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Remaining service life estimation of reinforced concrete buildings based on fuzzy approach

Hae-Chang Cho[1a], Deuck Hang Lee[1b], Hyunjin Ju[1c], Kang Su Kim[*1], Ki-Hyun Kim[2d] and Paulo J.M. Monteiro[2e]

[1]*Department of Architectural Engineering, University of Seoul, 163 Seoulsiripdaero, Dongdaemun-gu, Seoul 130-743, Republic of Korea*
[2]*Department of Civil & Environmental Engineering, University of California, 725 Davis Hall, Berkeley, CA 94720, USA*

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Abstract. The remaining service life (RSL) of buildings has been an important issue in the field of building and facility management, and its development is also one of the essential factors for achieving sustainable infrastructure. Since the estimation of RSL of buildings is heavily affected by the subjectivity of individual inspector or engineer, much effort has been placed in the development of a rational method that can estimate the RSL of existing buildings more quantitatively using objective measurement indices. Various uncertain factors contribute to the deterioration of the structural performance of buildings, and most of the common building structures are constructed not with a single structural member but with various types of structural components (e.g., beams, slabs, and columns) in multistory floors. Most existing RSL estimation methods, however, consider only an individual factor. In this study, an estimation method for RSL of concrete buildings is presented by utilizing a fuzzy theory to consider the effects of multiple influencing factors on the deterioration of durability (e.g., concrete carbonation, chloride attack, sulfate attack), as well as the current structural condition (or damage level) of buildings.

Keywords: remaining service life; chloride attack; concrete carbonation; sulfate attack; Choquet integral; λ- fuzzy measure

1. Introduction

In many developing and developed countries, various problems caused by deterioration of older reinforced concrete buildings are emerging as important social issues. Generally, a concrete structures should have adequate durability performance during its intended service life (ISL) (KCI 2009, ACI 365.1R-00 2000), where ISL is the duration in which a structure must maintain its specified durability performance based on consideration of the structures' importance, size, type,

[*]Corresponding author, Professor, E-mail: kangkim@uos.ac.kr
[a] Ph.D. Candidate, E-mail: chang41@uos.ac.kr
[b] Ph.D., Post-Doctoral Research Fellow, E-mail: dklee@uos.ac.kr
[c] Ph.D. Candidate, E-mail: fis00z@uos.ac.kr
[d] Ph.D., Post-Doctoral Research Fellow, E-mail: kimkh738@gmail.com
[e] Ph.D., Professor, E-mail: Monteiro@ce.berkeley.edu

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duration of use, maintenance and control level, economic performance, etc (KCI 2009). The Korean Concrete Institute (2009) and Japan Society of Civil Engineers (1995) categorize the durability grades of concrete buildings based on ISL into three levels: the highest grade is defined as 100 years. The design standard of structural concrete in the U.K. (BS 7543 2003) specifies 120 years as the highest grade. As such, while there are some differences depending on national codes, the ISL of concrete buildings is typically estimated to be 50 to 65 years, which can be extended to 100 to 120 years for special or important building, if necessary (KCI 2009, JSCE 1995, BS 7543 2003). Since the durability and structural performance of concrete buildings gradually decreases as the duration of their usage increases, continuous maintenance work, such as regular safety inspections and detailed safety diagnosis based on the facilities safety and maintenance plans (Ministry of Construction and Transportation of Korea 2008, International Code Council 2012), are required to meet the ISL. That being said, the diagnosis results of safety inspections often depend on the subjective decisions of inspectors as it is very difficult to obtain an objective quantification under practical conditions. Furthermore, the structural safety diagnosis tests currently being performed cannot evaluate the actual remaining service life (RSL) of structures where the RSL is given by the difference between the service life (SL) and the duration of usage of a building (i.e., the period from the completion of a structure to the present day). Here, SL is the amount of time in which the durability and mechanical performances of a structure are maintained above the minimum allowable value by regular maintenance work and is considered as that generally required by the national or local specifications since its completion of construction (ACI 365.1R-00 2000, ASTM E632-82 1996).

In general, a reinforced concrete structure has a long SL, but its RSL decreases as its durability continues to decline due to various factors such as environmental deterioration or change of external loads, for instance, caused by change of usage. Although the high alkalinity in the concrete pore solution creates a protective passive film against corrosion of the steel reinforcement (KCI 2009, Neville 1996), this passive film can be destroyed if the amount of chloride ion (Cl-) near the steel surface reaches a critical threshold: if that threshold is reached, the steel reinforcement becomes vulnerable to corrosion. Also, carbonation of concrete over time decreases the pH of the pore solution, which destroys the passive film and potentially leads to the corrosion of the steel bar. When the steel bar corrodes, its volume expands over two to three times its original volume, and the consequent the expanding pressure causes cracks in the concrete cover (Kim *et al*. 2010, Liu and Weyers 1998). Therefore, national design codes for concrete structures (KCI-M-07 2007, ACI Committee 318) regulates the crack size to avoid the penetration of harmful materials, such as sea sand, sea breeze, sulfate, etc.

Researchers (Schiessl 1988, Jung *et al.* 2000, Papadakis *et al.* 1996, AIJ 2004, Yoon 1994, Lee and Moon 2004, Ferraris *et al.* 1997, Atkinson and Hearne 1984) have studied the RSL of concrete structures with respect to individual durability deterioration factors such as chloride attack, concrete carbonation, and sulfate penetration. Concrete durability deterioration often occurs as a result of the combined action of these influential factors (Yoon 1994, Lee *et al*. 2013, Andrade *et al.*1997); therefore, it is not reasonable to evaluate the RSL considering only one deteriorating factor. Consideration of the interaction among these deterioration factors requires establishing a feasible methodology for the RSL estimation that takes into account the effect of multiple influencing factors. Therefore, this study introduces fuzzy theory to quantify the effect of multiple deterioration factors on the evaluation of the RSL of concrete building structures. The proposed evaluation method is presented in detail herein.

2. Factors affecting concrete durability deterioration

The durability performance of concrete buildings even with sufficient structural safety against design loads can be negatively affected by environmental condition, such as chloride attack, concrete carbonation, freezing and thawing, chemical corrosion, sulfate penetration, and alkali aggregate reaction, etc. The following sections will discuss the individual effects of chloride, concrete carbonation, and sulfate attack on the RSL of concrete structure, and in a latter section, the methodology that considers the effects of the multiple factors based on fuzzy theory is presented in detail.

2.1 Chloride attack

One of the key reasons for the performance deterioration of reinforced concrete building structures is the corrosion of steel bars caused by chloride attack. To prevent the corrosion of steel bars, the total chloride ions in fresh concrete is often limited to below 0.30 kg/m^3 (KCI-M-07 2007, ACI Committee 318 2008). As shown in Fig. 1, chloride ions together with moisture and oxygen can cause steel corrosion. Chloride attack may be triggered by sea sand or impurities contained in mixing water, by the direct contact of the concrete cracks or joints with seawater, or by the spraying of anti-freezes in cold weather. Halogen ion (Cl-, Br2- and I2-), sulfate ion (SO42-) and sulfide ion (S2-) are also the substances that destroy the passive iron oxide film (Monteiro 2006).

Schiessl (1988) reported that the mechanisms that facilitate the corrosion of steel bars (i.e., steel loss due to corrosion) depend largely on concrete cracks, cover depth, and level of chloride content in concrete and at member surface. Tuutti (1982) and Jung *et al*. (2000) described the corrosion behavior of a steel bar along the time as shown in Fig. 2, in which the steel bar corrosion due to the spread of chloride ion is activated after the initiation period (T_{ini}) during which there is no or little corrosion. It can be noted that the inclination of each curve in Fig. 2 represents the corrosion rate. In Fig. 2, Curve A represents a typical corrosion behavior of a steel bar placed in a concrete member. (Tuutti 1982) If the concrete cover depth becomes thinner or has cracks, however, the initiation period becomes shorter than Curve A as represented by the Curve C while its corrosion rate is identical to the Curve A. This is because two curves represent the corrosion behaviors of steel bars that are exposed to the same amount of chloride attack but are different only in their cover concrete depths or crack conditions. If the chloride ions exist initially in the concrete mixtures before the concrete hardening, its behavior curve will move from A to B or D, which shows that both of the corrosion rate and the initiation period (T_{ini}) largely depends on the initial amount of chloride ions.

Tuutti (1982) suggested that the service life of a concrete structure can be estimated as the sum of the initiation period (T_{ini}) and the propagation period (T_{pro}). The propagation period (T_{pro}), during which the corrosion rate increases rapidly, however, is shorter and more difficult to estimate than the initiation period (T_{ini}), and once the corrosion in the steel reinforcement is initiated, the durability of the concrete structure will rapidly deteriorate (Kim *et al*. 2010, Liu and Weyers). Therefore, it is commonly assumed, for the simplification and maintaining a margin of safety, that the initiation period (T_{ini}) can be taken as the SL of reinforced concrete structures (Schiessl 1988). As such, this study also assumed that the initiation period was considered to be the SL of concrete structures.

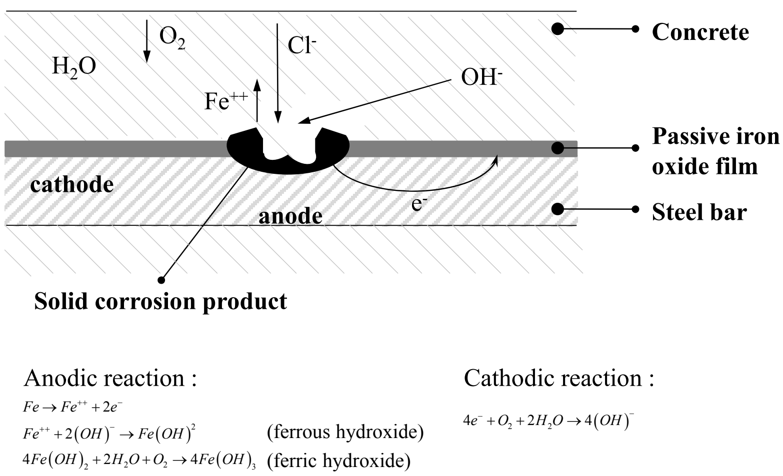


Fig. 1 Corrosion mechanism of steel bars embedded in concrete (Neville, 1996)

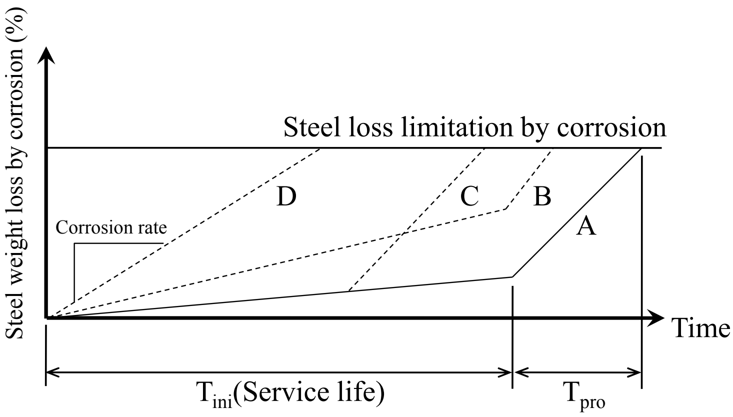


Fig. 2 Description of initiation and propagation period of corrosion
(Tutti 1982, Schiessl 1988, Jung *et al*. 2000)

The chloride ion concentration with respect to time can be calculated using Fick's Second Law (Poter and Easterling 1992), which describes the diffusion of ions, where the diffusion occurs from regions with a high chemical potential to those with a low chemical potential. The chloride ion concentration (C_c) is defined as follows

$$C_c(d_d, t) = C_i + (C_0 - C_i)\left[1 - erf\left(\frac{d_d}{2\sqrt{D_d t}}\right)\right] \quad (\mathrm{kg/m^3}) \tag{1}$$

where, C_i and C_c are the chloride ion concentration on the surface ($\mathrm{kg/m^3}$) at the initial time and at the present time, respectively, and erf is the Gauss error function, in which d_d is the diffusion distance (mm), D_d is the effective diffusion coefficient ($\mathrm{m^2/s}$), and t is the elapsed (year(s)). Ideally, the surface chloride ion concentration (C_0) should be measured by collecting specimens of the target structure from field inspections. When sampling of specimen is problematic, the approach proposed by Lee (2006) can be used to estimate C_0 using the airborne chloride concentration in the air, as follows

$$C_0 = 0.0944 C_a + 0.7645 \tag{2}$$

where, C_a is the estimated amount of flying chloride ($\mathrm{mdd,\ mg/100cm^2/day}$) considering the location, distance, and height of the structure, which is expressed as

$$C_a = s_f y_r h \tag{3}$$

where, s_f is the regional coefficient ($\mathrm{mdd, mg/100cm^2/day}$), y_r is airborne chlorides decreasing rate considering the distance from seacoast to the structure, and h is the height coefficient. The effective diffusion coefficient D_d in Eq. (1) is the function of the concrete curing temperature, concrete age, and relative humidity. The KCI (2009) Standard Specification for Concrete suggests that the effective diffusion coefficient can be calculated by multiplying the concrete material coefficient (γ_c) by the standard chloride ion diffusion coefficient (D_k), where γ_c is generally taken as 1.0, or 1.3 for a roof floor. Also, the standard chloride ion diffusion coefficient (D_k) can be estimated by the model proposed by Papadakis *et al.* (1996), which was derived based on the measured values from test specimens aging from 2 to 14 months, with the primary test variable of water-to-cement ratio, as follows

$$D_k = D_{H_2O} 0.15 \frac{1 + \rho_c \dfrac{\omega}{c_f}}{1 + \rho_c \dfrac{\omega}{c_f} + \dfrac{\rho_c}{\rho_a}\dfrac{a}{c_f}} \left(\frac{\rho_c \dfrac{\omega}{c_f} - 0.85}{1 + \rho_c \dfrac{\omega}{c_f}} \right)^3 \tag{4}$$

where, D_{H_2O} is the chloride ion diffusion coefficient (sodium chloride, $1.6 \times 10^{-9}\ \mathrm{m^2/s}$) in the infinite solution, ρ_c and ρ_a are the density of cement and aggregate, respectively, and ω, c_f, and a are the unit amount of water, cement, and aggregate ($\mathrm{kgf/m^3}$), respectively.

The application of Fick's Law to concrete has limitations: it does not take into account changes in material characteristics due to the aging of the concrete and the absorption of chlorides by hydration products, etc. Thus, the diffusion law should be applied carefully with recognition of its limitations. (Zhou 2014)

2.2 Concrete carbonation

Because the pore solution of the cement paste matrix is a strong alkaline at pH 13 to 15, it is unlikely that corrosion occurs on steel reinforcement compactly surrounded by concrete. However, the reaction of carbon dioxide (CO_2) contained in the air with calcium hydroxide in concrete generates calcium carbonate; thus the pH of concrete is gradually lowered to pH 8.5-10. (KCI

2009, Neville 1996) Once carbonation occurs, the pH of concrete is reduced to below pH 11, and the passive iron oxide film on the surface of the steel reinforcement is destroyed. In moderate environments, the reinforcing bar embedded in concrete is protected by the high pH of concrete. However, once the passive film of the reinforcement is destroyed by concrete carbonation, the corrosion mechanism is initiated. After the carbonation depth reaches the steel bar, corrosion rapidly progresses, and cross-section area of steel bar reduces, as shown in Fig. 3. (Han *et al.* 2014) Also, the corrosion products induce the dilation pressure to adjacent concrete due to their volume expansion, which causes radial cracks, resulting in the degradation of bond strength, the delamination of cover concrete, and contributes to the deterioration of the concrete durability. As the degree of concrete carbonation is a key factor in the durability and structural performance of concrete structures, the RSL evaluation model developed in this study considered the depth of carbonation.

Given that the carbonation depth increases in proportion to the square root of time in an environment with steady humidity, it can be expressed as follows (Neville 1996)

$$X_d = A_t\sqrt{t} \tag{5}$$

where, X_d is the standard depth (mm) of concrete carbonation, A_t is the carbonation velocity coefficient, which is a constant related to the type of cement or aggregate, environmental conditions, mixing materials, and surface finishing materials, and t is the elapsed time (years). In this study, the carbonation velocity coefficient (A_t) in Eq. (5) was estimated by applying the carbonation depth and concrete cover depth measured from field inspections (ACI 365.1R-00 2000, Papadakis *et al.* 1996) to Ang and Tang (1990)'s reliability model. That is, if the carbonation depth and concrete cover depth are assumed to be normal distributions, the limit state function of concrete carbonation depth [$M(X_d,t)$)] can be expressed, as follows

$$M(X_d,t) = D(X_d,t) - C(X_d,t) \tag{6}$$

where, $D(X_d,t)$ and $C(X_d,t)$ are the probability variables on the concrete cover depth and the carbonation depth, respectively. As shown in Fig. 4, the average and standard deviation of the concrete cover depth and the carbonation depth are defined as μ_d and σ_d, μ_c and σ_c, respectively, and the normal distribution function of the carbonation depth [$C(X_d,t)$] moves to the right side gradually as carbonation progresses with time. Therefore, the average (μ_m) and standard deviation (σ_m) of the limit state function of carbonation [$M(X_d,t)$] can be expressed, as follows

$$\mu_m = \mu_d - \mu_c \tag{7a}$$

$$\sigma_m = \sqrt{\sigma_d^2 + \sigma_c^2} \tag{7b}$$

Because the carbonation depth is proportional to the square root of time, the average [$\mu_c(t)$] and standard deviation [$\sigma_c(t)$] of the movement of carbonation depth in Fig. 4 can be expressed as follows

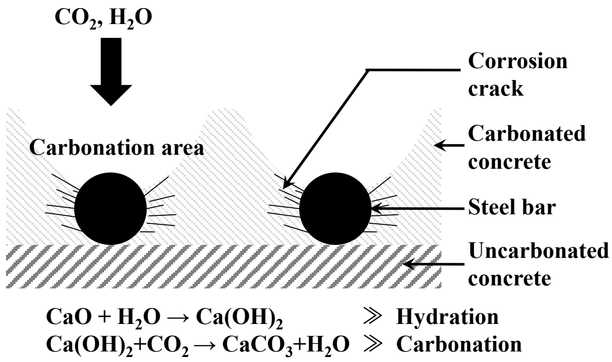


Fig. 3 Steel corrosion by concrete carbonation

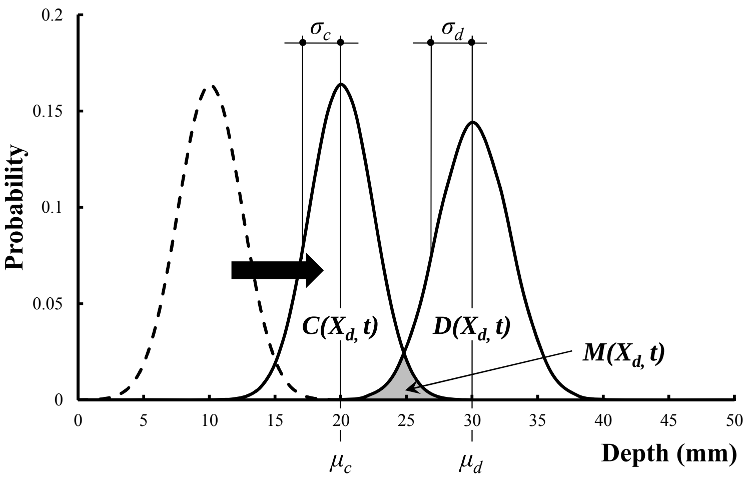


Fig. 4 Normal distribution of probabilistic variables of thickness of concrete carbonation and concrete cover

$$\mu_c(t) = A_t\sqrt{t} \tag{8a}$$

$$\sigma_c(t) = B_t\sqrt{t} \tag{8b}$$

and the carbonation velocity coefficients (A_t and B_t) can be determined using the carbonation depth measured at the present state. By substitution of Eq. (8) into Eq. (7), the average [$\mu_m(t)$] and standard deviation [$\sigma_m(t)$] of the limit state function can be estimated as follows

$$\mu_m(t) = \mu_d - A_t\sqrt{t} \tag{9a}$$

$$\sigma_m(t) = \sqrt{\sigma_d^2 + B_t^2 t} \tag{9b}$$

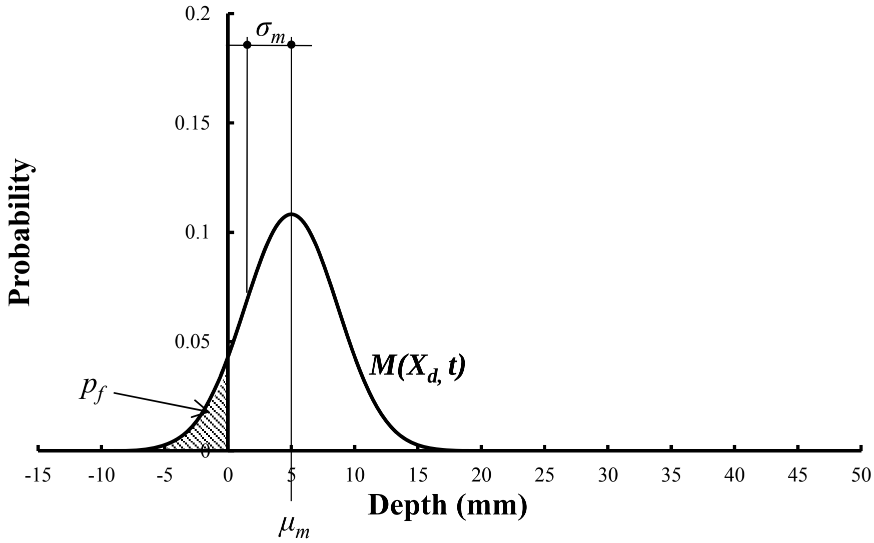


Fig. 5 Normal distribution of the limit state function of concrete carbonation depth

Therefore, the limit state function of the RSL against concrete carbonation [$M(X_d, t)$] can be expressed as a normal distribution function, as shown in Fig. 5, and the probability of the concrete carbonation reaching the steel bar surface is a cumulative distribution function (p_f) below 0 at the normal distribution of the limit state function, where p_f can be ranged from 0 to 1. In this study, the time (t, year) when the cumulative distribution function (p_f) reaches 0.5 was taken as the RSL with respect to concrete carbonation as a conservative estimate (Yoon 1994).

2.3 Sulfate attack

Concrete structures, such as marine substructures and bridge piers, are commonly exposed to hazardous chemical substances, such as acids or chloride contained in seawater or underground water. Among various chemical substances, sulfate is widely dissolved in soil as well as underground water or seawater (Lee and Moon 2004, Ferraris *et al*. 1997), and, therefore, concrete structures that make contact with soil may be exposed to sulfate attack, causing durability problems. This is especially true for concrete building structures constructed on reclaimed ground land, which are expected to have severe durability deterioration due to sulfate penetration. During sulfate attack, ettringite is often formed from the monosulfate hydrate ($Ca_4Al_2(OH)_{12}SO_4.6H_2O$). which is present in the cement paste. Other phases, such as $Ca_4Al_2O_{26}H_{38}$, $Ca_3Al_2O_6$, and $Ca_4Al_2Fe_2O_{10}$, can also be sources of the aluminate ions in the formation of ettringite (Monteiro 2006). Ettringite causes volume expansion in hardened concrete, resulting in concrete cracks and severe durability deterioration of the concrete after a certain point (Neville 1996). The sulfate penetrate velocity increases based on the sulfate concentration, and in cases with over 0.5% magnesium sulfate ($MgSO_4$) or over 1.0% sodium sulfate (Na_2SO_4), the sulfate penetrate velocity greatly increases. In this study, the sulfate penetration depth (d_{sp} in mm) proposed by Atkinson and Hearne (1984) was used, which is

$$d_{sp} = \frac{42}{5}\frac{\delta_c}{8}\frac{(Mg^{2+} + SO_4^{2-})}{0.19}t_{im} = 5.5\delta_c (Mg^{2+} + SO_4^{2-})t_{im} \tag{10}$$

where, δ_c is the weight ratio(%) of cement to $3CaO \cdot Al_2O_3 \cdot C_3A$, Mg^{2+} and SO_4^{2-} are the molar concentrations of magnesium and sulfate (mol/l), respectively, and t_{im} is the immersion time (year).

2.4 Factors related to concrete compressive strength and the structural condition

Because of factors such as aging, curing methods, or environmental conditions, the measured concrete compressive strength may differ from the design strength. According to the Safety Inspection Regulation (SIR) presented by Korea Infrastructure Safety and Technology Corporation (KISTEC 2009), the safety level of structures can be divided into five grades by the measured strength-to-design compressive strength ratio of concrete (α_c), as shown in Table 1. Grade A means that the concrete is in an excellent condition, whereas grade E means that it is in a dangerous state that may require structural strengthening or even restriction of use. This study also used the concrete strength coefficient (η_1) presented in Table 1, referring to the concrete strength evaluation standard in SIR (KISTEC 2009). The RSL of concrete structures is also affected by other current structural conditions. For example, cracks can make concrete structures vulnerable to chloride ion penetration and delamination reduces the concrete cover depth. To consider the current structural condition in evaluating the RSL of structures, this study adopted the condition evaluation system presented by KISTEC (2009), as shown in Table 2. With respect to the evaluation grade of the current structural condition, the structural condition coefficient (η_2) shown in Table 2, was applied to the estimation of RSL of structures.

Table 1 Evaluation grades based on the ratio of measured to design compressive strength of concrete (KISTEC 2009)

Grade	Ranges of grades*	Coefficient, η_1
A	$\alpha_c \geq 100\%$ (in perfect condition)	1
B	$\alpha_c \geq 100\%$ (with slight damage)	0.9
C	$85\% \leq \alpha_c \leq 100\%$	0.8
D	$70\% \leq \alpha_c \leq 85\%$	0.7
E	$\alpha_c < 70\%$	0.6

* α_c : (measured strength / design strength at 28 days) mode

Table 2 Evaluation grades based on structural condition (KISTEC 2009)

Grade	Condition	Coefficient, η_2
A	Good condition with no damage	1
B	Minor damage in subsidiary members	0.9
C	Minor damage in primary members or heavy damage in subsidiary members	0.8
D	Heavy damage in primary members	0.7
E	Serious damage in primary members and high risk condition	0.6

3. Remaining service life estimation method by fuzzy theory

The fuzzy theory, which was first introduced by Zadeh (1965), provides quantitative and numerical information from qualitative linguistic information whose boundary is unclear and vague. In this theory, fuzzy sets are defined by using fuzzy membership functions, and many practical problems can be resolved by using fuzzy relationships, compositions of fuzzy relationships, and fuzzy inference (Jang *et al*. 1997). After the fuzzy measure and the fuzzy integral were introduced by Sugeno (Grabisch *et al*. 2007), fuzzy theory began to be applied in industrial fields; recently, it has been widely used in state-of-the-art science and engineering fields as well as in construction area (Kim *et al*. 2007, Choi *et al*. 2007, Demir 2005, Ünal *et al*. 2005, Anoop and Rao 2007, Anoop *et al*. 2004, Mitra *et al*. 2010, Nehdi and Bassuoni 2009).

The fuzzy theory is a representative theory in measuring uncertainty, which can be divided into the fuzzy set and fuzzy measure methods (Klir and Folger 1988, Zimmermann 2001). The fuzzy set method processes uncertainty in human cognition, thought process, judgment, and language expression in a quantitative and reasonable manner, while the fuzzy measure method, a non-additive set function proposed by Sugeno (Grabisch *et al*. 2007), excludes additivity in the classic definitions of measurement. The fuzzy set, the basis of the fuzzy theory, is an expanded concept of a discrete set {0, 1} by granting a membership function to a fuzzy set [0, 1], which can have an arbitrary value between 0 and 1. The membership function defined in a fuzzy set A_x is the degree of how much the variable x belongs to A_x. Also, Sugeno (Grabisch *et al*. 2007), developed the λ-fuzzy measure that includes the probability measure, the belief measure, and the plausibility measure. Other measures include the possibility measure and the necessity measure, which are defined as a union and an intersection, respectively, of the membership function from the conventional fuzzy set theory. Banon (1981) defined the relationship among the fuzzy measures as shown in Fig. 6, which describes that the plausibility measure is a concept that includes both the possibility and probability measures, while the belief measure includes the necessity and probability measures. Also, probability can be defined as the intersection between the belief and plausibility measures. Since the fuzzy measure is not additive, Lebesgue's integral cannot be applied to fuzzy calculations (Grabisch *et al*. 2007). Therefore, for an integral method that is suitable for the fuzzy measures, the Sugeno integral (Grabisch *et al*. 2007) and the Choquet integral (Grabisch *et al*. 2007, Choquet 1954) are often adopted. In this study, the Choquet integral is used because it is considered to have a relatively clearer integral process and produces more accurate results.

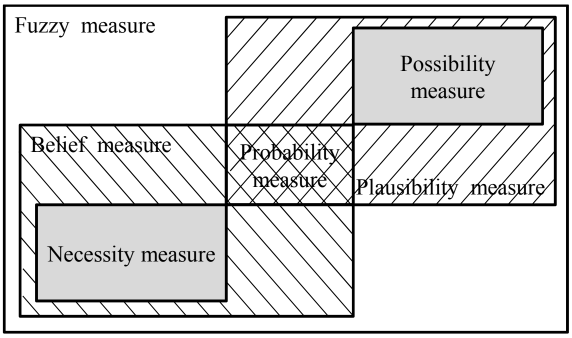


Fig. 6 Relationship of fuzzy measures (Banon 1981)

3.1 Estimation of remaining service life

As mentioned earlier, the durability deterioration of reinforced concrete structures is the result of the actions of various factors; this study applies fuzzy theory to analyze the effect of these factors over time. The ISL of reinforced concrete structures is assumed to be 100 years. As shown in Fig. 7, the fuzzy sets were defined by triangular membership functions, dividing the evaluation grade into six levels. The final RSL evaluation results were also expressed as fuzzy numbers with triangular shapes, where the membership function values over 0.5 were used based on the α-cut set concept (Klir and Folger 1988, Zimmermann 2001). Here the α -cut set is a set of membership functions with a certain possibility level or over (i.e., over the predefined α) (Klir and Folger 1988, Zimmermann 2001), in which the possibility is the irregular value of the uncertainty in language that is the upper-limit value of probability (Klir and Folger 1988, Zimmermann 2001). Since the additivity in mathematics is applicable for the probability, the sum of all probabilities should be 1.0, whereas this is not the case for the possibility; i.e., the sum of all possibilities need not to be 1.0. Therefore, if the possibility of an event is small, its probability can also be small, where as a small probability does not mean a small possibility.

Estimation of the RSL of concrete structures begins by calculating the RSL at a member level; Fig. 8 shows an example of the fuzzy calculation procedures on a sample floor. The RSL of each member with respect to chloride attack (CL), concrete carbonation (CA), and sulfate penetration (SU) is determined using the method described in the previous section. The calculated RSL of each member for each deterioration factor can be defined by the unified fuzzy set as follows

$$CL_n = \left\{CL_n^c, CL_n^g, CL_n^b, CL_n^s\right\} \tag{11a}$$

$$CA_n = \left\{CA_n^c, CA_n^g, CA_n^b, CA_n^s\right\} \tag{11b}$$

$$SU_n = \left\{SU_n^c, SU_n^g, SU_n^b, SU_n^s\right\} \tag{11c}$$

where, the subscript n is the sample floor number, which can include the underground floors, and the superscripts c, g, b, and s mean the column, girder, beam, and slab, respectively.

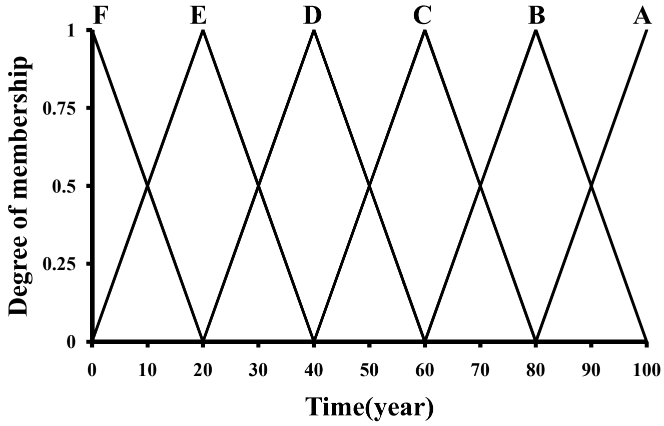


Fig. 7 Fuzzy sets for the estimation of RSL

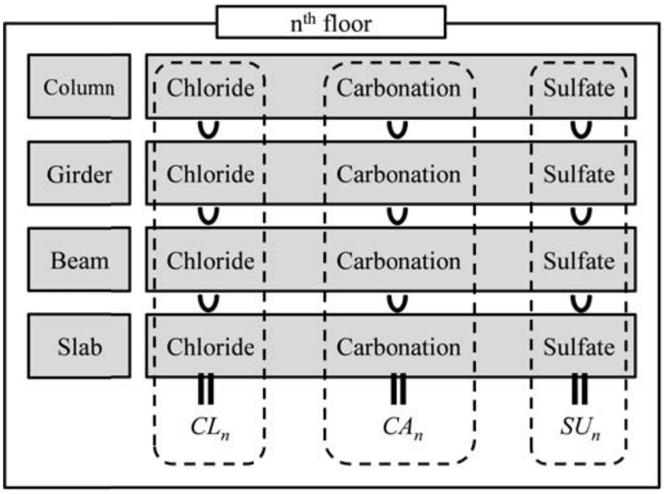


Fig. 8 Fuzzy calculation process for a sample story

Each member has a different level of effect on the RSL of the whole structure; the importance level of each member can be considered by introducing the weight factor of importance (I_m). The weight factor of importance (I_m) for each member can be determined by expert groups or theoretical bases, i.e., considering a contributory area of each member or collapse mechanism, etc (Kim *et al*. 2007, Choi *et al*. 2007, Demir 2005, Ünal *et al*. 2005). In this study, the weight factors of importance (I_m) were adopted from the specification of safety inspection presented by KITSEC (2009), as shown in Table 3, and the λ-fuzzy measure proposed by Sugeno (Grabisch *et al*. 2007) was used for the member level evaluation. Note that the λ-fuzzy measure becomes the probability measure when λ = 0, the belief measure when λ > 0, and the plausibility measure if -1 < λ < 0. While the mathematical sum of importance levels in Table 3 is larger than 1.0, it should satisfy following relationship

$$g(C \cup G \cup B \cup S) = 1 \tag{12}$$

Table 3 Weight factor of importance for members in RC frame structures (KISTEC 2009)

Members	Weight factor of importance (I_m)
Column	0.9
Girder	0.7
Beam	0.5
Slab	0.3

Thus, λ can be determined using the following equation

$$\begin{aligned}1 &= g(C)+g(G)+g(B)+g(S)\\&+\lambda[g(C)g(G)+g(C)g(B)+g(C)g(S)+g(G)g(B)+\\&\quad g(G)g(S)+g(B)g(S)]\\&+\lambda^2[g(C)g(G)g(B)+g(C)g(G)g(S)+g(C)g(B)g(S)+\\&\quad g(G)g(B)g(S)]\\&+\lambda^3[g(C)g(G)g(B)g(S)]\end{aligned} \tag{13}$$

where λ is -1 < λ < 0, which means that the λ-fuzzy measure is a plausibility measure, and g(C), g(G), g(B), and g(S) are the importance levels of the column, girder, beam, and slab, respectively. As λ value in the plausibility measure has a negative value, its result is more conservative than an average value resulting from a simple mathematical calculation (Grabisch *et al*. 2007). Using the λ value calculated using Eq. (14), each importance level can be combined. For example, the combined importance level of the column and girder can be calculated, as follows

$$g(C\cup G)=g(C)+g(G)+\lambda[g(C)g(G)] \tag{14}$$

Once the combined values of each importance level are determined, a fuzzy integral can be applied; this study used the Choquet integral (Choquet 1954). For better understanding of the application of Choquet method to the integration of importance levels, a simple example is shown in Fig. 9. The RSL of the nth floor with respect to chloride attack (CL_n) is calculated by the Choquet integral (Choquet 1954) as follows

$$\begin{aligned}CL_n=\int_x h(x_i)\cdot g(H_i)&=h(x_4)g(H_4)+[h(x_3)-h(x_4)]g(H_3)\\&+[h(x_2)-h(x_3)]g(H_2)\\&+[h(x_1)-h(x_2)]g(H_1)\end{aligned} \tag{15}$$

where x_i ($i=1,2,3,4$ in this example) is an arbitrary element in a member set x (i.e., x ={C, G, B, S}) sorted by their RSL values in a descending order such that the RSLs of x_i ($h(x_i)$) satisfy $h(x_1)\ge h(x_2)\ge h(x_3)\ge h(x_4)$. Also, $g(x_i)$ is the importance level of the member x_i, and $g(H_i)$ is the importance level of H_i using λ-fuzzy measure, in which H_i is the union of members x_i (i.e., $H_1=B\cup C\cup G\cup S$, $H_2=B\cup C\cup G$, $H_3=B\cup C$, $H_4=B$ in this example).

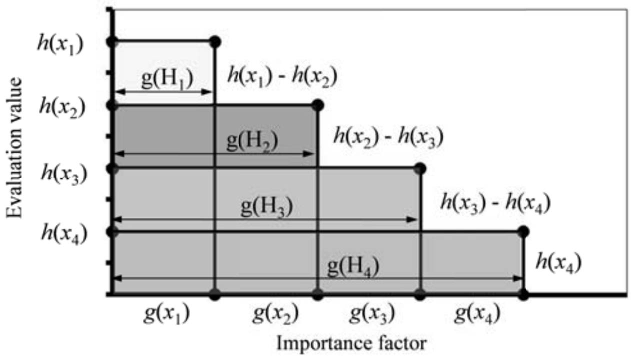


Fig. 9 Description of Choquet's integral (Choquet 1954)

The RSL of the nth floor against chloride penetration ($CL_n^{'}$) can be modified to reflect the effect of actual concrete compressive strength using the aforementioned concrete strength coefficient (η_1), as follows

$$CL_n^{'} = \eta_1 \times CL_n \tag{16}$$

The RSL of the nth floor against concrete carbonation and sulfate attack ($CA_n^{'}$, $SU_n^{'}$) also can be calculated in a similar manner, as follows

$$CA_n^{'} = \eta_1 \times CA_n \tag{17}$$

$$SU_n^{'} = \eta_1 \times SU_n \tag{18}$$

If n, $n+i$, and $n+j$-th floors of the target structure are sampled, the RSL of the structure with respect to each deterioration factor can be expressed as fuzzy sets, as follows

$$CL = \left\{CL_n^{'}, CL_{n+i}^{'}, CL_{n+j}^{'}\right\} \tag{19a}$$

$$CA = \left\{CA_n^{'}, CA_{n+i}^{'}, CA_{n+j}^{'}\right\} \tag{19b}$$

$$SU = \left\{SU_n^{'}, SU_{n+i}^{'}, SU_{n+j}^{'}\right\} \tag{19c}$$

As shown in Fig. 10, the RSL of the concrete structure with respect to each deterioration factor can be obtained by the union of the RSL of each sample floor with respect to each deterioration factor, for which the importance coefficient of each floor needs to be determined. According to KITSEC (2009), the floor importance factor (ζ_n) can be determined based on the load tributary ratio of the target floor. The load tributary ratio of the nth floor (ψ_n) is defined, as follows

$$\psi_n = \frac{N-(n-1)}{N} \tag{20}$$

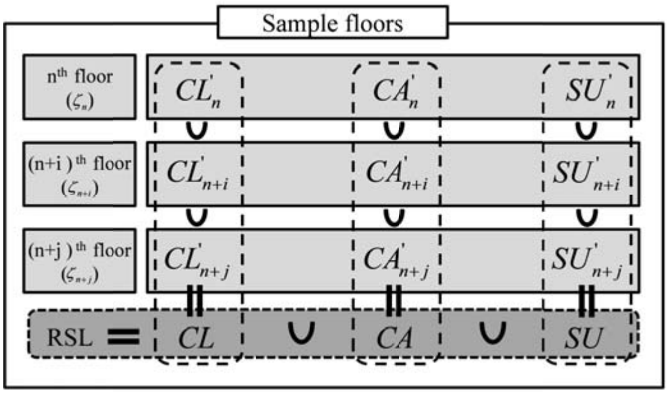


Fig. 10 Fuzzy calculation process for RSL of a multistory building

where, N is the total number of floors, including the number of underground floors, and n is the n^{th} floor number. In this study, the floor importance factor (ζ_n) was adopted by determining the tributary ratio of n^{th} floor (ψ_n) in an average manner, as shown in Table 4.

In order to evaluate the RSL of the whole structure, the RSL with respect to each deterioration factor need to be combined. Fig. 11 shows an example of fuzzy sets in which the RSL of the structure against chloride attack was assumed to be 53 years. In this example, the 53-year RSL belongs to the fuzzy sets C and D, and the RSL with respect to chloride attack (CL_C, CD_D) can be expressed in the form of fuzzy sets, as follows

$$CL_C = (53, 0.65) \tag{21a}$$

$$CL_D = (53, 0.35) \tag{21b}$$

Based on the α-cut method, fuzzy set D, which has the membership function below 0.5, can be removed. Next, the 53-year RSL against chloride attack belongs to fuzzy set C with the membership function of 0.65. For concrete carbonation and sulfate attack, the same procedures can be applied to obtain their fuzzy sets. Then, the RSL of the structure considering all the deteriorating factors can be expressed in fuzzy sets as follows

$$RSL = \{CL, CA, SU\} \tag{22}$$

Eq. (23) can then be operated by the union of all fuzzy set elements. For example, if the RSLs against each deterioration factor after applying the α-cut method are assumed to be

$$\begin{aligned} CL_C &= (53, 0.65) \\ CA_D &= (49, 0.55) \\ SU_D &= (32, 0.60) \end{aligned} \tag{23}$$

Then, the union of these fuzzy sets can be expressed as shown in Fig. 12. The RSL fuzzy set of the structure shown in Fig. 12 is, however, non-quantitative, and its defuzzification should be performed to use it as a quantitative value. The defuzzification of RSL (RSL_D) can be operated by

(Hamid and Pratap 1992, Jang and Sun 1995)

$$RSL_D = \frac{\sum X_k \chi(X_k)}{\sum \chi(X_k)} \tag{24}$$

Table 4 Importance grades and corresponding weight factors for floor levels (KISTEC 2009)

Importance grade	Important degree range (load tributary ratio, ψ_n)	Representative value (floor importance factor, ζ_n)
A	$0.8 \le \psi_n \le 1.0$	0.9
B	$0.6 \le \psi_n \le 0.8$	0.7
C	$0.4 \le \psi_n \le 0.6$	0.5
D	$0.2 \le \psi_n \le 0.4$	0.3
E	$0 \le \psi_n \le 0.2$	0.1

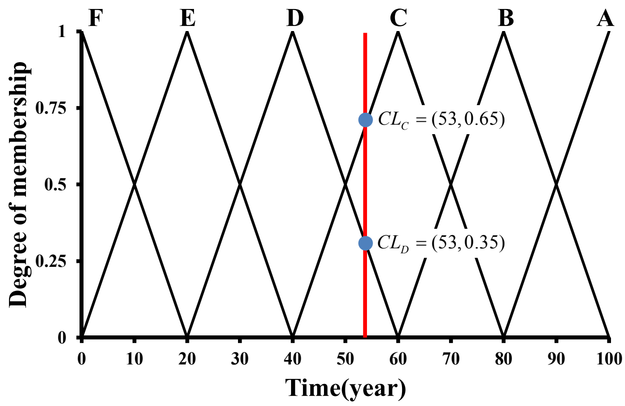


Fig. 11 Illustration of the fuzzy sets for chloride attack

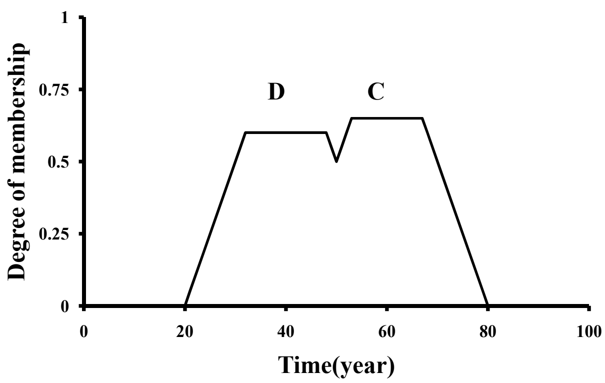


Fig. 12 Illustration of the union of membership degrees of deterioration factors

where, X_k is the RSL with respect to the deterioration factors (i.e., CL_k, CA_k, and SU_k) and the subscript k is the predefined fuzzy set, which is A, $B,\cdots$, or F as shown in Fig. 11. Also, $\chi(X_k)$ is the membership function of X_k. In addition, by considering the structural condition evaluation coefficient (η_2) in Table 2, the RSL of the concrete structure (RSL_F) can be expressed in a final form, as follows

$$RSL_F = \eta_2 \times RSL_D \tag{25}$$

In summary, Fig. 13 shows the flowchart of calculation procedures for determining the RSL of concrete structures.

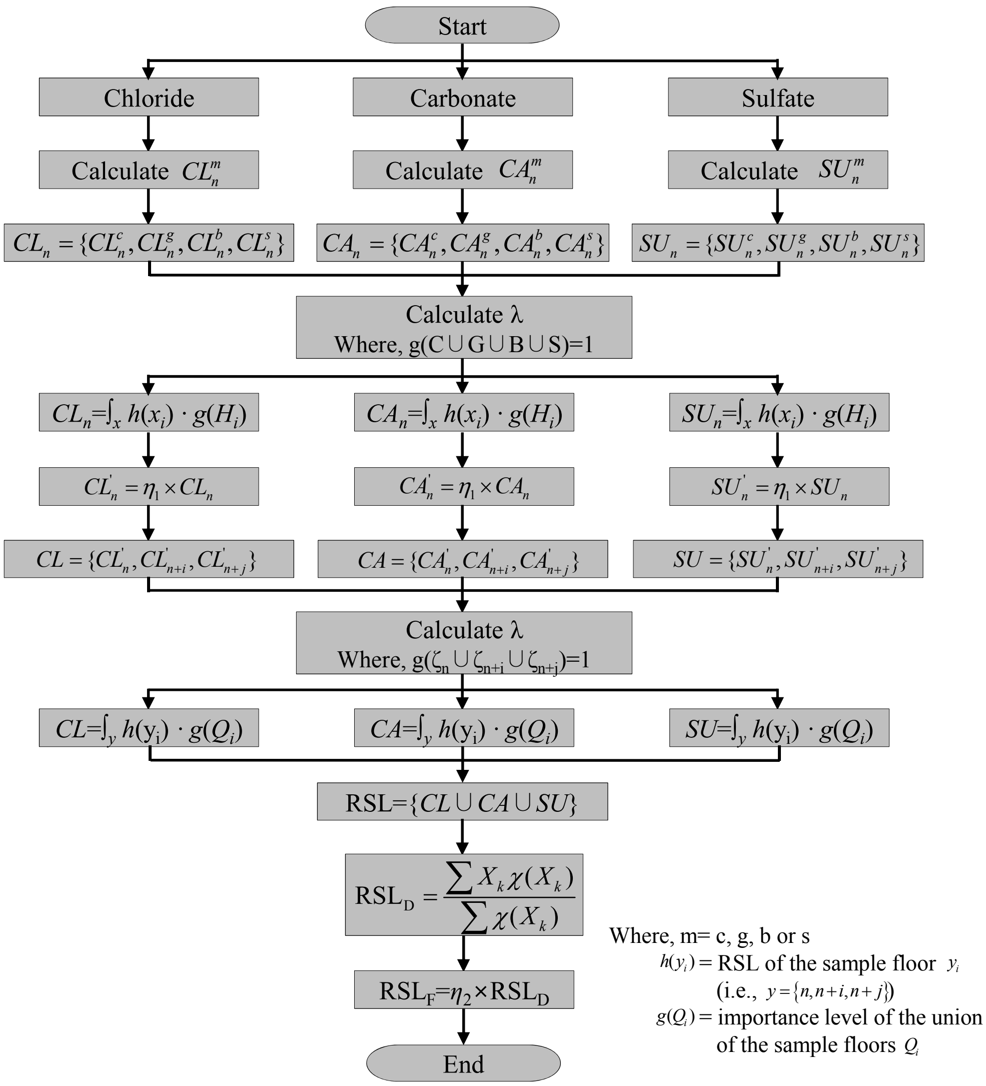


Fig. 13 Flow chart of the proposed evaluation method for RSL

3.2 Example of application

The proposed RSL evaluation method has been applied to estimate the RSLs of a concrete building structure built in 1986 and shown in Fig. 14. The inspection was completed in 2004; thus it had been occupied for 18 years at the time of inspection. The target structure is a school with five stories and a basement floor. The design compressive strength of concrete (f_{ck}) was 24 MPa, and there was available records of previous safety inspections (KISTEC 2005). In this study, the basement, 2nd and 5th floors were chosen as the sample floors; the concrete cover depths of the sample floors were shown in Table 5.

For illustration purposes on the effect of different deterioration factors, this study set three virtual sites (i.e., sites A, B, and C); i.e., the RSLs of three concrete building structures are estimated and compared here; all details are identical except for the virtually-imposed deterioration conditions. While the depths of concrete carbonation in the three sites were assumed to be identical, the conditions related to chloride and sulfate penetrations were applied differently to these sites, as shown in Table 6. The effective chloride diffusion coefficients (D_d) were assumed to be 4.26×10^{-5} m^2/s for sites A and C, and 3.77×10^{-5} m^2/s for site B, respectively. Then, the surface chloride ion concentration was calculated using Eq. (2), based on the airborne chloride concentration. The amount of sulfate was selected, as shown in Table 6, by referring to the sulfate-resisting erosion standard specified in ACI Committee 201.2R-08 (2008). Based on the amount of $MgSO_4$ for each site, sites A and B were set to a moderate level, and site C was set to a severe level, as shown in Table 7. In the ACI Committee 201.2R-08 (2008) it is recommended using sulfate-resisting cement or reducing $Ca(OH)_2$ in the cement paste in cases of severe exposure

Table 5 Thickness of concrete covers of members measured from sample floors (mm) (KISTEC 2005)

Sample floor levels	Members	No.1	No.2	No.3	No.4	No.5	Mean
basement	column	23	21	19	22	20	21
	girder	30	30	32	31	32	31
	beam	31	33	31	29	31	31
	slab	22	25	25	24	24	24
2nd floor	column	33	37	35	31	39	35
	girder	28	31	30	29	32	30
	beam	25	31	32	33	29	30
	slab	26	26	30	29	29	28
5rd floor	column	36	35	32	34	33	34
	girder	40	44	42	41	43	42
	beam	36	37	37	38	37	37
	slab	28	30	28	26	28	28

Table 6 Site condition assumed for analysis

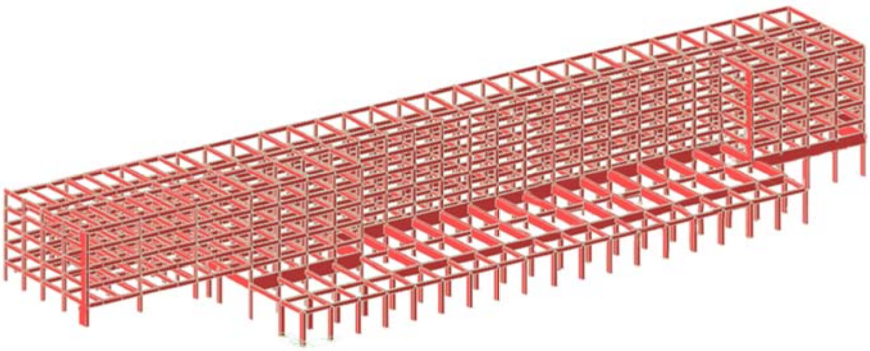
Site	$C_0(\text{kg/m}^3)$	$D_d(\text{m}^2/\text{h})$	M_gSO_4(mol/l)
A	1.86	4.26×10^{-5}	0.01
B	1.86	3.77×10^{-5}	0.01
C	1.86	4.26×10^{-5}	0.15

Table 7 Classification of severity of sulfate environment according to ACI 201.2R-08 (2008)

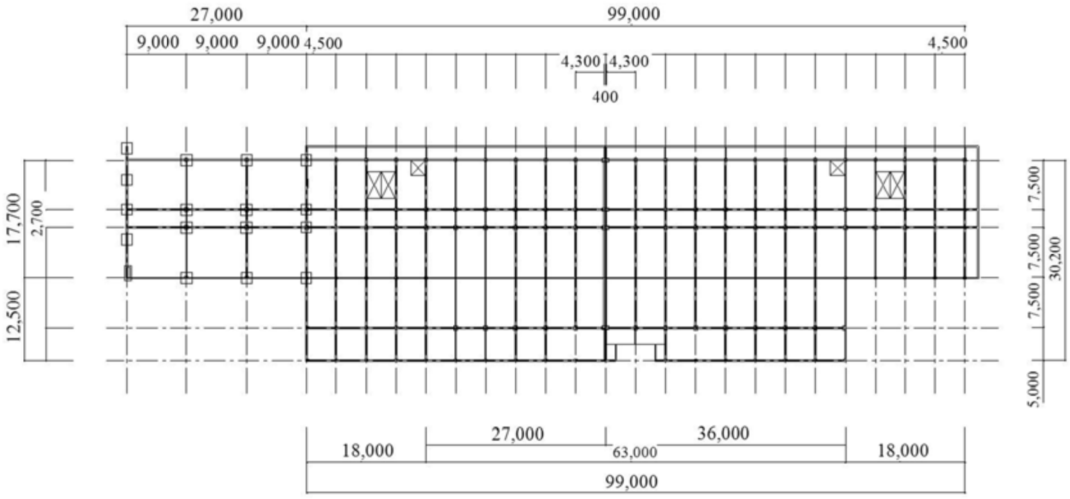
Exposure	Concentration of soluble sulfates expressed as SO_4	
	in soil percent	in water ppm
mild (case 1)	< 0.1	< 150
moderate (case 2)	0.1 to 0.2	150 to 1500
severe (case 3)	0.2 to 2.0	1500 to 10000
very severe (case 4)	> 2.0	> 10000

to the sulfate penetration. In this example, however, it was assumed that the sulfate-resisting cement was not used to clearly show the effect of sulfate penetration on the RSL of the concrete building structure.

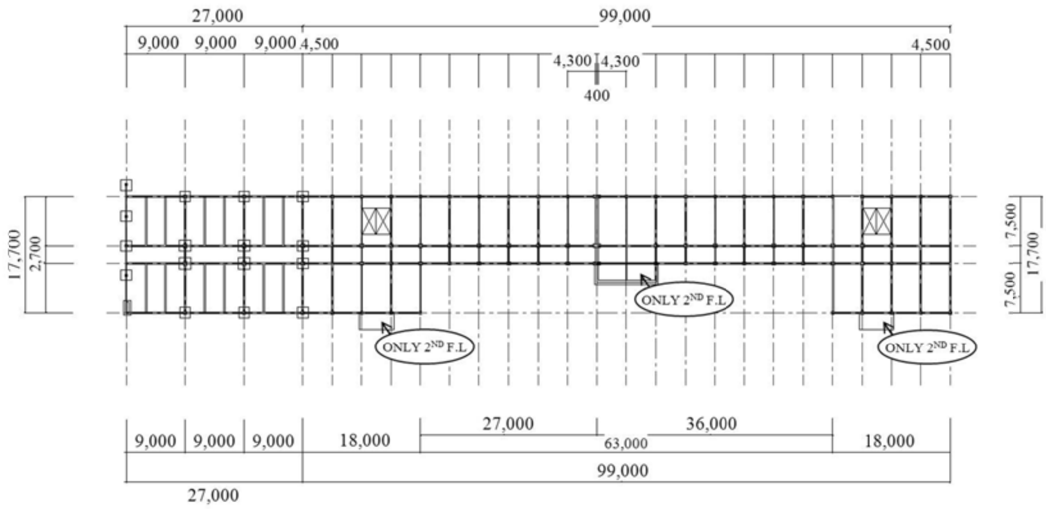
According to the analysis process shown in Fig. 13, the RSL in the level of member, floor, and whole structure were evaluated by the site conditions; the estimated results are shown in Table 8. While the estimated RSL on chloride attack was about 14 years higher for the structure in site B than compared to site A, due to its smaller effective chloride ion diffusion coefficient (D_d), the proposed model calculated a final RSL of the site B structure as 50 years, which is about 10 years longer than that of the site A structure. Note that the proposed model also reflects the effect of not only chloride attack but also sulfate penetration and concrete carbonation. Also, for the structure located at the site C, whose sulfate concentration is higher than that found at site A with the same conditions for other factors, the estimated RSL against sulfate attack was 11 years smaller than that located in the site A, and its final RSL was estimated as 30 years that was about 10 years less than the structure located at site A. Indeed, the estimated RSL values of previously illustrated example are meaningful in and of itself, but it is also important to recognize that the proposed evaluation method reasonably reflects the effect of multiple deteriorations on the RSL of the structures using the fuzzy theory. In addition, if there is any advancement in the deterministic RSL evaluation methods on deterioration factors in near future, it can be immediately applied to the proposed method to account for the effect of combined deterioration factors on the RSL of concrete structures, which would lead to provide an enhanced accuracy.



(a) 3D elevation view



(b) Plan view for 1st floor



(c) Plan view for 2nd~5th floors

Fig. 14 An example building (a high-school building) (KISTEC 2005)

Table 8 Comparisons of evaluation results on example buildings under different site conditions (unit: year)

Sample floor levels	Members	Site A chloride	Site A carbonate	Site A sulfate	Site B chloride	Site B carbonate	Site B sulfate	Site C chloride	Site C carbonate	Site C sulfate
		severe	moderate	severe	moderate	moderate	severe	severe	moderate	very severe
basement	column	6	18	20	9	18	20	6	18	9
	girder	35	49	30	42	49	30	35	49	19
	beam	35	49	30	41	49	30	35	49	19
	slab	13	17	23	17	17	23	13	17	12
	overall	31	45	28	37	45	28	31	45	17
2nd floor	column	42	50	34	58	50	34	42	50	23
	girder	28	45	29	38	45	29	28	45	18
	beam	27	54	29	38	54	29	27	54	18
	slab	30	44	27	30	44	27	30	44	16
	overall	40	51	33	56	51	33	40	51	22
5rd floor	column	44	46	33	54	46	33	44	46	22
	girder	59	42	41	92	42	41	59	42	30
	beam	46	27	36	67	27	36	46	27	25
	slab	20	25	27	30	25	27	20	25	16
	overall	55	45	39	82	45	39	55	45	28
RSL for each deterioration factor		39	49	32	53	49	32	39	49	21
RSL (defuzzification)			40			50			30	

4. Conclusions

This study proposed using a fuzzy theory-based method to estimate the RSL of concrete structures to properly account for multiple factors in concrete durability, such as concrete carbonation, chloride penetration, and sulfate attack. The presented model introduced weight factors according to member types, such as beam, girder, column, and slab, and the importance levels were defined for floor levels based on their load tributary ratios. The RSLs for each durability degradation factor were first expressed in the form of Fuzzy sets and then were quantified by the defuzzification procedure. The current structural condition was also considered by the condition coefficient in the proposed RSL evaluation model. For better understanding of the proposed model, an example has been provided demonstrating that the proposed model can provide comprehensive estimation of the RSL of concrete structures influenced by multiple durability deterioration factors. Yet, further studies would be still required for more objective verification of this RSL model and also for updating to consider in more detail the interactive effects of the combined actions.

Acknowledgments

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