when evaluating the effect of Preventive type I on lifetime structural performance by using CBDAS (Tian 2009), the only factor that needs to be taken into account is the modification of the three critical times (i.e., corrosion initiation time t_1 , cracking initiation time t_2 and concrete cover spalling time t_3) during the degradation process (i.e., adding the delayed period to the original critical times). The delayed period ΔT_1 is defined as a random variable

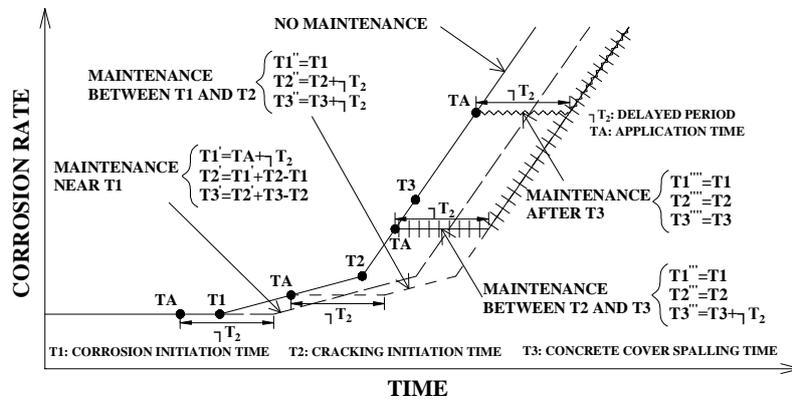


Fig. 2 The effects of Preventive type II on the critical times

in this study. Note that the corrosion initiation times of concrete edges in the same section may be not identical due to the different values of design parameters, such as the concentration of chloride, concrete cover thickness, and reinforcing steel diameter. For the purposes of this paper, this maintenance action only needs to be applied to those edges where the corrosion initiation times are shorter than the prescribed service life. This is very useful in reducing the total maintenance cost.

2.2 Preventive type II

When Preventive type II is utilized, the corrosion process of reinforcing steel can be suppressed for a period of time. Cathodic protection and painting are two typical maintenance actions that slow down the corrosion process. When the former is applied, anodes are replaced at regular time intervals and the application of cathodic protection and timely replacement of anodes almost completely suppress the corrosion of reinforcing steels, which consequently leads to a delay of the corrosion process. If the latter is used, the paint or coating completely prevents the corrosion of reinforcing steels, and therefore can also delay the corrosion process over a period of time. As with Preventive type I, there is no improvement in structural performance and the reduction of structural resistance is only delayed by using this maintenance type. Accordingly, the only factor that needs to be considered is the modification of the three critical times in relation to the maintenance application time TA and the delayed period ΔT_2 . The effects of this maintenance type on the critical times, with different maintenance application times, are shown in Fig. 2. In this study, ΔT_2 and TA are assumed to be a random variable and a deterministic parameter, respectively. Similar to Preventive type I, Preventive type II should be performed on the designated edges of a concrete section when the corrosion initiation times are shorter than the prescribed service life.

2.3 Strengthening

Strengthening is used to improve the bridge's structural performance by increasing its structural resistance with various maintenance materials when the structural deterioration is significant and the performance thresholds have been reached. A bonding steel plate or FRP (Fiber Reinforced Plastic) is commonly employed in the concrete section to increase the resistance of deteriorating RC bridges in China (2008). The steel plate or FRP is bonded onto the tension zone and (or) the web zone of the concrete section, which means that the bending capacity and (or) the shear capacity of the original component can be increased and thus the structural performance is improved immediately.

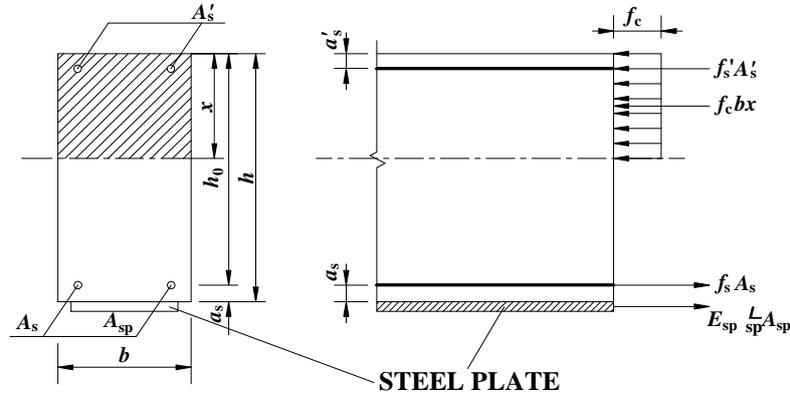


Fig. 3 The calculation of bending capacity with a steel plate

Furthermore, strengthening not only improves structural resistance but also has an impact on the structural load effect. The approaches for considering the effects of this maintenance type on structural resistance and load effect are now discussed, with both the steel plate and FRP being considered.

2.3.1 Improvement of resistance

A steel plate can be bonded onto the tension zone of a concrete section to increase bending resistance when the deterioration of a bridge's structural bending capacity is significant. The bending capacity of a rectangular reinforced concrete section after the bonding of a steel plate can be expressed as in Fig. 3 (2008).

$$R_b = \left(f_c b x \left(h_0 - \frac{x}{2} \right) + f_s' A_s' (h_0 - a_s') + E_{sp} \epsilon_{sp} A_{sp} a_s \right) \times 10^3 \quad (1)$$

Furthermore, the height of the concrete compression zone can be determined by (2008)

$$f_c b x = f_s A_s + E_{sp} \epsilon_{sp} A_{sp} - f_s' A_s' \quad (2)$$

where R_b = the bending capacity (kNm), f_c = the concrete compressive stress (MPa), b = the width of the concrete section (m), x = the height of the equivalent rectangular concrete compression zone (m), h_0 = the effective height of the concrete section (m), f_s and f_s' = the tensile and compressive strength of reinforcing steel, respectively (MPa), A_s and A_s' = the reinforcing steel areas in the tension and compression zones of the concrete section, respectively (m^2), a_s and a_s' = the distances between the centroids of reinforcing steels and the concrete edges in the tension and compression zones of the concrete section, respectively (m), E_{sp} = the elastic modulus of the steel plate (MPa), A_{sp} = the section area of the steel plate (m^2), and ϵ_{sp} = the tensile strain of the steel plate when the limit state is reached, which can be calculated by (2008)

$$\epsilon_{sp} = \frac{\epsilon_{cu} (0.8h - x)}{x} - \frac{\epsilon_{c1} (h - x_1)}{x_1} \quad (3)$$

in which ϵ_{cu} = the ultimate compressive strain of concrete, ϵ_{c1} = the compressive strain of concrete induced by the loads before maintenance, h = the height of the concrete section (m), and x_1 = the height of the equivalent rectangular concrete compression zone before maintenance (m).

If $x < 2a_s'$, (i.e., the reinforcing steels in the compression zone cannot reach their compressive

strengths when the limit state is reached), it is assumed that the centroid of the compressive concrete section is also the centroid of reinforcing steels in the compression zone. Thus, the bending capacity can be approximately computed by (2008)

$$R_b = f_s A_s (h_0 - a'_s) + E_{sp} \varepsilon_{sp} A_{sp} (h - a'_s) \quad (4)$$

Eqs. (1)-(4) are employed to calculate the bending capacity of the rectangular section. In the case of a T-section and box section, only a few modifications to these equations are needed and are they not explained herein.

When the deterioration of the structural shear capacity is significant, the steel plate can also be bonded onto the web zone of the concrete section to increase the shear resistance. The shear capacity of a reinforced concrete section after the bonding of a steel plate can be expressed as (2008)

$$R_s = 0.43 \times 10^{-3} \alpha_1 \alpha_3 b h_0 \psi_{cs} \sqrt{(2 + 0.6P)} \sqrt{f_{cu,k} \rho_{sv} f_{sv}} + 0.75 \times 10^{-3} f_s \sum A_{sb} \sin \theta_b + \psi_{vb} V_{d2} \quad (5)$$

where R_s = the shear capacity (kN), α_1 = the influence coefficient of contrary sign moments, $\alpha_1 = 1.0$ for simply supported bridges and the individual components on the near side abutments of continuous bridges, whereas $\alpha_1 = 0.9$ for the individual components near the middle abutments of continuous and cantilever bridges, α_3 = the influence coefficient of the compression flange, $\alpha_3 = 1.0$ for the rectangular section, whereas $\alpha_3 = 1.1$ for the T-section, I-section and box section, ψ_{cs} = the modified coefficient when considering the diagonal cracks in the original structure that has not undergone maintenance, P = the ratio of the longitudinal reinforcing steel in the concrete section, $f_{cu,k}$ = the concrete compressive grade strength (MPa), ρ_{sv} = the ratio of the stirrup in the concrete section, f_{sv} = the tensile strength of the stirrup (MPa), $\sum A_{sb}$ = the area of all the bent-up reinforcing steels that are intersected with diagonal cracks (mm^2), θ_b = the included angle between the tangent of bent-up reinforcing steels and the horizontal line, V_{d2} = the shear force induced by the loads after maintenance (kN), and ψ_{vb} = the modification coefficient that can be computed by (2008)

$$\psi_{vb} = \frac{0.8 A_{spv} E_{sp}}{A_{sv} E_{sv} + 0.707 A_{sb} E_{sb} + A_{spv} E_{sp}} \quad (6)$$

in which A_{spv} = the area of the bonded steel plate in the web zone (mm^2), A_{sv} = the area of all the stirrups in a concrete section (mm^2), E_{sv} = the elastic modulus of the stirrup (MPa), A_{sb} = the area of the bent-up reinforcing steels, and E_{sb} = the elastic modulus of bent-up reinforcing steel (MPa).

It can be seen in Eq. (5) that the total shear capacity R_s is composed of three sections: (1) the comprehensive shear resistance induced by the concrete and stirrups; (2) the shear resistance derived from the bent-up reinforcing steels; and (3) the shear resistance caused by the bonded steel plate. When compared with the mathematical formula that is related to shear capacity without maintenance, the first section is modified and the third section is a new addition. For the first section, the modified coefficient ψ_{cs} is introduced to consider the effect of the diagonal cracks in the web zone of a concrete section, before maintenance, on shear capacity. For the third section, shear force V_{d2} is assumed to be sustained by the stirrups, the bent-up reinforcing steels and the bonded steel plate, which are allocated according to their sectional areas. The shear force distributed on the bonded steel plate is assumed to be the additional shear capacity. Note that concrete is not involved in the allocation and thus 0.8 is used to modify ψ_{vb} .

When the loss of structural bending capacity is evident, bonding FRP onto the concrete tension zone is a feasible maintenance action that improves structural resistance. When the bending capacity of a maintained reinforced concrete section with FRP is calculated, two failure modes should be taken into account (see Fig. 4) (2008).

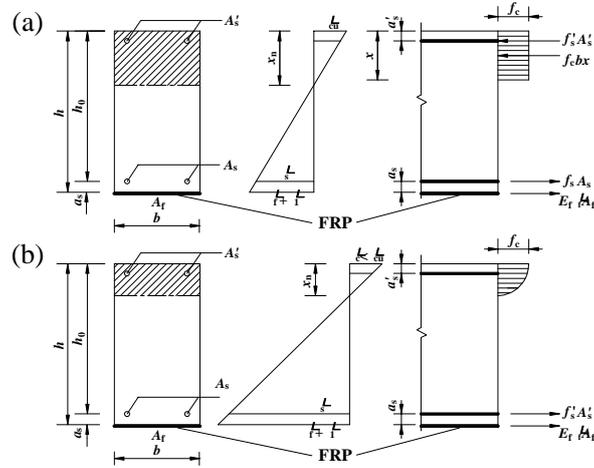


Fig. 4 The calculation of bending capacity with FRP: (a) failure mode 1, and (b) failure mode 2

The first failure mode (Fig. 4(a)) can be interpreted as occurring when the tensile steels initially yield and the concrete in the compression zone is thereafter crushed. In this case, the allowable tensile strain of FRP cannot be reached. Furthermore, there is a condition that $x > \xi_{fb} h$ should be satisfied, in which ξ_{fb} is the relative critical height of the concrete compression zone when FRP reaches its allowable tensile strain and the concrete is crushed simultaneously. The bending capacity can be expressed as (2008)

$$R_b = \left(f_c b x \left(h_0 - \frac{x}{2} \right) + f_s' A_s' (h_0 - a_s') + E_f \epsilon_f A_f a_s \right) \times 10^3 \tag{7}$$

Furthermore, the height of the concrete compression zone and tensile strain of FRP can be calculated by (2008)

$$f_c b x + f_s' A_s' = f_s A_s + E_f \epsilon_f A_f \tag{8}$$

$$(\epsilon_{cu} + \epsilon_f + \epsilon_1) x = 0.8 \epsilon_{cu} h \tag{9}$$

where E_f = the elastic modulus of FRP (MPa), ϵ_f = the tensile strain of FRP when the bending capacity threshold is reached, A_f = the section area of FRP (m^2), and ϵ_1 = the initial strain of the concrete at the tensile edge induced by the loads before maintenance.

When $x \leq \xi_{fb} h$, the second failure mode appears (Fig. 4(b)), which can be characterized as occurring when the tensile steels initially yield and the allowable tensile strain of FRP is subsequently reached. However, the concrete in the compression zone is not crushed at this point. The bending capacity can be computed by (2008)

$$R_b = f_s A_s (h_0 - 0.5 \xi_{fb} h) + E_f \epsilon_f A_f (h - 0.5 \xi_{fb} h) \tag{10}$$

Additionally if the height of the concrete compression zone further reduces to $x < 2a_s'$, the bending capacity can be calculated by (2008)

$$R_b = f_s A_s (h_0 - a_s') + E_f \epsilon_f A_f (h - a_s') \tag{11}$$

As with the steel plate, Equations (7)-(11) are performed to calculate the bending resistance of the

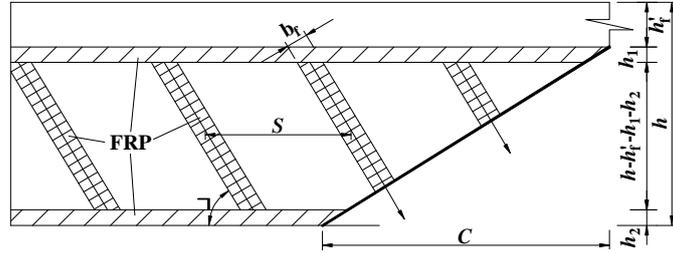


Fig. 5 The calculation of the shear capacity with FRP

rectangular section. In the case of the T-section and box section, it is only necessary to make a few modifications to these equations that are not explained herein.

When FRP is used to improve the shear resistance, the shear capacity can be formulated as in Fig. 5. (2008)

$$R_s = 0.43 \times 10^{-3} \alpha_1 \alpha_3 b h_0 \psi_{cs} \sqrt{(2 + 0.6P)} \sqrt{f_{cu,k} \rho_{sv} f_{sv}} + 0.75 \times 10^{-3} f_s \sum A_{sb} \sin \theta_b + V_f \quad (12)$$

$$V_f = D_{sh} \kappa_m f_f n_f t_f b_f \frac{C - C_1}{S} \sin \alpha \times 10^{-3} \quad (13)$$

$$D_{sh} = 1 - \frac{L_e}{h - h'_f - h_1} \sin \alpha \quad (14)$$

$$L_e = \sqrt{\frac{E_f n_f t_f}{\sqrt{1.18} f_c}} \quad (15)$$

$$C_1 = \frac{C(h_1 + h_2)}{h - h'_f} \quad (16)$$

in which, V_f = the additional shear capacity after the maintenance when FRP has been used on the concrete web zone (kN), D_{sh} = the distribution coefficient of FRP stress, κ_m = the reduction coefficient of FRP strength, f_f = the tensile strength of FRP (MPa), n_f = the number of layers of FRP, t_f = the thickness of the FRP banding (mm), b_f = the width of the FRP banding (mm), C = the horizontal projected length of the concrete diagonal crack (mm), L_e = the effective bonded length of FRP (mm), h'_f = the distance between the top of the section and the upper anchorage zone of FRP (mm), h_1 = the width of the FRP layer in the upper anchorage zone (mm), h_2 = the width of the FRP layer in the lower anchorage zone (mm), α = the included angle between the forced direction of FRP and the longitudinal direction ($\leq 90^\circ$), and S = the distance between the two adjacent FRP bandings (mm), which should be less than S_{max} that is provided by (2008)

$$S_{max} = \frac{h - h'_f - h_1}{2 \tan \alpha} \quad (17)$$

The total shear capacity R_s shown in Eq. (12) is composed of three sections. The first two sections are derived from the original structure and the third section is provided by FRP. The non-uniform stress

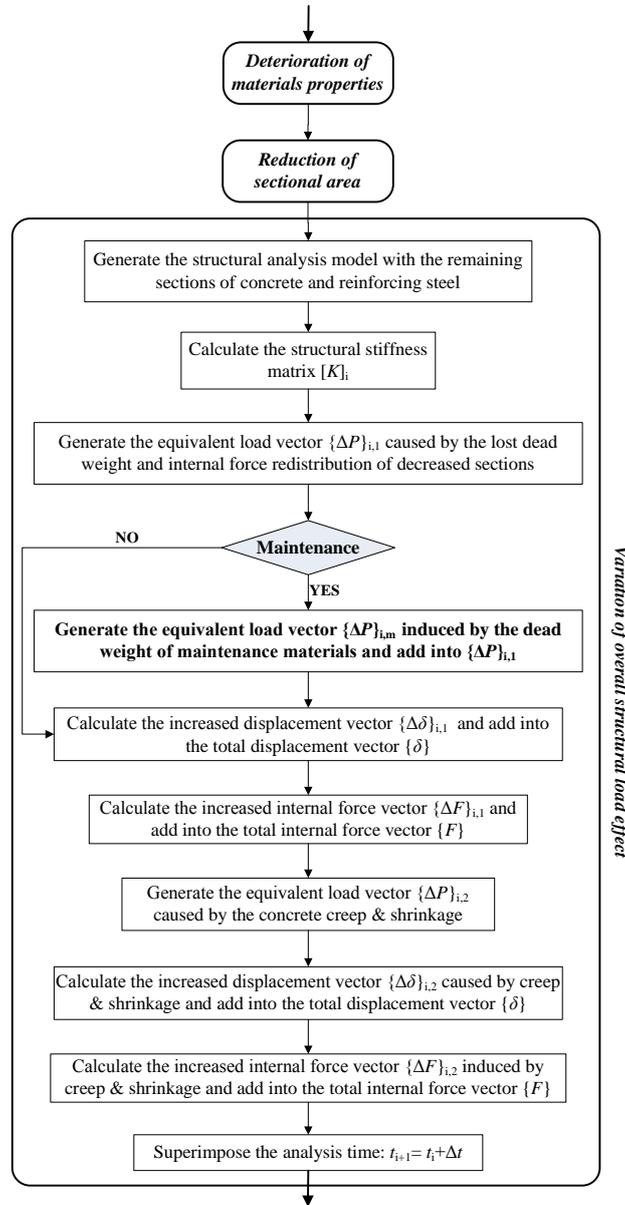


Fig. 6 The schematic for evaluating the lifetime load effect with Strengthening

distribution among the FRP bandings is taken into account, and consequently, coefficients D_{sh} and κ_m are introduced. To guarantee the effectiveness of the bonded FRP, the distance between two adjacent FRP bandings S should be less than half of the horizontal projected length of the concrete diagonal crack. In this case, at least two FRP bandings can be intersected with the concrete diagonal crack and one FRP banding is effective under any circumstances.

2.3.2 The influence on the load effect

Table 1 The random variables involved in strengthening

Maintenance material	Variable	Parameter description
Steel plate	E_{sp}	Elastic modulus
	A_{sp}	Area in tension zone
	f_{sp}	Tensile strength
	A_{spv}	Area in web zone
	V_{d2}	Shear force after maintenance
	γ_{sp}	Unit weight
FRP	E_f	Elastic modulus
	A_f	Area in tension zone
	f_f	Tensile strength
	$[\epsilon_f]$	Allowable tensile strain
	γ_f	Unit weight

Table 2 The maintenance types and their associated effects on performance

Maintenance type	Action	Structural state	Effect on performance
Preventive type I	Silane treatment	The onset of corrosion not reached	Delay the corrosion initiation time
Preventive type II	Cathodic protection Painting	Reinforcing steel has corroded	Suppress the corrosion process
Strengthening	Steel plate FRP	Structural performance deteriorates significantly	Improve the structural resistance
Replacement	Component Overall structure	The limit state is about to be reached	Restore to the original state

The load effect of an overall structure due to the bonded steel plate or FRP mainly stems from: (1) the dead weight of maintenance materials; and (2) the internal force redistribution of the combined section. The effect of the second ingredient on structural performance is usually advantageous, as the maintenance material can share a portion of the internal force that originally impacted on the structure without any maintenance. Thus, only the first problem is given careful consideration in this study. In the CBDAS program (Tian 2009), the analysis module which considers the load effect caused by the dead weight of maintenance materials is written and added into subsystem 4 (degradation process analysis). Fig. 6 shows the schematic for evaluating the lifetime load effect of this maintenance type.

When this maintenance type is used, there are some associated design parameters that are regarded as random variables and these are listed in Table 1. The application time of maintenance action TA is assumed to be a deterministic parameter.

2.4 Replacement

If the damage to a RC bridge is so significant that the structural performance improvements brought about by the above three maintenance types are insufficient or the costs are too high, Replacement can be used. When this maintenance type is performed, either some apparently decayed individual

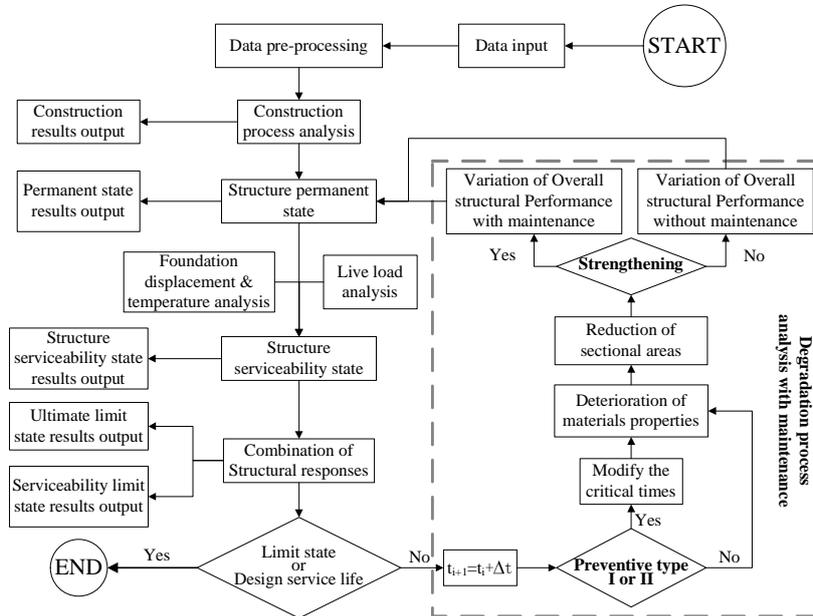


Fig. 7 The analysis procedure of the updated CBDAS

components or the overall structure can be replaced (Okasha and Frangopol 2009) or restored to their original states. In this maintenance type, the design variable that should be considered is the maintenance application time TA , which is assumed to be a deterministic parameter.

The four maintenance types and their associated effects on structural performance are summarized in Table 2.

3. Time-variant system reliability with maintenance

Based on the abovementioned simulation approaches, the analysis modules for considering the effects of the various maintenance types on structural performance are written in FORTRAN 95 and added to the computer program CBDAS(Tian 2009). The analysis procedure of the updated CBDAS is demonstrated in Fig. 7.

The time-variant system reliability with maintenance can be calculated by the following steps: (1) evaluate the lifetime performance of deteriorating RC bridges under the maintenance interventions that are associated with the updated CBDAS and Monte Carlo simulations, based on which the time-variant resistances $R(t)$ and load effect $S(t)$, with respect to the individual components, can be obtained; (2) perform the statistical analysis with respect to the simulation results of $R(t)$ and $S(t)$ and obtain their distribution types and statistical descriptors (i.e., mean value and coefficient of variation); and (3) calculate the time-variant system reliability according to the distribution types, the statistical descriptors of $R(t)$ and $S(t)$ and the structure failure mode that is sufficient to reflect the relationship between the individual components and the overall structure (Estes and Frangopol *et al.* 1998, Estes and Frangopol 1999, Nowak 2000). Fig. 8 shows the schematic for computing the time-variant system reliability with maintenance. In this study, distribution fitting, which is a procedure for selecting the most appropriate distribution type for the simulation result, is performed to obtain the best-fit distributions with the Kolmogorov-Smirnov test (Corder and Foreman 2009).

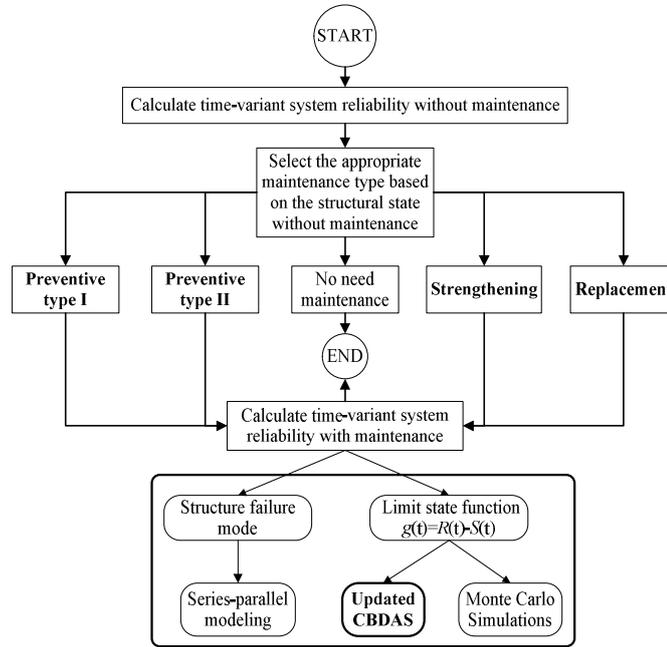


Fig. 8 The schematic for computing time-variant system reliability with maintenance

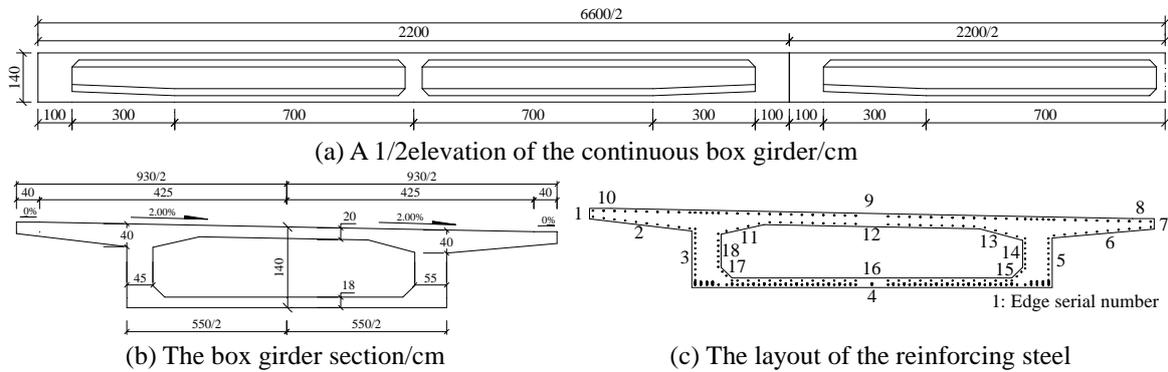


Fig. 9 The profile of the reinforced concrete continuous bridge

4. An illustrative example

4.1 An introduction to the case study

The effects of the various maintenance types on an existing RC bridge located in Shanghai, China, are investigated by using the proposed approach. The case study bridge, built in 2007, is a 3×22m three-span reinforced concrete continuous box girder bridge as shown in Fig. 9. The chloride-induced corrosion of the reinforcing steel is considered herein, since the case study bridge is near the East China Sea and thus the chloride ion concentration in the atmosphere may be high. The details of this example are demonstrated in five cases. Case 1 shows the variation of structural performance without any maintenance interventions. Case 2 demonstrates the effect of Preventive type I on the lifetime structural

Table 3 The statistical parameters of random variables

Variable	Nominal	Mean	COV ^a	Distribution type	Variable description
E_c	34,500MPa	34,500MPa	0.10	Normal ^b	Concrete elastic modulus
f_c	32.4MPa	40.5MPa	0.12	Normal ^b	Concrete compressive strength
E_s	200,000MPa	200,000MPa	0.06	Normal ^b	Reinforcing steel elastic modulus
f_s	335MPa	375MPa	0.065	Normal ^b	Reinforcing steel tensile strength
C	C_{nom}^c	$1.02C_{nom}$	0.05	Normal ^b	Concrete cover thickness
d_s	$d_{s,nom}^d$	$1.0d_{s,nom}$	0.02	Normal ^b	Reinforcing steel diameter
γ_c	25kN/m ³	26kN/m ³	0.06	Normal ^b	In-situ concrete unit weight
γ_s	78.5kN/m ³	80.1kN/m ³	0.05	Normal ^b	Reinforcing steel unit weight
γ_a	23kN/m ³	23kN/m ³	0.04	Normal ^b	Deck paving unit weight
Q	10.5kN	9.03kN	0.08	Extreme value type I	Uniform live load
P	250kN	215kN	0.08	Extreme value type I	Concentrated live load
M_s	6.9kg/m ³	6.9kg/m ³	0.1	Normal ^b	Chloride concentration at surface
M_{cr}	1.5kg/m ³	1.5kg/m ³	0.15	Normal ^b	Critical chloride concentration
D_c	22.5mm ² /year	22.5mm ² /year	0.45	Lognormal	Diffusion coefficient
ΔT_1	40 years	40 years	0.125	Normal ^b	Delayed time of Preventive type I
ΔT_2	40 years	40 years	0.125	Normal ^b	Delayed time of Preventive type II
E_{sp}	200,000MPa	200,000MPa	0.06	Normal ^b	Steel plate elastic modulus
f_{sp}	300MPa	336MPa	0.065	Normal ^b	Steel plate tensile strength
A_{sp}	0.015m ²	0.015m ²	0.0125	Normal ^b	Steel plate area in the tension zone
A_{spv}	0.015m ²	0.015m ²	0.0125	Normal ^b	Steel plate area in the web zone
γ_{sp}	78.5kN/m ³	80.1kN/m ³	0.05	Normal ^b	Steel plate unit weight
V_{d2}	$V_{d2,com}^e$	$V_{d2,com}^e$	0.1	Normal ^b	Shear force after maintenance
E_f	140,000MPa	140,000MPa	0.06	Normal ^b	FRP elastic modulus
f_f	1700MPa	1785MPa	0.065	Normal ^b	FRP tensile strength
A_f	0.005m ²	0.005m ²	0.0125	Normal ^b	FRP area in tension zone
γ_f	17.5kN/m ³	17.85kN/m ³	0.05	Normal ^b	FRP unit weight
$[\epsilon_f]$	0.007	0.007	0.1	Normal ^b	FRP allowable tensile strain

a. COV is the coefficient of variation;
 b. Truncated distributions with non-negative outcomes are adopted in the simulation process;
 c. C_{nom} is the cover thickness of any concrete edge in the section;
 d. $d_{s,nom}$ is the reinforcing steel diameter of any concrete edge in the section;
 e. $V_{d2,com}$ is computed by CBDAS.

performance. Two subcases are investigated, in which the maintenance action is used on all and some of the concrete edges. Case 3 introduces the effect of Preventive type II. Four subcases are considered, in which the application times of maintenance are not identical. Case 4 shows the influence of strengthening and a steel plate and FRP are both taken into account. Finally, the effect of Replacement is studied in case 5. A summary of the statistical descriptors of the random variables involved in the structural configuration, materials properties, live loads, environmental conditions and various maintenance types are provided in Table 3; these have been obtained from structural drawings and certain references (Tabsh and Nowak 1991, Nowak and Yamani *et al.* 1994, 1999, Nowak and Szerszen

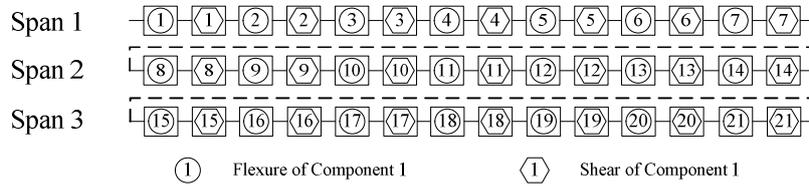


Fig. 10 The structure failure mode

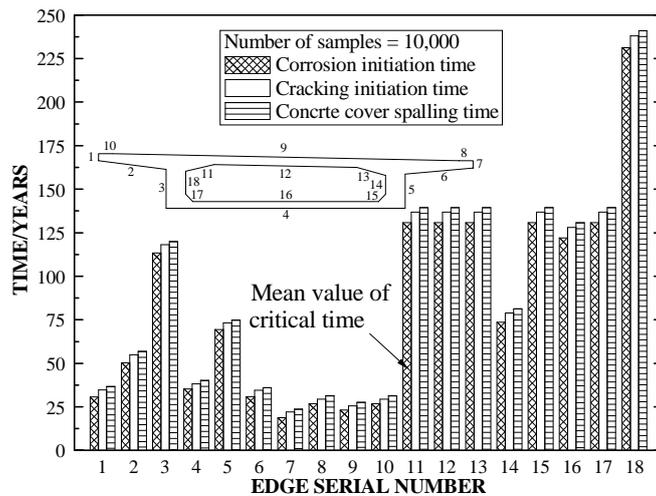


Fig. 11 The mean values of the critical times without maintenance

2003, 2004, 2004, 2008). Note that the chloride ion concentrations at different surfaces of the concrete section may not be identical. Accordingly, the following assumptions are used here in: (1) the chloride ion concentrations on the surfaces of concrete edges 5, 6 and 7 (Fig. 9(c)) that face the sea are 100% of the value in Table 3; (2) the chloride ion concentrations on the surfaces of concrete edges 1, 2 and 3 (Fig. 9(c)) that do not face the sea are 70%; (3) the chloride ion concentrations on the surfaces of concrete edges 4, 8, 9 and 10 (Fig. 9(c)) are 85%; and (4) the chloride ion concentrations on the surfaces of interior concrete edges are assumed to be half of the values of those on the surfaces of the related exterior concrete edges. In the following results, the prescribed service life of the bridge and unit calculating time periods are taken as 100 years and 10 years, respectively. The number of samples in the Monte Carlo simulations is defined as 10,000. Statistical independence is assumed between the components' resistances and load effects as well as among the resistances. Finally, the correlation coefficients of the load effects between two adjacent components are assumed to be 0.5.

4.2 The structure failure mode

Based on the structure's decomposition technique developed by the authors (Tian 2009), the case study bridge is divided into 21 components, with 7 components in each span. Only the ultimate limit state is discussed in this study, as the structural performance at the ultimate limit state is more likely to deteriorate significantly during the service life, mainly due to the area loss of the reinforcing steel. In each component, two failure modes, the flexure failure and the shear failure, are taken into account. To evaluate the structural system reliability, the structure failure mode of this case study is defined and a series system is considered as shown in Fig. 10. In this failure mode, the exceedance probability

Table 4 The distribution types of bending moments

Time/years Component	0	10	20	30	40	50	60	70	80	90	100
1	W	W	W	W	W	W	W	W	W	W	W
2	N	N	N	N	N	N	N	N	N	N	N
3	N	N	N	N	N	N	N	N	N	N	N
4	N	N	N	N	N	N	N	N	N	N	N
5	W	W	W	W	N	N	N	W	W	W	W
6	N	N	N	N	N	N	N	G	LN	LN	W
7	\overline{N}^*	N	N	\overline{N}	N	N	N	\overline{G}	G	LN	\overline{LN}
8	N	N	N	N	N	N	N	G	G	LN	LN
9	N	N	N	N	N	N	N	N	LN	LN	LN
10	W	W	W	W	W	W	G	G	G	G	G
11	W	W	W	W	W	W	W	W	W	W	W

*: A detailed description is shown in Fig. 12.

$P(R < S)$ represents the probabilities that the load effect of any individual component exceeds its resistance.

4.3 Simulation results

4.3.1 No maintenance

Fig. 11 shows the mean values of the three critical times with respect to all concrete edges of the middle-span-center section in the case study bridge when no maintenance takes place. It is clear that the critical times of the different edges in the same concrete section are not identical due to the different values of various design parameters, such as: the concrete cover thickness, the diameter of reinforcing steel, and the chloride ion concentration. The shortest and longest critical times appear in edge 7 and 18, respectively, which result from the highest (lowest) chloride ion concentration, the thinnest (thickest) concrete cover depth, and the largest (smallest) reinforcing steel diameter. In addition, the two time intervals between the critical times of all the concrete edges are very short, which denotes that the corrosion rate of the reinforcing steel under chloride penetration is very fast.

The best-fit distribution types of the time-variant bending moments of the components are summarized in Table 4. Only half of the components are listed due to the structural symmetry. Different probability distribution types appear in the table and a similar phenomenon is also found in the simulation results of resistance. Accordingly, the following conclusions can be made: (1) the resistances and load effects of the individual components possibly follow the different distribution types at certain points in time; and (2) for the same component the distribution types may be not identical at different points in time. Fig. 12 shows the histograms and associated best-fit distributions of the bending moment of component 7 at 0, 30, 70, and 100 years. The best-fit distribution patterns at the four points in time are Normal, Normal, Gamma and Lognormal, respectively.

The time-variant critical components and system reliabilities without maintenance interventions are shown in Fig. 13. The minimum component reliability for flexure $\beta_{min,2}$ is smaller than that for shear $\beta_{min,1}$ throughout the service life, and thus the system reliability is consistently dominated by $\beta_{min,2}$ during the service life. The structural system reliability is about 5.8 at the beginning of the service life; however, it decreases to a negative value at the end due to the significantly deteriorated structural

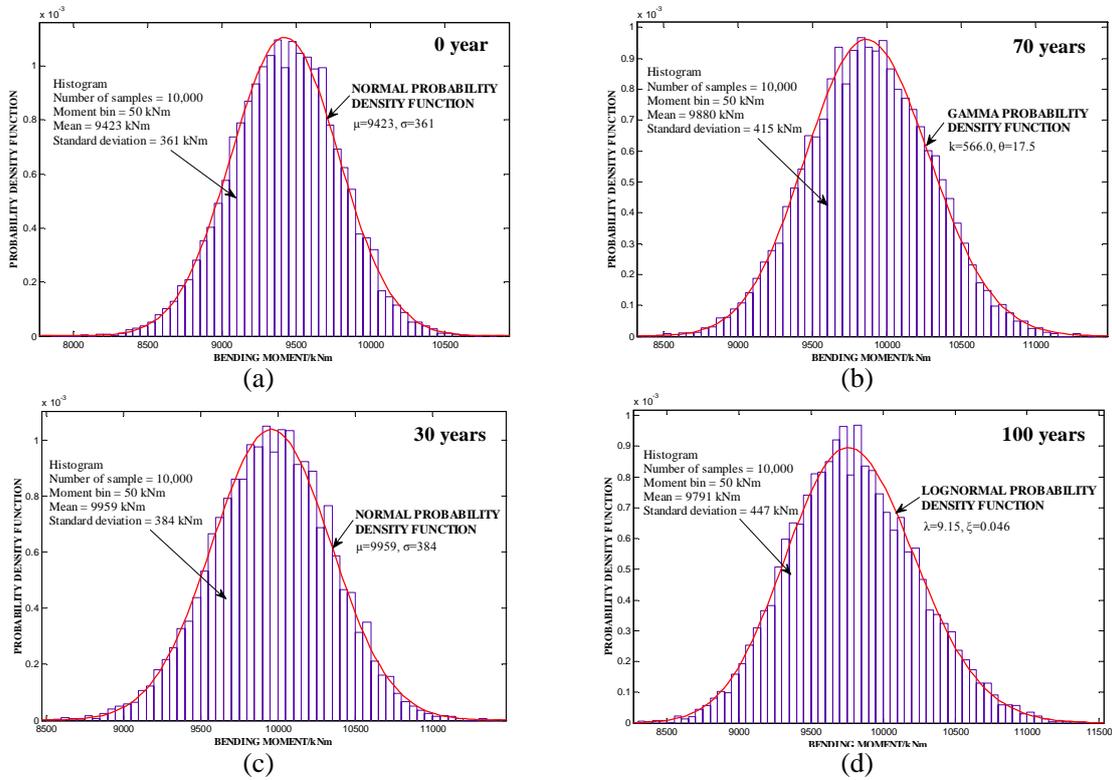


Fig. 12 The histograms and associated PDFs of bending moments in component 7

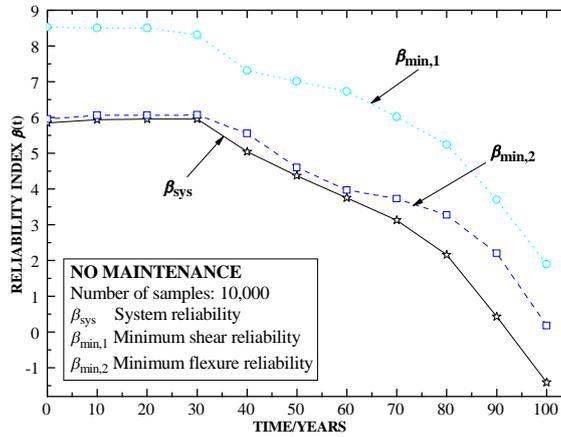


Fig. 13 The variation of the components and system reliabilities without maintenance

performance resulting from the chloride-induced corrosion. Therefore, effective maintenance interventions are required to improve the structural performance or reduce its deterioration rate.

4.3.2 Maintenance with Preventive type I

The delayed time period ΔT_1 should be determined by the volume of maintenance material. Due to a

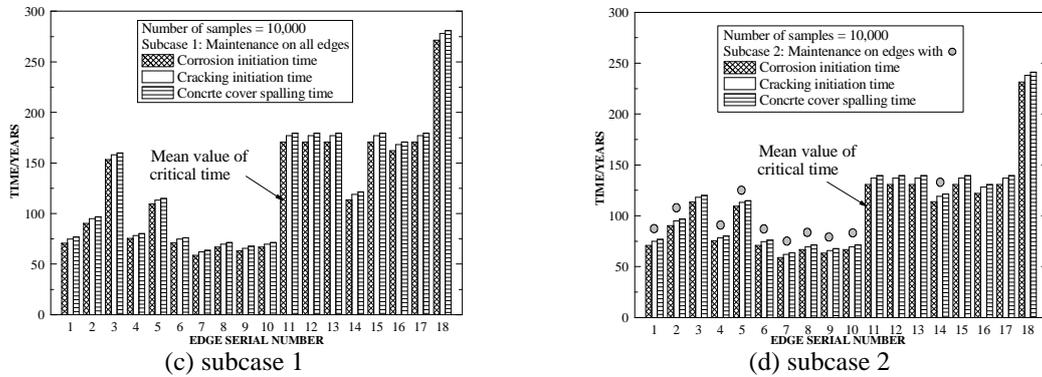


Fig. 14 The mean values of the critical times with Preventive type I

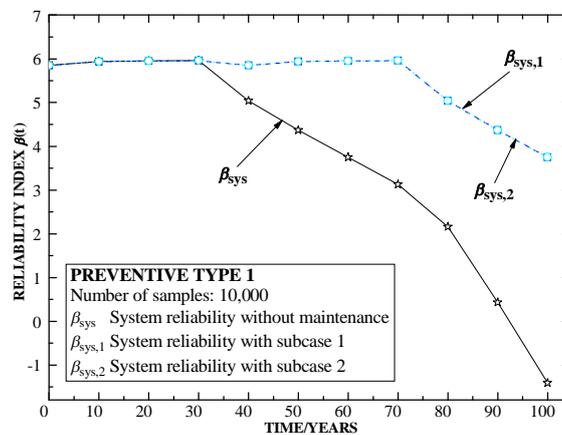


Fig. 15 The variation of system reliabilities with no maintenance or Preventive type I

lack of actual data, ΔT_1 is assumed to follow a normal distribution in this case, with the mean value and standard deviation equal to 40 and 5 years, respectively. Two subcases are considered: (1) applying the maintenance action on all the concrete edges; and (2) applying the maintenance action on the concrete edges where the first critical times are shorter than 100 years (i.e., the prescribed service life). Fig. 14 shows the mean values of the three critical times with respect to all the concrete edges of the middle-span-center section with this maintenance type. In Fig. 14(a) (i.e., subcase 1), the delayed time period ΔT_1 is added to the critical times of all the concrete edges, while in Fig. 14(b) (i.e., subcase 2), the delayed time period ΔT_1 is added to the critical times of ten concrete edges where the corrosion initiation times are shorter than 100 years. The time-variant system reliabilities with no maintenance or Preventive type I are shown in Fig. 15. It is clear that with this maintenance type, the system reliability is nearly invariant in the first 70 years and decreases to about 3.8 at the end of the service life, which means that the structure will still be in an operating state. Furthermore, although the two maintenance schedules have a different influence on the critical times of the various concrete edges (Fig. 14), their effects on the structural performance are identical (i.e., $\beta_{sys,1} = \beta_{sys,2}$). Thus, compared with subcase 1, subcase 2 should be the optimal solution when the maintenance cost is taken into account.

4.3.3 Maintenance with Preventive type II

Preventive type II is performed to delay the deterioration of structural performance by suppressing

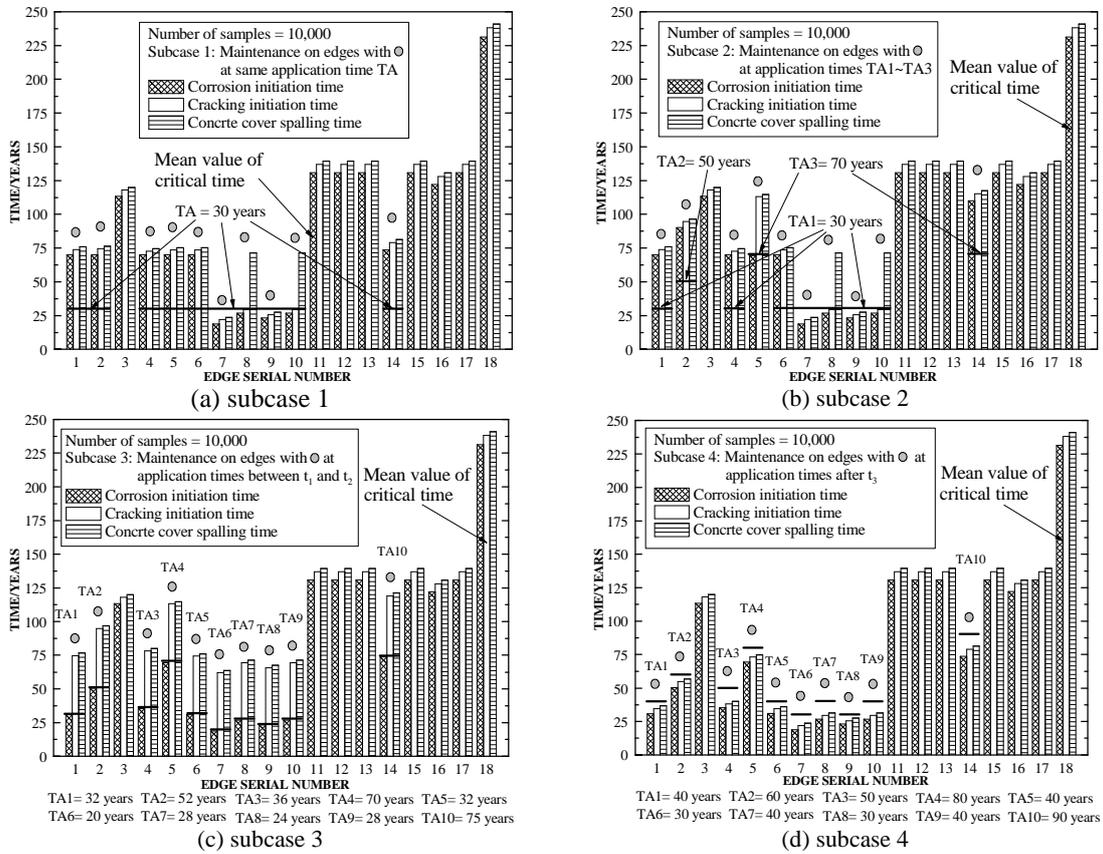


Fig. 16 The mean values of the critical times with Preventive type II

the corrosion process over a period of time, the effect of which is similar to Preventive type I. In this case, the delayed time period ΔT_2 is also assumed to follow a normal distribution, with a mean value of 40 years and a standard deviation of 5 years. Due to the simulation results that were derived from the evaluation of Preventive type I, the maintenance action will be merely applied on the ten concrete edges where the first critical times are shorter than 100 years. Four subcases are discussed as follows

Subcase 1: the maintenance application times of the ten concrete edges are all equal to 30 years;

Subcase 2: the maintenance application times of the ten concrete edges are 30 years (edges 1, 4, and 6~10), 50 years (edge 2), and 70 years (edges 5 and 14);

Subcase 3: the maintenance application times of the ten concrete edges are between the corrosion initiation time t_1 and the cracking initiation time t_2 of each concrete edge; and

Subcase 4: the maintenance application times of the ten concrete edges are longer than the concrete cover spalling time t_3 of each concrete edge.

Fig. 16 shows the mean values of the three critical times related to all the concrete edges of the middle-span-center section when Preventive type II is used. In Fig. 16(a), the effects of maintenance action on concrete edges are not identical due to their various critical times. For example, the critical times of edges 7 and 9 are invariant with the maintenance action; nevertheless, the corrosion processes of reinforcing steels on the two edges are still delayed after the third critical time t_3 . In the case of edges 5 and 14, the variations of the critical times are small following the maintenance action, because their first critical times are close to $\Delta T_2 + TA$. Thus, the effects of maintenance action on these edges are not

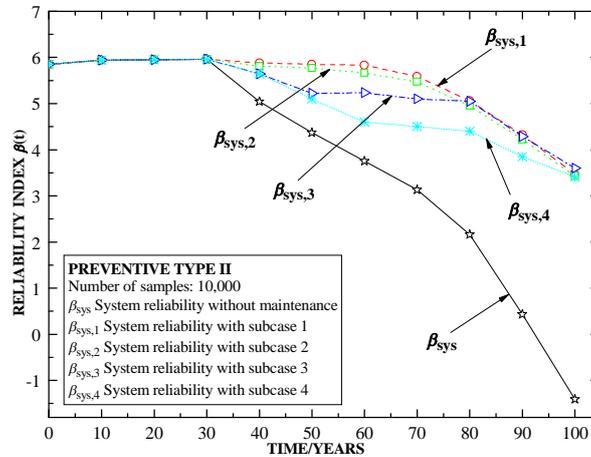


Fig. 17 The variation of system reliabilities with no maintenance or Preventive type II

evident. In Fig. 16(b), three application times of the maintenance action are assumed to be performed. Compared with subcase 1, the corrosion processes of reinforcing steels on edges 5 and 14 are postponed for a long period of time, since the maintenance application time TA_3 is close to their first critical times. In subcase 3, the maintenance application time of each concrete edge is defined to be between its first critical time t_1 and second critical time t_2 , which denotes that ten different application times (i.e., $TA_1 \sim TA_{10}$ in Fig. 16(c)) are used. Thus, the corrosion processes of reinforcing steels on the ten edges are all delayed ΔT_2 between t_1 and t_2 , and the expected effect of the maintenance action is completely achieved. The maintenance schedule in subcase 4 (Fig. 16(d)) is similar to that of subcase 3, and the only difference is that the maintenance application time of each concrete edge is assumed to be after the third critical time t_3 .

Fig. 17 displays the time-variant system reliabilities with no maintenance or Preventive type II. The variations of the system reliabilities in the different subcases are not identical due to the various maintenance schedules. For example, the deterioration of system reliability with maintenance subcase 1 (i.e., $\beta_{\text{sys},1}$ in Fig. 17) is not apparent between 30 to 70 years, but gradually accelerates after 70 years. However, with the various developing processes, the system reliabilities under four maintenance schedules are nearly identical at the end of the service life. This is because the maintenance application times of the concrete edges are mostly between 30 to 50 years in Figs. 16(b)-(d), which are similar to the ones in Fig. 16(a). Accordingly, subcase 1 should be the optimal maintenance plan out of the four subcases when cost is one of the objectives, as the maintenance actions in the other three need to be applied more than once and thus higher costs are incurred.

4.3.4 Maintenance with strengthening

When the deterioration of structural performance is significant, Strengthening may be an appropriate maintenance action. The effects of both the bonded steel plate and FRP on lifetime structural performances are investigated in this case. Note that the steel plate and FRP are assumed to be completely protected and thus their deteriorations are not considered.

When the steel plate is used, two subcases are taken into account

Subcase 1: only increasing the structural bending capacity, thus the steel plate is assumed to be bonded onto the tension zone of the concrete section after 60 years;

Subcase 2: increasing both the structural bending capacity and shear capacity, and thus the steel plates are assumed to be bonded onto the tension and web zones of the concrete section after 60 years

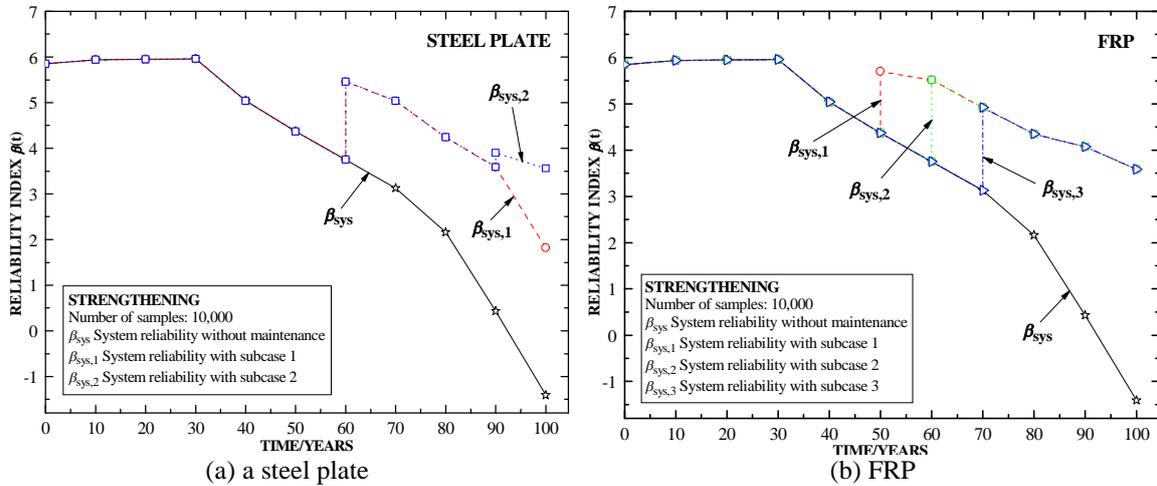


Fig. 18 The variation of system reliabilities with no maintenance or Strengthening using different materials

and 90 years.

Fig. 18(a) shows the time-variant system reliabilities with no maintenance or a bonded steel plate. In subcase 1, the system reliability is improved from 3.75 to 5.46 and the structural performance is restored to a good condition immediately after 60 years. However, it still decreases to 1.82 after 100 years. This is because the system reliability is dominated by the shear failure at the end of the service life, as it is reduced to 1.89 after 100 years (Fig. 13) without an improvement in the shear resistance. Accordingly, the structural bending and shear capacities are both improved in subcase 2, where the system reliability increases from 3.59 to 3.90 after 90 years and the structure is still in an operating state at the end of its service life.

With regard to the aforementioned simulation results of the maintenance action of using the steel plate, the improvement of bending and shear capacities are now both considered if FRP is used. Three subcases are discussed herein, in which FRP is bonded onto the tension and web zones of the concrete section after 50 years, 60 years and 70 years. Fig. 18(b) shows the variations of the system reliabilities with no maintenance or FRP. Although the application times are different, the system reliabilities under the three maintenance schedules are identical at the end of the service life. Thus, 60 years should be the optimal application time for maintenance action due to the following reasons: (1) the structure with no maintenance is about to decrease into a non-operating state after 60 years; and (2) the other maintenance actions with the same effect on structural performance may be more economical than FRP after 50 years. Note that the system reliabilities with the maintenance actions of using the steel plate and FRP are almost identical at the end of the service life, and the area of a steel plate is three times that of FRP. This is mainly due to the high tensile strength and low unit weight of FRP. However, the price of FRP is much higher than that of a steel plate and therefore maintenance optimization is required to select the optimum maintenance plan.

4.3.5 Maintenance with replacement

Finally, the use of Replacement is considered for the case study bridge. Only the replacement of the overall structure is discussed herein, since the structural failure mode is a series system and the replacement of the individual components is impossible. Three subcases, in which the maintenance actions are applied after 50, 60, and 70 years, are taken into account in this case. The time-variant system reliabilities with no maintenance or replacement are shown in Figure 19. It is clear that 60 years

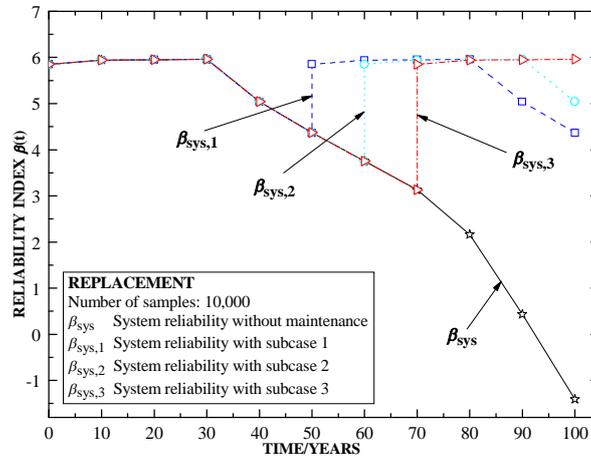


Fig. 19 The variation of system reliabilities with no maintenance or Replacement

is the best application time for maintenance for the same reason it was optimal for the FRP-based maintenance action. It should be noted that in this case the life time performance of the new structure is assumed to be the same as that of the original one from the beginning of its service life, as no other protection measures (e.g., the reinforcing steels are coated by anti-corrosive paint) are applied to the new structure. When these protection measures are utilized, the lifetime performance of the new structure during its remaining service life would need to be re-evaluated.

5. Conclusions

In this paper, a probabilistic and finite element-based approach to evaluate and predict the lifetime performance of RC bridges undergoing various maintenance types was proposed with the time-variant system reliability being used as the performance indicator. Depending on the structural state during the degradation process, the classical maintenance actions commonly used in RC bridges were firstly categorized as four types: Preventive type I, Preventive type II, Strengthening and Replacement. The quantitative effects of these maintenance types on structural performance were investigated, and subsequently the analysis modules to consider these effects were written and added to the computer program CBDAS. Consequently, the time-variant system reliability could be calculated using Monte Carlo simulations and the updated CBDAS. Finally, the lifetime structural performance with and without each of the maintenance types with respect to a RC continuous bridge subject to chloride-induced corrosion was discussed. As a result of this study the following conclusions can be drawn

(1) The effects of the Preventive type I and Preventive type II maintenance actions on structural performance are null when they are performed on concrete edges where the corrosion initiation times are longer than the prescribed service life. Thus, when Preventive type I or Preventive type II is used, it is necessary to initially ascertain which concrete edges have critical times that are shorter than the prescribed service life.

(2) When Preventive type II is applied, the system reliabilities under the four maintenance subcases are almost same at the end of the service life, although their developing processes are different. Therefore, subcase 1 (the application times with respect to all the maintain-needed concrete edges are after 30 years) should be the optimal maintenance schedule if the maintenance cost is taken into account, since the other maintenance subcases are all required to be performed more than once with

higher costs.

(3) When compared with the steel plate, the effect of the maintenance action of using FRP is preferable (i.e., the same effect as a steel plate with a less area), because of its relatively high tensile strength and light unit weight. However, the price of FRP is far more expensive than that of a steel plate, and thus maintenance optimization is necessary to select an appropriate maintenance schedule.

(4) When Replacement is used, the best application time is when the structure is about to enter into a non-operating state. However, this maintenance type may be still not be the best choice, since the structure may be maintained in a good condition throughout its remaining service life by using the other maintenance types, especially the third maintenance type, Strengthening, at less cost.

(5) In general, Preventive type I and Preventive type II are time-based while Strengthening and Replacement are performance-based. The maintenance strategy of a RC bridge should mainly be selected based on its structural state, that is: (1) Preventive type I is an appropriate choice if the corrosion initiation time has not been reached; (2) Preventive type II should be used if the reinforcing steel has corroded and the deterioration of the structural performance is not evident; and (3) Strengthening or Replacement should be utilized if the structural performance decreases significantly and is about to reach the limit state.

It should be noted that in the present study, only the effect of each single maintenance type on structural performance was investigated, and the maintenance cost was not taken into account. Therefore, functions for considering the effect of the combined maintenance types and the optimization of maintenance schedules should be added to the proposed approach in the future.

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