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A damage model formulation: unilateral effect and RC structures analysis

José J.C. Pituba*

Department of Civil Engineering, Campus Catalão, Federal University of Goiás,
Av. Dr Lamartine Pinto de Avelar, 1120, Catalão, Goiás 75740-020, Brazil

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Abstract. This work deals with a damage model formulation taking into account the unilateral effect of the mechanical behaviour of brittle materials such as concrete. The material is assumed as an initial elastic isotropic medium presenting anisotropy, permanent strains and bimodularity induced by damage evolution. Two damage tensors governing the stiffness in tension or compression regimes are introduced. A new damage tensor in tension regimes is proposed in order to model the diffuse damage originated in prevails compression regimes. Accordingly with micromechanical theory, the constitutive model is validate when dealing with unilateral effect of brittle materials, Finally, the proposed model is applied in the analyses of reinforced concrete framed structures submitted to reversal loading. The numerical results have shown the good performance of the modelling and its potentialities to simulate practical problems in structural engineering.

Keywords: damage mechanics; unilateral effect; concrete structures; structural failure; constitutive model

1. Introduction

The Continuum Damage Mechanics (CDM) has already proved to be a suitable tool for simulating the material deterioration in equivalent continuous media due exclusively to microcracking process. Many constitutive models for brittle and ductile media taking into account anisotropic characteristics of those media have been proposed. It can be mentioned: Cauvin and Testa (1999) have suggested a damage model applied to the transverse isotropy case; Lemaitre *et al.* (2000) and Brünig (2004) have developed constitutive models to study the damage processes in ductile media; Ibrahimbegovic *et al.* (2008) have proposed a coupled damage-plasticity model analyzing materials that present irreversible plastic deformation, change of elastic response and the localized failure; On the other hand, the localized failure process has been frequently studied by researchers (Brancherie and Ibrahimbegovic 2009, Kucerova *et al.* 2009, Hervé *et al.* 2005) in order to ensure finite element mesh objectivity. Also, some simplified damage models have been proposed in order to evaluate the mechanical properties of the brittle materials in practical situations of the Structural Engineering (Mohammed and Parvin 2013, Cao and Ronagh 2013, Esposito and Hendriks 2014).

*Corresponding author, Professor, E-mail: julio.pituba@pq.cnpq.br

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On the other hand, for modeling the unilateral effect present in brittle media subject to reversal loading, many possible strategies have been proposed in the literature to model the stiffness recovery as described in Comi (2001), Carol and Willam (1996), Welemane and Comery (2002), Bielski *et al.* (2006), Liu *et al.* (2008) and Araújo and Proença (2008). For more details, in Bielski *et al.* (2006) is presented a summary of some constitutive model formulations which take into account the unilateral effect of the damage process, such as: the use of fourth-rank projection operators for the decomposition of the stress and strain tensors into the positive and negative projections, besides the use of the generalized projection operators.

Despite the progress in the macroscopic modelling of the unilateral effect (in particular, the continuity problems that arise when the induced anisotropy is simultaneously described), this subject still remains as an open research field when it deals with induced anisotropy damage models, even when the micromechanical theory has been used to justify the proposal of constitutive models dealing with damaged media, Welemane and Comery (2002), Zhu *et al.* (2008), Zhu *et al.* (2009), Deudé *et al.* (2002), Pensée *et al.* (2002) and Pichler and Dormieux (2009). Pensée *et al.* (2002) have developed an Eshelby-type approach for open and closed mesocracks leading to a more realistic description of the macromechanical behavior in brittle materials in three-dimensional continuum damage mechanics. The development has led to a general expression of the free energy of the microcracked material that includes unilateral effects. The authors have proposed a formulation considering closed penny-shaped cracks using the Eshelby homogenization techniques. The modelling proposed satisfies the requirements at the micromechanical theory: convexity and continuity of the potential, continuity of the mechanical response and symmetry of the damage elastic stiffness tensor, Pensée *et al.* (2002). This requirements is also investigated when dealing with the damage model proposed here (please, see section 3). On the other hand, Deudé *et al.* (2002) has developed a micromechanical approach to deal with the macroscopic modelling of the nonlinear poroelastic behaviour present in cracked rocks.

The unilateral effect is a research field still opened even nowadays when dealing with more actual approaches based on multiscale analysis procedures, Skarzynski and Tejchman (2012), Pituba and Souza Neto (2012) and Fernandes *et al.* (2015).

In this work, a damage model taking into account the unilateral effect of the mechanical behaviour of brittle materials such as concrete is proposed. An induced anisotropy, permanent strains and bimodularity are considered. The author intends to contribute to discussion about the unilateral effect in brittle materials considering the assessment of some requirements of the micromechanical theory. Besides, the work intends to present a simplified version of the proposed damage model in order to apply to practical situations of Structural Engineering. Indeed, this work intends to contribute to the modelling of damage unilateral effect applied to concrete structures. However, it must be noted that the proposed model is not capable to take into account the friction effects, namely blocking and dissipative sliding of closed microcrack lips. This feature can be discussed in future works.

This work is divided into five sections. In section 2, the damage model formulation is briefly described. Then, some aspects regarding to unilateral effect as well as the validation of the proposed model is presented in section 3. After that, some numerical analyses of RC framed structures are shown in section 4. Finally, in section 5 some concluding remarks, limitations and possible extensions are discussed.

2. Damage model for brittle materials

In this work, for modelling the concrete behaviour, it can be assumed that the concrete belongs to the category of materials which can be considered initially isotropic and unimodular presenting different behaviours in tension and compression regimes when the media presents damage process. A formulation of constitutive laws for isotropic and anisotropic elastic materials presenting different behaviours in tension and compression predominant regimes under small deformations has been proposed by Curnier *et al.* (1995) for two and three-dimensional cases. The authors have considered a bimodular hyperelastic material defining an elastic potential energy density W which must be once continuously differentiable (whole wise), but only piecewise twice continuously differentiable. In this way, the model is able to produce different response in tension and compression predominant regimes. Pituba (2006) has extended that formulation in order to take into account the damage effects. Accordingly with, the bulk (λ_{ab}) and shear (μ_a) module are considered such as functions of the damage state, so that the stress-strain relationship would be influenced by damage variables. Moreover, the hypersurface $g(\varepsilon, D_i)$ adopted as the criterion for identification of the constitutive responses in compression or tension predominant regimes would be also influenced by the damage variables. Then, a damage constitutive model accounting for induced anisotropy and bimodular elastic response for the concrete has been derived from Pituba (2006) and its potentialities for 1D and 2D analyses have been discussed in Pituba (2010), Pituba and Fernandes (2011), Pituba and Lacerda (2012) and Pituba *et al.* (2012). Also, the numerical simulations of uniaxial, biaxial and triaxial stress experimental tests are reported in Pituba and Fernandes (2011). The original version of the damage model is bimodular in the sense that presents different elasticity tensors in tension and compression predominant regimes. Thus, the model is potentially capable to simulate the stiffness recovery when the medium is submitted to a reversal loading that evidences the transition from of tension to compression predominant regimes, i.e., the so-called unilateral behaviour of the damaged concrete. However, the model is not capable to simulate the influence of the previous damage processes in compression predominant regimes (diffuse damage) when a transition from compression to tension predominant regimes takes place, Comi (2001). From a micromechanics point of view, this feature is due to the partial closure process of microcracks subject to compression loading which affects less the elasticity module in compression predominant regimes than in tension ones, Desmorat (2000). Therefore, to avoid this problem a new elasticity tensor is proposed in this work and some numerical analyses are performed to simulate practical problems in structural engineering.

The original damage model formulation has been developed in Pituba and Fernandes (2011) considering the formalism presented in Pituba (2006). To obtain the class of anisotropy induced and considered in the model one has assumed that locally the loaded concrete presents damage distribution oriented diffusely as appointed by experimental observations (Willam *et al.* 1988, Kupfer *et al.* 1969, Van Mier 1997). One has also considered that the oriented damage is responsible for the changes on the material characteristics leading to a transverse isotropic medium. Note that a constitutive model with orthotropy induced by damage could be proposed. However, the parametric identification could become infeasible at the practical point of view. Moreover, the model respects the principle of energy equivalence between damaged real medium and equivalent continuum medium established in the CDM leading to the guarantee of the major symmetry of the constitutive tensor even if non-symmetric damage tensors are used throughout the formulation, Lemaitre (1996). More details can be found in Pituba and Fernandes (2011).

Initially, for dominant tension states, a damage tensor is proposed

$$\boldsymbol{D_T}=f_1(D_1,D_4,D_5)\,(\boldsymbol{A}\otimes\boldsymbol{A})+2f_2(D_4,D_5)\,[(\boldsymbol{A}\,\underline{\overline{\otimes}}\,\boldsymbol{I}+\boldsymbol{I}\,\underline{\overline{\otimes}}\,\boldsymbol{A})-(\boldsymbol{A}\otimes\boldsymbol{A})] \tag{1}$$

where $f_1(D_1, D_4, D_5) = D_1 - 2\, f_2(D_4, D_5)$ and $f_2(D_4, D_5) = 1 - (1\text{-}D_4)\,(1\text{-}D_5)$. The variable D_1 represents the damage in the direction orthogonal to the transverse isotropy local plane of the material, while D_4 is representative of the damage due to the sliding movement between the crack faces. The third damage variable, D_5, is only activated if a previous compression state accompanied by damage has occurred. In Curnier *et al.* (1995), the tensor **I** is the second-order identity tensor and the tensor **A**, is formed by dyadic product of the unit vector perpendicular to the transverse isotropy plane for himself. Those products are given in Pituba (2006). For dominant compression states, it is proposed another damage tensor

$$\boldsymbol{D_C}= f_1^*(D_2,D_4,D_5)\,(\boldsymbol{A}\otimes\boldsymbol{A})+f_2(D_3)\,[(\boldsymbol{I}\,\overline{\otimes}\,\boldsymbol{I})-(\boldsymbol{A}\otimes\boldsymbol{A})]+2f_3(D_4,D_5)$$
$$[(\boldsymbol{A}\,\underline{\overline{\otimes}}\,\boldsymbol{I}+\boldsymbol{I}\,\underline{\overline{\otimes}}\,\boldsymbol{A})-(\boldsymbol{A}\otimes\boldsymbol{A})] \tag{2}$$

where $f_1^*(D_2, D_4, D_5) = D_2 - 2f_3(D_4, D_5)$,$f_2(D_3) = D_3$ and $f_3(D_4,D_5)=1-(1\text{-}D_4)(1\text{-}D_5)$. Note that the compression damage tensor introduces two additional scalar variables in its composition: D_2 and D_3. The variable D_2 (damage perpendicular to the transverse isotropy local plane) reduces the Young's modulus in that direction and in conjunction to D_3 (that represents the damage in the transverse isotropy plane) degrades the Poisson's ratio throughout the perpendicular planes to the one of transverse isotropy.

On the other hand, the constitutive tensor is written as

$$\mathbf{E}\,(\varepsilon) := \begin{cases} E_-(\varepsilon)\ \text{if}\ \ g\ (\varepsilon, D_T, D_C) < 0, \\ E_+(\varepsilon)\ \text{if}\ \ g\ (\varepsilon, D_T, D_C) > 0, \end{cases} \tag{3}$$

$$\boldsymbol{E}_+(\varepsilon)=\lambda_{11}[\boldsymbol{I}\otimes\boldsymbol{I}]+2\mu_1[\boldsymbol{I}\,\underline{\overline{\otimes}}\,\boldsymbol{I}]-\lambda_{22}^+(D_1,D_4,D_5)\,[\boldsymbol{A}\otimes\boldsymbol{A}]-\lambda_{12}^+(D_1)\,[\boldsymbol{A}\otimes\boldsymbol{I}+\boldsymbol{I}\otimes\boldsymbol{A}]$$
$$-\mu_2(D_4,D_5)\,[\boldsymbol{A}\,\underline{\overline{\otimes}}\,\boldsymbol{I}+\boldsymbol{I}\,\underline{\overline{\otimes}}\,\boldsymbol{A}] \tag{4}$$

$$\boldsymbol{E}_-(\varepsilon)=\lambda_{11}[\boldsymbol{I}\otimes\boldsymbol{I}]+2\mu_1[\boldsymbol{I}\,\underline{\overline{\otimes}}\,\boldsymbol{I}]-\lambda_{22}^-(D_2,D_3,D_4,D_5)\,[\boldsymbol{A}\otimes\boldsymbol{A}]-\lambda_{12}^-(D_2,D_3)\,[\boldsymbol{A}\otimes\boldsymbol{I}+\boldsymbol{I}\otimes\boldsymbol{A}]$$
$$-\lambda_{11}^-(D_3)\,[\boldsymbol{I}\otimes\boldsymbol{I}]-\frac{(1-2\nu_0)}{\nu_0}\lambda_{11}^-(D_3)\,[\boldsymbol{I}\,\overline{\otimes}\,\boldsymbol{I}]-\mu_2(D_4,D_5)\,[\boldsymbol{A}\,\underline{\overline{\otimes}}\,\boldsymbol{I}+\boldsymbol{I}\,\underline{\overline{\otimes}}\,\boldsymbol{A}] \tag{5}$$

The remaining parameters will only exist for no-null damage evidencing the anisotropy and bimodularity induced by damage process. Those parameters are given by

$$\lambda_{22}^+(D_1,D_4,D_5)=(\lambda_0+2\mu_0)(2D_1-D_1^2)-2\lambda_{12}^+(D_1)-2\mu_2(D_4,D_5)$$
$$\lambda_{12}^+(D_1)=\lambda_0 D_1;\ \ \mu_2(D_4,D_5)=2\mu_0[1-(1-D_4)^2(1-D_5)^2]$$
$$\lambda_{22}^-(D_2,D_3,D_4,D_5)=(\lambda_0+2\mu_0)(2D_2-D_2^2)-2\lambda_{12}^-(D_2,D_3)+\frac{(\nu_0-1)}{\nu_0}\lambda_{11}^-(D_3)-2\mu_2(D_4,D_5)$$

$$\lambda_{12}^{-}(D_2, D_3) = \lambda_0[(1-D_3)^2 - (1-D_2)(1-D_3)]; \lambda_{11}^{-}(D_3) = \lambda_0(2D_3 - D_3^2) \tag{6}$$

Therefore, the constitutive model includes two damage tensors in order to take into account the bimodularity induced by damage in the concrete behaviour. Therefore, it is necessary a criterion to define the tension and compression dominant states to indicate what damage tensor should be used.

The criterion has been extended in Pituba (2006) to deal with damaged media. This extension influences the hyperplane definition. Therefore, the following relationship has been proposed

$$g(\boldsymbol{\varepsilon}, \boldsymbol{D_T}, \boldsymbol{D_C}) = \boldsymbol{N}(\boldsymbol{D_T}, \boldsymbol{D_C}) \; . \; \boldsymbol{\varepsilon_e} \tag{7}$$

In Pituba and Fernandes (2011), a particular form is adopted for the hypersurface in the strain space: a hyperplane $g(\varepsilon)$ defined by the unit normal $\mathbf{N}$ ($\|\mathbf{N}\| = 1$) and characterized by its dependence of the strain and damage states. Accordingly with Eq. (7) and referring to general cases of loading, the following relationship has been proposed for the hyperplane

$$g(\boldsymbol{\varepsilon}, \boldsymbol{D_T}, \boldsymbol{D_C}) = \boldsymbol{N}(\boldsymbol{D_T}, \boldsymbol{D_C}) \; . \; \boldsymbol{\varepsilon_e} = \gamma_1(D_1, D_2) \;\; \varepsilon_V^e \; + \gamma_2(D_1, D_2) \;\; \varepsilon_{11}^e \tag{8}$$

where $\gamma_1(D_1,D_2) = \{1+H(D_2)[H(D_1)-1]\}\eta(D_1)+\{1+H(D_1)[H(D_2)-1]\}\eta(D_2)$ and $\gamma_2(D_1,D_2) = D_1+D_2$.

The Heaveside functions employed above are given by

$$H(Di) = 1 \text{ for } Di > 0; H(Di) = 0 \text{ for } Di = 0 \qquad (i = 1, 2) \tag{9}$$

The $\eta(D_1)$ and $\eta(D_2)$ functions are defined, respectively, for the tension and compression cases, assuming for the first one that there was no previous damage in compression affecting the present tension damage variable D_1 and analogously, for the second one that has not had previous tension damage affecting variable D_2. The proposed functions are

$$\eta\,(D_1) = \frac{-D_1 + \sqrt{3 - 2D_1^2}}{3} \tag{10}$$

$$\eta\,(D_2) = \frac{-D_2 + \sqrt{3 - 2D_2^2}}{3} \tag{11}$$

Note that if the damage process in the material is not activated ($D_1 = D_2 = 0$), the Eq. (8) recovers the equation proposed by Comi (2001), thus the formulation satisfies the proposed condition of initially isotropic material. On the other hand, if the material is totally damaged, $D_1 = D_2 = 1$ ($\eta\,(D_1) = \eta\,(D_2) = 0$) and $\gamma_2 = 2$, the hyperplane $g(\boldsymbol{\varepsilon})$ is coincident to the transverse isotropy local plane of the material and, therefore, the normal vector to the hyperplane is given by the transverse isotropy tensor **A**.

Regarding to anisotropy induced by damage, it is convenient to separate the damage criterion into two: the first one is only used to indicate damage beginning what means that the material is no longer isotropic; the second one is used for loading and unloading, when the material is already considered as transverse isotropic. This second criterion identifies if there is or not evolution of the damage variables. That division is justified by the difference between the complementary elastic strain energies of isotropic and transverse isotropic material.

If there is damage evolution, i.e., when $\dot{\mathbf{D}}_{\mathbf{T}} \neq \mathbf{0}$ or $\dot{\mathbf{D}}_{\mathbf{C}} \neq \mathbf{0}$, the evolution laws of the damage variables are written as associated variables functions. Considering only the case of monotonic loading, the evolution laws proposed for the scalar damage variables are resulting of fittings on experimental results. The general form proposed is

$$D_i = 1 - \frac{1 + A_i}{A_i + \exp\left[B_i(Y_i - Y_{0i})\right]} \quad i = 1,\, 5 \tag{12}$$

where A_i, B_i and Y_{0i} are parameters of the model that must be identified through the uniaxial tension and compression tests and biaxial compression tests.

When the damage process is activated, the formulation starts to involve the tensor **A** that depends on the normal to the transverse isotropy plane. Therefore, it is necessary to establish some rules to identify its location for an actual strain state. Initially, it is established a general criterion for the existence of the transverse isotropy plane. In Pituba and Fernandes (2011) is proposed that the transverse isotropy due to damage only arises if positive strain rates exist at least in one of the principal directions. After assuming such proposition as valid, some rules to identify its location are defined.

However it is necessary to take into account the diffuse damage generated in previous compression dominant regimes. This problem can be solved by introduction of a new elasticity tensor in tension dominant regimes. Taking as example, an uniaxial modelling and respecting the principle of energy equivalence, the constitutive tensor is written as

$$\boldsymbol{E_T} = \boldsymbol{E}_0(1 - \boldsymbol{D}_1)^2(1 - \boldsymbol{D}_2)^2 \tag{13}$$

The relationship above shows that in a situation where a tension dominant state is prevailing with occurrence of previous compression damage process, it is possible to solve the problem discussed here. By analogy, under multiaxial stress states it can be concluded that the damage tensor in compression $\mathbf{D_C}$ should composes the expression of the constitutive tensor in tension dominant states. Therefore, respecting the principle of energy equivalence, the constitutive tensor is now written as

$$\boldsymbol{E_T} = (\boldsymbol{I} - \boldsymbol{D_C})(\boldsymbol{I} - \boldsymbol{D_T})\boldsymbol{E}_0(\boldsymbol{I} - \boldsymbol{D_T})(\boldsymbol{I} - \boldsymbol{D_C}) \tag{14}$$

Considering a matrix representation and assuming, for instance, that the transversal isotropy local plane is coincident to the 2–3 plane, the constitutive tensor $\mathbf{E_T}$ may be described as follows

$$\boldsymbol{E_T} = \begin{bmatrix} (\lambda_0 + 2\mu_0)(1-\boldsymbol{D}_1)^2(1-\boldsymbol{D}_2)^2 & \lambda_0(1-\boldsymbol{D}_1)(1-\boldsymbol{D}_2)(1-\boldsymbol{D}_3) & \lambda_0(1-\boldsymbol{D}_1)(1-\boldsymbol{D}_2)(1-\boldsymbol{D}_3) & 0 & 0 & 0 \\ \lambda_0(1-\boldsymbol{D}_1)(1-\boldsymbol{D}_2)(1-\boldsymbol{D}_3) & (\lambda_0 + 2\mu_0)(1-\boldsymbol{D}_3)^2 & \lambda_0(1-\boldsymbol{D}_3)^2 & 0 & 0 & 0 \\ \lambda_0(1-\boldsymbol{D}_1)(1-\boldsymbol{D}_2)(1-\boldsymbol{D}_3) & \lambda_0(1-\boldsymbol{D}_3)^2 & ((\lambda_0 + 2\mu_0)(1-\boldsymbol{D}_3)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\mu_0(1-\boldsymbol{D}_4)^4(1-\boldsymbol{D}_5)^4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu_0(1-\boldsymbol{D}_4)^4(1-\boldsymbol{D}_5)^4 \end{bmatrix} \tag{16}$$

It can be noted that the Eqs. (5) and (15) present different values for the shear components in compression and tension dominant states, respectively. Therefore, this alternative formulation does not respect the Curnier´s condition about the tangential continuity (Curnier *et al.* 1995) when diffuse damage takes place. To avoid this problem, another expression for the damage tensor in compression dominant states $\mathbf{D}_\mathbf{C}^*$ is proposed. This tensor is given by

$$\boldsymbol{D}_C^* = f_1(D_2,)\,(\boldsymbol{A}\otimes\boldsymbol{A}) + f_2(D_3)\,[(\boldsymbol{I}\,\bar{\otimes}\,\boldsymbol{I}) - (\boldsymbol{A}\otimes\boldsymbol{A})] \tag{16}$$

where $f_1(D_2) = D_2$ and $f_2(D_3) = D_3$. It is important to observe that the damage tensor $\mathbf{D}_\mathbf{C}^*$ provides the diffuse damage in previous compression states by means the changing of the volumetric modulus, as proposed in Comi (2001). For simplicity, considering a matrix representation and assuming, for visualization proposals, that the transversal isotropy local plane is coincident to the 2–3 plane, Eq. (16) is written as

$$\boldsymbol{D}_C^* = \begin{bmatrix} \boldsymbol{D}_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{D}_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{D}_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

Finally, take into account the principle of energy equivalence, the constitutive tensor for tension dominant states is given by

$$\boldsymbol{E}_T = (\boldsymbol{I} - \boldsymbol{D}_C^*)(\boldsymbol{I} - \boldsymbol{D}_T)\boldsymbol{E}_0(\boldsymbol{I} - \boldsymbol{D}_T)(\boldsymbol{I} - \boldsymbol{D}_C^*) \tag{18}$$

$$\boldsymbol{E}_T = \begin{bmatrix} (\lambda_0+2\mu_0)(1-\boldsymbol{D}_1)^2(1-\boldsymbol{D}_2)^2 & \lambda_0(1-\boldsymbol{D}_1)(1-\boldsymbol{D}_2)(1-\boldsymbol{D}_3) & \lambda_0(1-\boldsymbol{D}_1)(1-\boldsymbol{D}_2)(1-\boldsymbol{D}_3) & 0 & 0 & 0 \\ \lambda_0(1-\boldsymbol{D}_1)(1-\boldsymbol{D}_2)(1-\boldsymbol{D}_3) & (\lambda_0+2\mu_0)(1-\boldsymbol{D}_3)^2 & \lambda_0(1-\boldsymbol{D}_3)^2 & 0 & 0 & 0 \\ \lambda_0(1-\boldsymbol{D}_1)(1-\boldsymbol{D}_2)(1-\boldsymbol{D}_3) & \lambda_0(1-\boldsymbol{D}_3)^2 & ((\lambda_0+2\mu_0)(1-\boldsymbol{D}_3)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\mu_0(1-\boldsymbol{D}_4)^2(1-\boldsymbol{D}_5)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu_0(1-\boldsymbol{D}_4)^2(1-\boldsymbol{D}_5)^2 \end{bmatrix} \tag{19}$$

Then, following the formalism presented in Pituba (2006), the bi-dissipative anisotropy damage model taking into account the unilateral effect in brittle materials is written as

$$\mathrm{W}(\varepsilon)=\rho\psi(\varepsilon) := \begin{cases} W_-\ (\boldsymbol{\varepsilon})\ if\ \ \boldsymbol{g}\ (\boldsymbol{\varepsilon}, \boldsymbol{D_T}, \boldsymbol{D_C}) < 0, \\ W_+\ (\boldsymbol{\varepsilon})\ if\ \ \boldsymbol{g}\ (\boldsymbol{\varepsilon}, \boldsymbol{D_T}, \boldsymbol{D_C}) > 0, \end{cases} \tag{20}$$

$$\mathrm{W}_+=\rho\psi_+(\varepsilon)=\frac{\lambda_{11}}{2}tr^2(\varepsilon)+\mu_1\mathrm{tr}(\varepsilon^{\mathbf{2}})-\frac{\lambda_{22}^+(D_1,D_2,D_3,D_4,D_5)}{2}\mathrm{tr}^2(\mathbf{A}\varepsilon)-$$

$$\lambda_{12}^+(\mathrm{D}_1,\mathrm{D}_2,\mathrm{D}_3)\mathrm{tr}(\varepsilon)\mathrm{tr}(\mathbf{A}\varepsilon)-\frac{\lambda_{11}^-(D_3)}{2}\,tr^2(\boldsymbol{\varepsilon})-\frac{(1-2v_0)}{2v_0}\lambda_{11}^-(D_3)tr[(\,\boldsymbol{I}\,\bar{\otimes}\,\boldsymbol{I})\boldsymbol{\varepsilon}]^2-\mu_2(D_4,D_5)\,\boldsymbol{tr(A\varepsilon^2)} \tag{21}$$

$$\mathrm{W}_-=\rho\psi_-(\varepsilon)=\frac{\lambda_{11}}{2}tr^2(\varepsilon)+\mu_1 tr(\varepsilon^{\mathbf{2}})-\frac{\lambda_{22}^-(D_2,D_3,D_4,D_5)}{2}\mathrm{tr}^2(\mathbf{A}\varepsilon)-\lambda_{12}^-(\mathrm{D}_2,\mathrm{D}_3)\mathrm{tr}(\varepsilon)\mathrm{tr}(\mathbf{A}\varepsilon)-\frac{\lambda_{11}^-(D_3)}{2}$$

$$tr^2(\boldsymbol{\varepsilon}) - \frac{(1-2\nu_0)}{2\nu_0}\lambda_{11}^-(D_3)tr[(\mathbf{I}\overline{\otimes}\mathbf{I})\boldsymbol{\varepsilon}]^2 - \mu_2(D_4,D_5)\,tr(\mathbf{A}\boldsymbol{\varepsilon}^2) \tag{22}$$

Now, the parameters λ_{ij} and μ_i are given by

$$\lambda_{22}^+(D_1,D_2,D_3,D_4,D_5) = (\lambda_0 + 2\mu_0)(2D_1 - D_1^2)$$
$$-2\lambda_{12}^+(D_1,D_2,D_3) - 2\mu_2(D_4,D_5) + \frac{(\nu_0-1)}{\nu_0}\lambda_{11}^-(D_3) + (\lambda_0+2\mu_0)\left[(1-D_1)^2 - (1-D_1)^2(1-D_2)^2\right]$$
$$\lambda_{12}^+(D_1,D_2,D_3) = \lambda_0\left[(1-D_3)^2 - (1-D_1)(1-D_2)(1-D_3)\right]$$
$$\mu_2(D_4,D_5) = 2\mu_0[1-(1-D_4)^2(1-D_5)^2]$$
$$\lambda_{22}^-(D_2,D_3,D_4,D_5) = (\lambda_0+2\mu_0)(2D_2 - D_2^2) - 2\lambda_{12}^-(D_2,D_3) + \frac{(\nu_0-1)}{\nu_0}\lambda_{11}^-(D_3) - 2\mu_2(D_4,D_5)$$
$$\lambda_{12}^-(D_2,D_3) = \lambda_0[(1-D_3)^2 - (1-D_2)(1-D_3)]$$
$$\lambda_{11}^-(D_3) = \lambda_0(2D_3 - D_3^2) \tag{23}$$

The stress tensor is obtained from the gradient of the elastic potential, as follows

$$\boldsymbol{\sigma}(\varepsilon) = \begin{cases} \sigma_-(\varepsilon) = \nabla_\varepsilon \rho\psi_-(\varepsilon) & if \quad g(\boldsymbol{\varepsilon}, \mathbf{D_T}, \mathbf{D_C}) < 0, \\ \sigma_+(\varepsilon) = \nabla_\varepsilon \rho\psi_+(\varepsilon) & if \quad g(\boldsymbol{\varepsilon}, \mathbf{D_T}, \mathbf{D_C}) > 0, \end{cases} \tag{24}$$

$$\boldsymbol{\sigma}_+(\varepsilon) = \lambda_{11} tr(\varepsilon)\mathbf{I} + 2\mu_1\varepsilon - \lambda_{22}^+(D_1,D_2,D_3,D_4,D_5)\,\mathrm{tr}(\mathbf{A}\varepsilon)\mathbf{A} - \lambda_{12}^+(D_1,D_2,D_3)(\mathrm{tr}(\varepsilon)\mathbf{A} + \mathrm{tr}(\mathbf{A}\varepsilon)\mathbf{I})$$
$$-\lambda_{11}^-(D_3)\,tr(\boldsymbol{\varepsilon})\mathbf{I} - \frac{(1-2\nu_0)}{\nu_0}\lambda_{11}^-(D_3)(\mathbf{I}\overline{\otimes}\mathbf{I})\boldsymbol{\varepsilon} - \mu_2(D_4,D_5)\,(\mathbf{A}\boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}\mathbf{A}) \tag{25}$$

$$\boldsymbol{\sigma}_-(\varepsilon) = \lambda_{11} tr(\varepsilon)\mathbf{I} + 2\mu_1\varepsilon - \lambda_{22}^-(D_2,D_3,D_4,D_5)\,\mathrm{tr}(\mathbf{A}\varepsilon)\mathbf{A} - \lambda_{12}^-(D_2,D_3)(\mathrm{tr}(\varepsilon)\mathbf{A} + \mathrm{tr}(\mathbf{A}\varepsilon)\mathbf{I})$$
$$-\lambda_{11}^-(D_3)\,tr(\boldsymbol{\varepsilon})\mathbf{I} - \frac{(1-2\nu_0)}{\nu_0}\lambda_{11}^-(D_3)(\mathbf{I}\overline{\otimes}\mathbf{I})\boldsymbol{\varepsilon} - \mu_2(D_4,D_5)\,(\mathbf{A}\boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}\mathbf{A}) \tag{26}$$

The constitutive tensor is also obtained from the elastic potential, i.e.

$$\mathbf{E}(\varepsilon) := \begin{cases} \mathbf{E}_-(\boldsymbol{\varepsilon}) = \nabla_\varepsilon^2 \rho\psi_-(\boldsymbol{\varepsilon}) & if \quad g(\boldsymbol{\varepsilon}, \mathbf{D_T}, \mathbf{D_C}) < 0, \\ \mathbf{E}_+(\boldsymbol{\varepsilon}) = \nabla_\varepsilon^2 \rho\psi_+(\boldsymbol{\varepsilon}) & if \quad g(\boldsymbol{\varepsilon}, \mathbf{D_T}, \mathbf{D_C}) > 0, \end{cases} \tag{27}$$

$$\mathbf{E}_+(\boldsymbol{\varepsilon}) = \mathbf{E_T} = \lambda_{11}[\mathbf{I}\otimes\mathbf{I}] + 2\mu_1[\mathbf{I}\overline{\underline{\otimes}}\mathbf{I}] - \lambda_{22}^+(D_1,D_2,D_3,D_4,D_5)[\mathbf{A}\otimes\mathbf{A}] - \lambda_{12}^+(D_1,D_2,D_3)$$
$$[\mathbf{A}\otimes\mathbf{I} + \mathbf{I}\otimes\mathbf{A}] - \lambda_{11}^-(D_3)[\mathbf{I}\otimes\mathbf{I}] - \frac{(1-2\nu_0)}{\nu_0}\lambda_{11}^-(D_3)[\mathbf{I}\overline{\otimes}\mathbf{I}] - \mu_2(D_4,D_5)[\mathbf{A}\overline{\underline{\otimes}}\mathbf{I} + \mathbf{I}\overline{\underline{\otimes}}\mathbf{A}] \tag{28}$$

$$\mathbf{E}_-(\boldsymbol{\varepsilon}) = \mathbf{E_C} = \lambda_{11}[\mathbf{I}\otimes\mathbf{I}] + 2\mu_1[\mathbf{I}\overline{\underline{\otimes}}\mathbf{I}] - \lambda_{22}^-(D_2,D_3,D_4,D_5)[\mathbf{A}\otimes\mathbf{A}] - \lambda_{12}^-(D_2,D_3)[\mathbf{A}\otimes\mathbf{I} + \mathbf{I}\otimes\mathbf{A}]$$
$$-\lambda_{11}^-(D_3)[\mathbf{I}\otimes\mathbf{I}] - \frac{(1-2\nu_0)}{\nu_0}\lambda_{11}^-(D_3)[\mathbf{I}\overline{\otimes}\mathbf{I}] - \mu_2(D_4,D_5)[\mathbf{A}\overline{\underline{\otimes}}\mathbf{I} + \mathbf{I}\overline{\underline{\otimes}}\mathbf{A}] \tag{29}$$

Taking into account the unilateral effects and focussing in the case that the direction 1 is perpendicular to the transverse isotropy local plane, the complementary elastic energy of the damaged medium in tension dominant states is now expressed by

$$W_{e+}^{*} = \frac{\sigma_{11}^{2}}{2E_0(1-D_1)^2(1-D_2)^2} + \frac{(\sigma_{22}^{2}+\sigma_{33}^{2})}{2E_0(1-D_3)^2} - \frac{\nu_0(\sigma_{11}\sigma_{22}+\sigma_{11}\sigma_{33})}{E_0(1-D_1)(1-D_2)(1-D_3)} - \frac{\nu_0\sigma_{22}\sigma_{33}}{E_0(1-D_3)^2}$$
$$+ \frac{(1+\nu_0)}{E_0(1-D_4)^2(1-D_5)^2}(\sigma_{12}^{2}+\sigma_{13}^{2}) + \frac{(1+\nu_0)}{E_0}\sigma_{23}^{2} \quad (30)$$

The variables associated to damage variables in tension with damage activated in previous compression will also be modified, because they are obtained from the elastic potential (Eq. (20)). Therefore, the following relationships are valid

$$Y_T = \frac{\partial W_{e+}^{*}}{\partial D_1} + \frac{\partial W_{e+}^{*}}{\partial D_4} = Y_1 + Y_4 \quad (31)$$

$$Y_1 = \frac{\sigma_{11}^{2}}{E_0(1-D_1)^3(1-D_2)^2} - \frac{\nu_0(\sigma_{11}\sigma_{22}+\sigma_{11}\sigma_{33})}{E_0(1-D_1)^2(1-D_2)(1-D_3)} \quad (32)$$

$$Y_4 = \frac{(1+\nu_0)}{E_0(1-D_4)^3(1-D_5)^2}(2\sigma_{12}^{2}+2\sigma_{13}^{2}) \quad (33)$$

Note that only Y_1 must to take into account the diffuse damage represented by D_2 and D_3. In this case, those damage variables are constants because there is no energy release rates during the damage evolution in tension dominant states related to D_2 and D_3.

In the case of tension dominant states without activation of damage processes in previous compression the original version of the damage model is recovered.

It can be verified that the unilateral damage model satisfies two basic requirements of this modelling kind:

The model does not produce spurious energy dissipation upon closed load paths which do not activate damage, Matallah and La Borderie (2009).

The continuity of the stress-strain law across the tension-compression interface is assured (hiperplano $g(\varepsilon,\mathbf{D_T},\mathbf{D_C})$), because the damage model is derived from the formulation proposed in Pituba (2006), following the requirements of Curnier *et al.* (1995) and Welemane and Comery (2002). The continuity of the stress-strain law between two damage states imposes that the elastic potential must be once continuously differentiable (whole wise), but only piecewise twice continuously differentiable.

Accordingly with Curnier *et al.* (1995), other problem related to this kind of modelling is concerned to the loss of isotropy of the elasticity tensor in the transition through the tension-compression interface. The isotropy is preserved only if the interface is defined in the same group of symmetry of the elasticity tensor. In the proposed model discussed here, the hyperplane and elasticity tensor belong to the group of isotropic material if there is not damage process. On the other hand, if there is activation of damage processes, the hyperplane starts to present the symmetry of the transverse isotropic material as well as the elasticity tensor. Anyway, the model always preserves the isotropy of the elasticity tensor.

3. Some remarks about the micromechanical theory

Although the damage model has been based on the macromechanical behaviour of the concrete, this item intends to show the strong connection between the model and the micromechanical theory. The description of the damage activation-deactivation process as part of macroscopic modelling requires knowing when the transition between these two states of damage occurs and how damage deactivation affects the elastic properties of the material, Welemane and Comery (2002), Pensée *et al.* (2002). Moreover, there is a difficulty in recognizing tension and compression states in 3D microscale analysis in order to adopt a differentiable Gibbs potential. It is noted that the formulation for bimodular anisotropic damaged media proposed in Pituba (2006) replies the first question (see Eq. (7)). Besides, the continuity of the stress-strain law has been assured. In this context, this section aims to point out the influence of the opening-closure of microdefects on the elastic properties of the microcracked concrete.

Following Welemane and Comery (2002), consider a RVE (representative volume element) of an homogeneous isotropic elastic linear matrix (Young modulus E_0 and Poisson rate ν_0) weakened by an array of N randomly distributed flat penny-shaped microcracks (unit normal n_k, radius a_k), whose radii are very small in comparison with the size of the RVE. Assuming non-interaction among microcracks and sliding without friction of their lips, the free enthalpy of the microcracked medium is given by

$$\text{u}=\frac{(1+\nu_0)}{2E_0}tr(\sigma\cdot\sigma)-\frac{\nu_0}{2E_0}(tr\sigma)^2+\frac{8}{3V}\frac{(1-\nu_0^2)}{(2-\nu_0)E_0}\sigma:\sum_{K=1}^{N}a_K^3$$

$$\left[n_k^{\otimes 2}\,\overline{\underline{\otimes}}\,I+I\,\overline{\underline{\otimes}}\,n_k^{\otimes 2}-\left[2-(2-\nu_0)H\left(\sigma_n^k\right)\right]n_k^{\otimes 4}\right]:\sigma \tag{34}$$

The Heaviside function H depending on the normal stress to each microcrack is open ($\sigma_n^k \geq 0$) or closed ($\sigma_n^k < 0$).

Consider the simple case of a material weakened by a single array of parallel microcracks with unit normal **n** as described in Fig. 1 and parameter $A = 16(1-\nu_0^2)/(6-3\nu_0)$. This case is interesting for the damage model proposed in this work because the effective medium exhibits the symmetry associated with the geometric shape of the microcracks with the privileged direction **n** (transverse isotropic material).

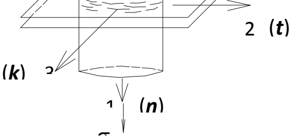


Fig. 1 Parallel microcracks on concrete submitted to uniaxial tension stress

Then, the elastic modules are fully determined by five independent coefficients E(**n**), E(**t**), ν(**n**,**t**), ν(**t**,**k**) and μ(**n**,**t**), for any vectors **t** and **k** forming with n an orthonormal basis of R^3. Using Eq. (34), it can be obtained the elastic modules mentioned above.

$$E(n) = E_0 \left[1 + \frac{A}{V} \sum_{K=1}^{N} a_k^3 (2 - \nu_0) H\left(\sigma_n^k\right) \right]^{-1} \tag{35}$$

$$\nu(n,t) = \nu_0 \left[1 + \frac{A}{V} \sum_{K=1}^{N} a_k^3 (2 - \nu_0) H\left(\sigma_n^k\right) \right]^{-1} \tag{36}$$

$$E(\mathbf{t}) = E_0 \tag{37}$$

$$\nu(\mathbf{t},\mathbf{k}) = \nu_0 \tag{38}$$

$$\mu(n,t) = \mu_0 \left[1 + \frac{A}{(1+\nu_0)V} \sum_{K=1}^{N} a_k^3 \right]^{-1} \tag{39}$$

In Welemane and Comery (2002) are described some conclusions about Eqs. (35)-(39) that are useful for a discussion about the proposed model. In general way, a macroscopic approach of the unilateral effect in brittle materials should no longer be considered only by the single restoration of the Young modulus in the direction normal to closed microcracks. Therefore, based on micromechanical observations, some important aspects related to unilateral effect of damage processes can be pointed out:

- The elastic modules E(**n**) and Poisson ratio ν(**n**,**t**), related to normal direction to parallel microcracks, are affected by the evolution of the microdefects. In particular, those modules recover their initial values (E_0 and ν_0) when the microcracks are closed.
- In the other hand, the shear modulus μ(**n**,**t**) remains the same when the microcracks are closed (partial deactivation of damage). This behaviour is consistent with the hypothesis about tangential jump null of the constitutive tensor. However, the elastic modules E(**m**), ν(**m**,**p**) and μ(**m**,**p**) related to directions with different orientations at principal axes (**n**, **t** and **k**) are partially recovered when the microcracks close.

The particular nature of the microdefects contribution allows extending these considerations for any of *N* microcracks with different normal vectors. In this context, let us compare the damaged elastic modules given by the proposed model to those ones given by the micromechanical equations. Then, considering Fig. 1 and assuming, for instance, that the transversal isotropy local plane is coincident with the 2-3 plane, the elastic modules given by the proposed model in dominant compression (subscript *C*) and in tension (subscript *T*) regimes are written as

$$E_{T1} = E_0 (1 - D_1)^2 (1 - D_2)^2 ; \;\; E_{C1} = E_0 (1 - D_2)^2 \tag{40}$$

$$\nu_{T12} = \nu_{T13} = \nu_0 \frac{(1 - D_1)(1 - D_2)}{(1 - D_3)} ; \nu_{C12} = \nu_{C13} = \nu_0 \frac{(1 - D_2)}{(1 - D_3)} \tag{41}$$

$$E_{T2} = E_{T3} = E_0 (1 - D_3)^2 ; \;\; E_{C2} = E_{C3} = E_0 (1 - D_3)^2 \tag{42}$$

$$\nu_{T23} = \nu_{C23} = \nu_0 \tag{43}$$

$$\mu_{T12} = \mu_{T13} = \mu_{C12} = \mu_{C13} = \mu_0 (1 - D_4)^2 (1 - D_5)^2 \tag{44}$$

The longitudinal elastic modules in tension and in compression in the direction 1 depend on the dominant state, i.e., of the opening-closure criterion. This is also valid for the Poisson ratio in the 1-2 and 1-3 planes. On the other hand, the Poisson ratio in the 2-3 plane (transversal isotropy local plane) is not affected by the damage process. The shear modules are not changed in the transition from the tension to compression regimes and vice-versa. Observe Eq. (42) and consider the transition from dominant tension regime (damage process in tension activated or not) to the compression regime without previous compression. In this case, $E_{T2} = E_{T3} = E_{C2} = E_{C3} = E_0$. This result is in correspondence with the form described by Eq. (37). Indeed, the coefficient $(1 - D_3)^2$ is necessary to take into account the diffuse damage in previous compression when the current dominant state is tension.

Obviously, in general cases, when the damage process is activated, the formulation starts to involve the tensor **A**, which depends on the knowledge of the normal to the transverse isotropy plane, Pituba and Fernandes (2011). Therefore, the discussion about elastic modules presented above is valid but those moduli are dependents of the tensor **A**, as described in section 2.

Finally, it is observed that despite the proposed model have macromechanical motivations in the macroscopic behaviour of the concrete, the model assists to the requirements suggested by Welemane and Comery (2002) for the micromechanical analysis of the unilateral effect in materials.

4. Numerical analyses of framed RC structures

This work intends to show the capabilities of the modified damage model to simulate the mechanical behaviour of reinforced concrete structures submitted to reversal loading in possible practical situations of structural engineering. So, it is necessary that the model presents efficient numerical responses, i.e., numerical analyses with low computational cost and a few parameters of the model to be identified. In this context, the one-dimensional version of the damage model has been implemented in a finite element code for bar structures analysis with finite layered elements in order to model the reinforced concrete framed structures. For the longitudinal reinforcement bars, standard elasto-plastic behaviour is admitted. In the transversal section, a certain layer can contain steel and concrete, see Fig. 2. A perfect adherence between materials is adopted and an equivalent elasticity modulus and inelastic strain are defined for each layer by using homogenization rule

$$E_k = (1 - C_{sk}) E_{ck} + C_{sk} E_{sk} \tag{45}$$

$$\varepsilon_{ink} = (1 - C_{sk}) \varepsilon_{cink} + C_{sk} \varepsilon_{psk} \tag{46}$$

where,

- C_{sk} is the volumetric rate of steel in the layer N° k
- E_{sk} is the elasticity modulus of steel in the layer N° k
- E_{ck} is the elasticity modulus of concrete in the layer N° k
- ε_{psk} is the plastic strain of steel in the layer N° k
- ε_{ink} is the homogenised inelastic strain in the layer N° k

- ε_{cink} is the inelastic strain of concrete in the layer N° k
- E_k is the homogenised elasticity modulus in the layer N° k

Considering the direction 1 as the longitudinal direction of the finite element, the formulation presented in the previous item is simplified and presented as follows

$$\boldsymbol{E}\,(\boldsymbol{\varepsilon}) := \begin{cases} \mathbf{E}_-(\boldsymbol{\varepsilon}) = \nabla^2_{\varepsilon}\rho\psi_-\,(\boldsymbol{\varepsilon}) & \textit{if} \quad g\,(\boldsymbol{\varepsilon}, \mathbf{D_T}, \mathbf{D_C}) < 0, \\ \mathbf{E}_+(\boldsymbol{\varepsilon}) = \nabla^2_{\varepsilon}\rho\psi_+\,(\boldsymbol{\varepsilon}) & \textit{if} \quad g\,(\boldsymbol{\varepsilon}, \mathbf{D_T}, \mathbf{D_C}) > 0, \end{cases} \tag{47}$$

$$E_T = E_0(1-D_1)^2(1-D_2)^2 \tag{48}$$

$$E_C = E_0(1-D_2)^2 \tag{49}$$

$$W^*_{e\,+} = \frac{\sigma^2_{11}}{2E_0(1-D_1)^2(1-D_2)^2} \tag{50}$$

$$W^*_{e\,-} = \frac{\sigma^2_{11}}{2E_0(1-D_2)^2} \tag{51}$$

$$Y_T = \frac{\partial W^*_{e+}}{\partial D_1} = Y_1 \tag{52}$$

$$Y_C = \frac{\partial W^*_{e-}}{\partial D_2} = Y_2 \tag{53}$$

$$Y_1 = \frac{\sigma^2_{11}}{E_0(1-D_1)^3(1-D_2)^2} \tag{54}$$

$$Y_2 = \frac{\sigma^2_{11}}{E_0(1-D_2)^3} \tag{55}$$

The one-dimensional version of the model takes into account permanent strains induced by damage evolution. Assuming, for simplicity, that the permanent strains are composed exclusively by volumetric strains, as it has already been considered in Comi (2001), and taking into account the unilateral effect, the evolution law results

$$\dot{\varepsilon}^p = \left(\frac{\beta_1}{(1-D_1)^2}\dot{D}_1 + \frac{\beta_2}{(1-D_2)^2}\dot{D}_2 \right)\mathbf{I} \tag{56}$$

Observe that β_1 and β_2 are parameters directly related to the evolutions of permanent strains induced by damage in tension and in compression, respectively. The consideration of the permanent strains improves the capture of the transverse strains by the model, as it can see in Pituba and Fernandes (2011). Besides, the model predicts the change in sign of the volumetric strain.

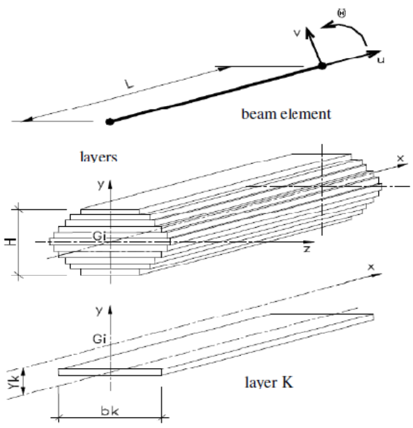


Fig. 2 Finite layered element

Here and after, some numerical applications are performed in order to show the potentialities of the proposed model when dealing with RC structures analysis. Initially, the constitutive model is used in the simulation of an uniaxial test in concrete specimens upon reversal load in order to show the qualitative numerical response. Observe that the permanent strains are important in the definition of the hyperplane, in the sense that the total strains start to compose the criterion, Eq. (8). The initial stiffness recovery can be clearly observed taking into account permanent strain in the dominant tension regime. It is noted the contribution of the diffuse damage generated in previous compression regimes when dealing to tension regimes.

Now, a reinforced concrete beam with symmetric reinforcement is analysed. This test corresponds to a reinforced concrete beam in a configuration of three points cyclic flexion. For more details, see La Borderie (1991) and Matallah and La Borderie (2009). The beam is subject to cyclic loading at the mid span. The concrete used in the beam has elasticity modulus E_c = 31,800 MPa; the steel has E_s = 210,000 MPa, yielding stress of 445 MPa and ultimate stress of 540 MPa. In the experimental test, the beam is subjected to two loading cycles of amplitude, the first one is 1mm and 2 mm the second one (see Fig. 4). The beam geometry and its reinforcement distribution are illustrated in Fig. 4.

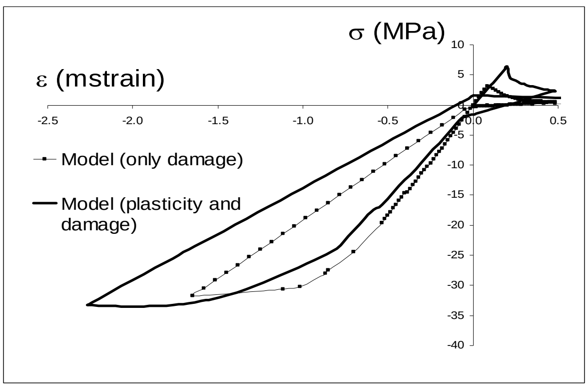


Fig. 3 Concrete specimens submitted to reversal loading

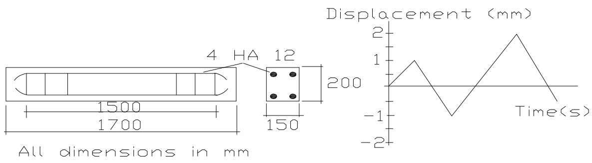


Fig. 4 Geometry, reinforcement details and loading history

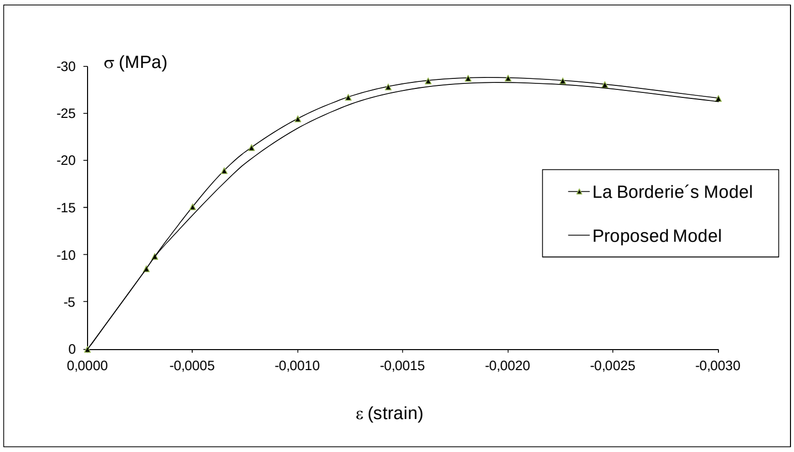


Fig. 5 Parametric identification in uniaxial compression test – La Borderie's RC Beam

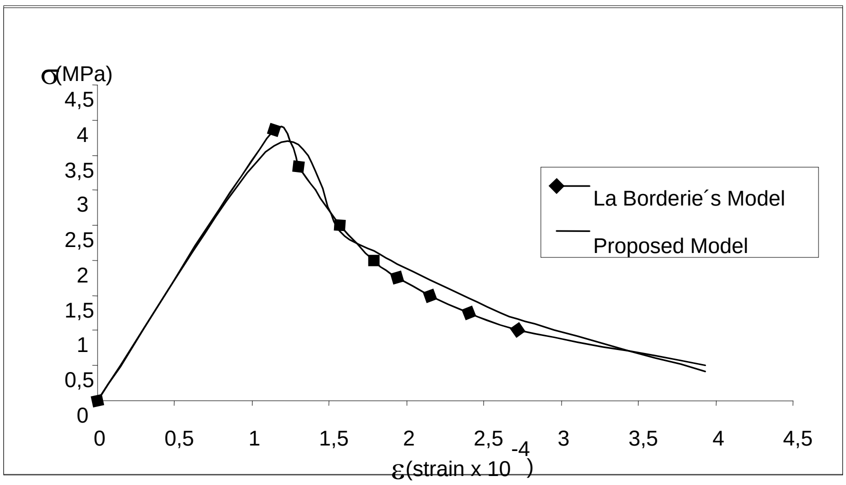


Fig. 6 Parametric identification in uniaxial tension test – La Borderie's RC Beam

The parametric identification of the proposed damage model is presented in the Figs. 5 and 6. The parameters used by La Borderie (1991) were taken as reference in the simulation of uniaxial tension and compression tests. Table 1 presents the parameter values. It is important to note that the experimental tests do not present loading/unloading paths. Therefore, the parameters β_1 and β_2 have been adopted without interference in the maximum stress of the concrete.

In the numerical analysis, displacements increments were enforced in the mid span. Using the advantage of symmetry, only half of the beam is discretised into 20 finite elements. The transversal sections were divided into 16 layers where the reinforcement layers are located in the medium planes of the second and fifteenth layers. In Fig. 7 are shown the numerical and experimental responses of the vertical force and displacement in the middle of the span related to the first stage of the loading. It is noted the good precision of the numerical response.

Table 1 Parameters for the proposed damage model – La Borderie's RC Beam

Tension	Compression
$Y_{01}=6.0x10^{-5}MPa$	$Y_{02}=3.0x10^{-3}MPa$
$A_1=-0.93$	$A_2=1.50$
$B_1=110\ MPa^{-1}$	$B_2=10.01\ MPa^{-1}$
$\beta_1=8x10^{-5}MPa$	$\beta_2=1.0x10^{-3}MPa$

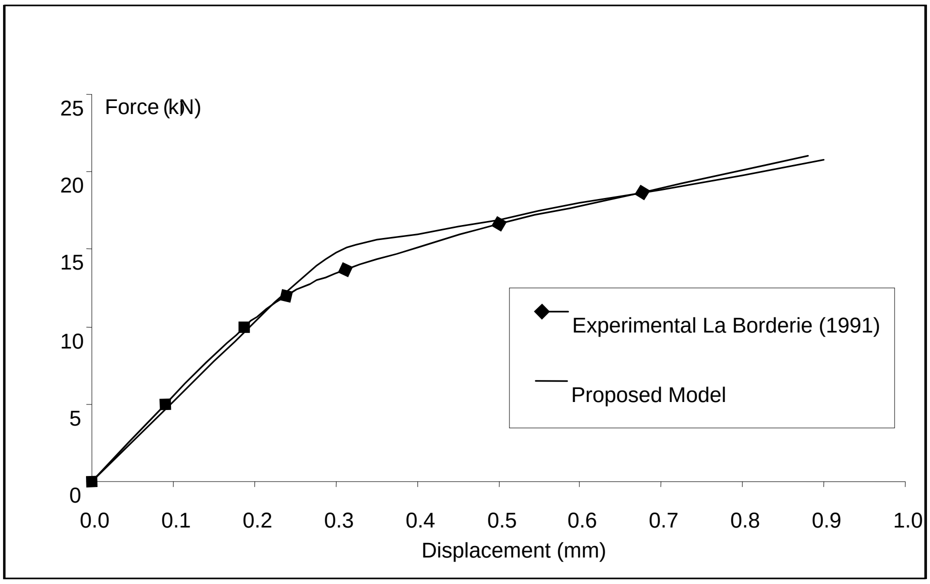


Fig. 7 Experimental and numerical responses – first loading

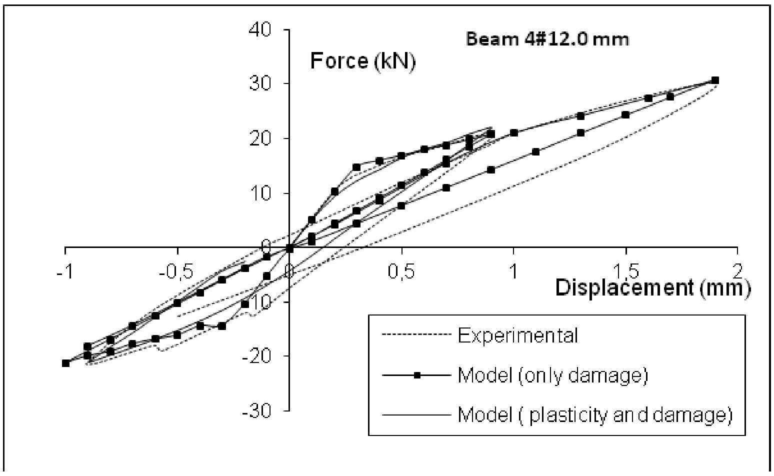
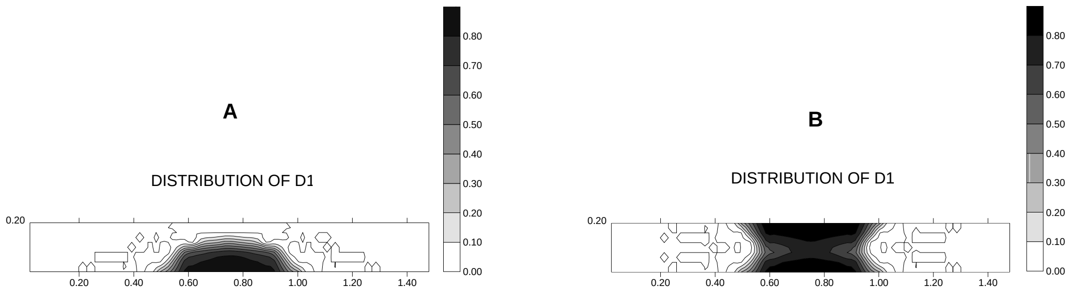


Fig. 8 Global response of the reinforced concrete beam



(a) End of first loading (b) End of second loading (reversal loading)

Fig. 9 Damage distribution in tension (D_1)

In the other hand, in the Fig. 8 is illustrated the global response of whole test. The results obtained by the model have shown to be satisfactory despite the limited parametric identification of the parameters related to permanent strains. Therefore, the ultimate loads are computed more accurately than the permanent strains in the unloading processes. In general way, the model reproduces satisfactorily the cyclic behaviour of the beam.

Besides, the damage profile is also close to experimental test observations, see Matallah and La Borderie (2009). In Fig. 09 is shown the damage distribution in tension regimes at two points of the curve illustrated in Fig. 08. The first point is located at the end of the first loading and the second one is located in the end of the second loading (reversal loading). These distributions have shown the opening/closure cracks process. The first damage in tension zone (D_1) occurs in the bottom of the beam. On the other hand, when the load is inverted, the damage in tension zone (D_1) appears in the upper zone of the beam, but the D_1 distribution in the bottom of the beam remains the same, although there is no increasing of values. Therefore, the cracks previously open are now closed. Note that the damage processes in the compression regimes (D_2) are not so important in this numerical application, according to observations in La Borderie (1991). It can be observed that the symmetric arrangement of the reinforcement leads to an additional support to compression stresses in the concrete.

Another numerical application discussed in this work deals with an experimental test originally performed by Vecchio and Emara (1992) taking into account just proportional loading/unloading, but without reversal loading. However, in this work, the reinforced concrete frame is submitted to loading/unloading and then a reversal loading is applied in order to show the potentialities of the proposed model to simulate the collapse of frames in cyclic loading conditions.

The frame geometry and its reinforcement distribution are illustrated in Fig. 10. The concrete used has the elasticity modulus E_c=30,400 MPa and the steel has E_s=192,500 MPa, yielding stress of 418 MPa and ultimate stress of 596 MPa. A bilinear elasto-plastic model has been adopted with a reduced elasticity modulus in the second branch (E_{s2}=0.009 E_s). Also, Table 1 contains the parameters values for the concrete as well as in the Fig. 11 is illustrated the parametric identification by fitting experimental curve on compression test given in Vecchio and Emara (1992). However, the parameters in tension regime have been obtained using the La Borderie´s model response given by Pituba (2010).

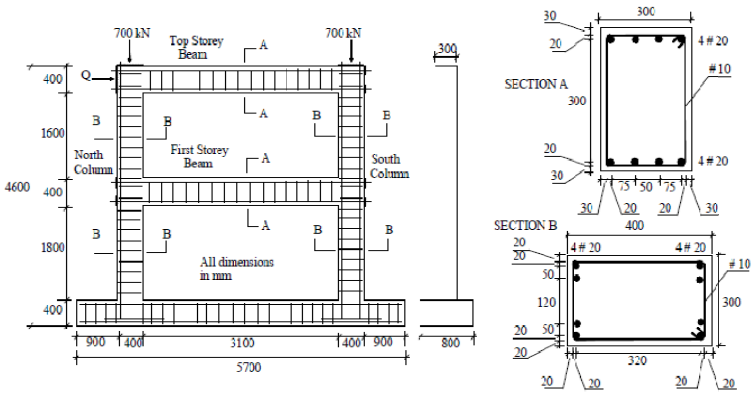
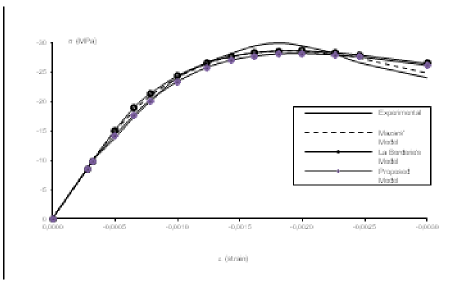
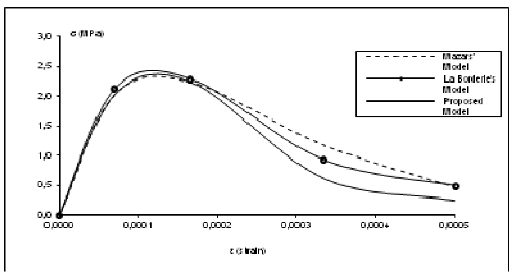


Fig. 10 Geometry and reinforcement details of the frame



(a) Uniaxial compression test



(b) Uniaxial tension test

Fig. 11 Parametric identification

Table 2 Parameters for the proposed damage model – La Borderie's RC Beam

Tension	Compression
$Y_{01} = 0{,}72x10^{-4}MPa$	$Y_{02} = 0{,}17x10^{-2}MPa$
$A_1 = 49$	$A_2 = 0{,}30$
$B_1 = 6560\ MPa^{-1}$	$B_2 = 5{,}13\ MPa^{-1}$
$\beta_1 = 1x10^{-6}\ MPa$	$\beta_2 = 1x10^{-3}\ MPa$

In the experimental test, it was initially applied an axial load of 700 kN for each column, which was maintained constant during all the lateral load application. The lateral force was applied increments up to the frame ultimate load. In the numerical analysis originally performed by Pituba (2010), displacements increments were enforced in the application point of the horizontal force up to the frame ultimate load. In that work, it has been performed loading and unloading trying to simulate the experimental behaviour of the frame. The numerical results were very satisfactory simulating the ultimate load as well as the residual strains.

Now, in this work, the frame was discretised into 30 finite elements, 10 of which were used in the discretisation of each column and 5 in each beam. The transversal sections were divided into 10 layers. The numerical and experimental responses are illustrated in Fig. 12, where the graphs represent the applied horizontal force versus horizontal displacement computed at the superior floor of the frame (see Fig. 10).

In order to investigate the potentialities of the improvement of the damage model proposed in section 2.2, the framed structure has been analyzed attempting to perform an unloading of the horizontal force Q, including reversal loading. The goal is to observe the consistency of the qualitative response provided by damage model.

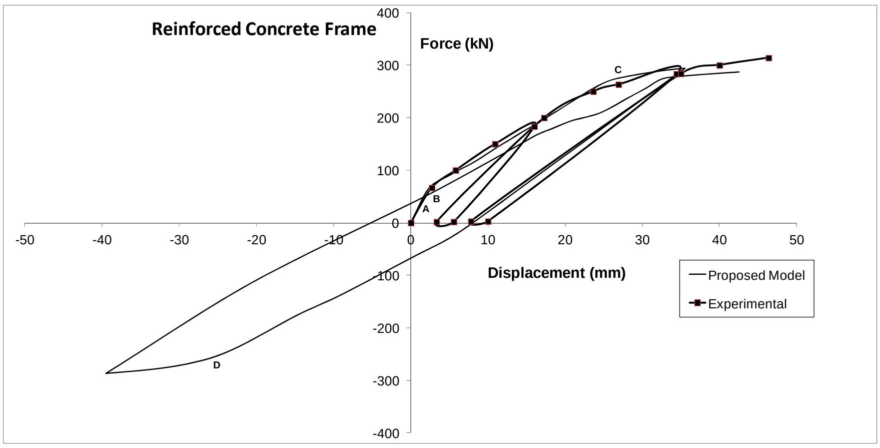
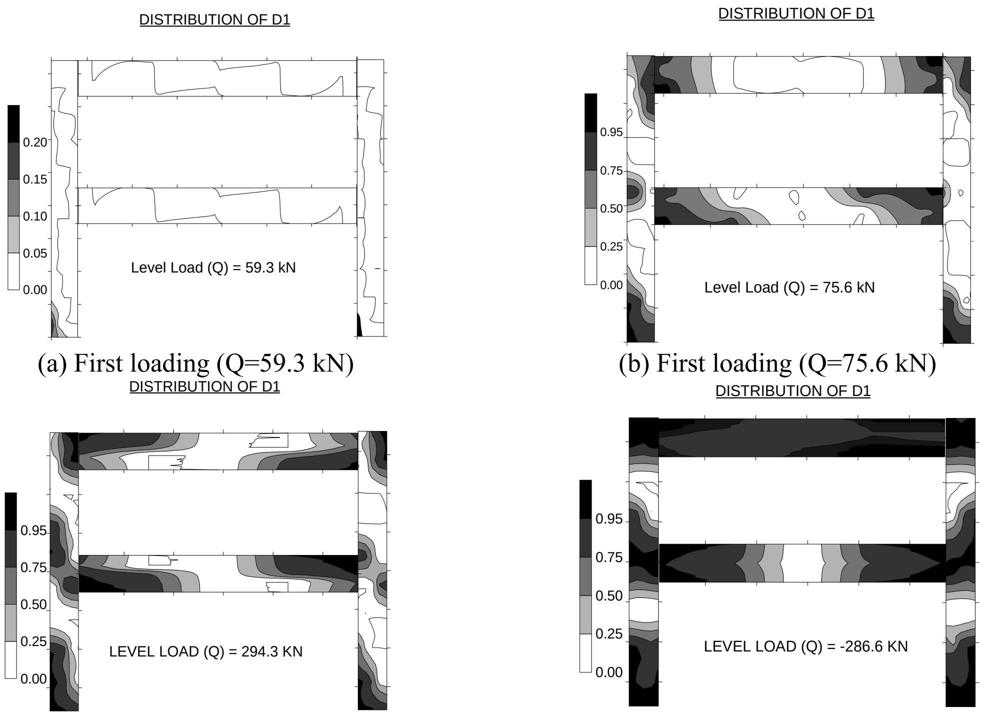


Fig. 12 Numerical and experimental results of reinforced concrete frame

It can be noted the agreement between numerical and experimental responses during the unloading process. This evidences the good performance of the damage model to capture residual strains. In this stage, the loading capacity of the frame has been achieved and the damage level is high in most zones of the frame, as it can see in Fig. 13. Note that the figure presents the damage distribution related to tension regimes (D_1) because the analysis has shown the importance of that variable. This is related to the concept of the damage model proposed in this work. It is possible to observe the evolution of the damage processes within the stages displayed in Fig. 13.

Besides, in Fig. 12 the symmetric behaviour of the frame related to load capacity when the horizontal force Q is applied to right direction and then it has been changed to left direction. In the first case, the load level capacity was about 294.3 kN. On the other hand, the load level capacity was 286.6 kN for the second case. Note yet, the capability of the model to simulate the recovery of the load capacity when the first cycle of loading is complete.

There are some parts of the frame with high values of damage variable D_1 that together with the yielding of the reinforcement bars contribute to concentrate damage-plastic zones like plastic joints, Araújo and Proença (2008). It can be observed these zones in first and second floor beam/column junctions and, mainly, in the supports of the frame. These observations are in agreement with described in Vecchio and Emara (1992).



(a) First loading (Q=59.3 kN) (b) First loading (Q=75.6 kN)

(c) First loading (Q=294.3 kN) (d) Reversal loading (Q=-286.6 kN)

Fig. 13 Damage distribution in tension (D_1)

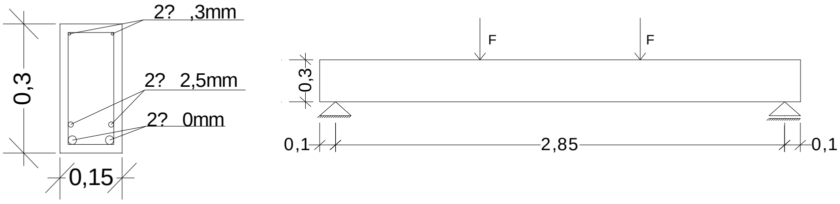


Fig. 14 Geometry and reinforcement details

Table 3 Parameter values of the proposed damage model – RC Beam

Tension	Compression
$Y_{01} = 6.0x10^{-5} MPa$	$Y_{02} = 1.0x10^{-3} MPa$
$A_1 = 0.3$	$A_2 = 1.5$
$B_1 = 195\ MPa^{-1}$	$B_2 = 10.2\ MPa^{-1}$
$\beta_1 = 5x10^{-5}\ MPa$	$\beta_2 = 3x10^{-4}\ MPa$

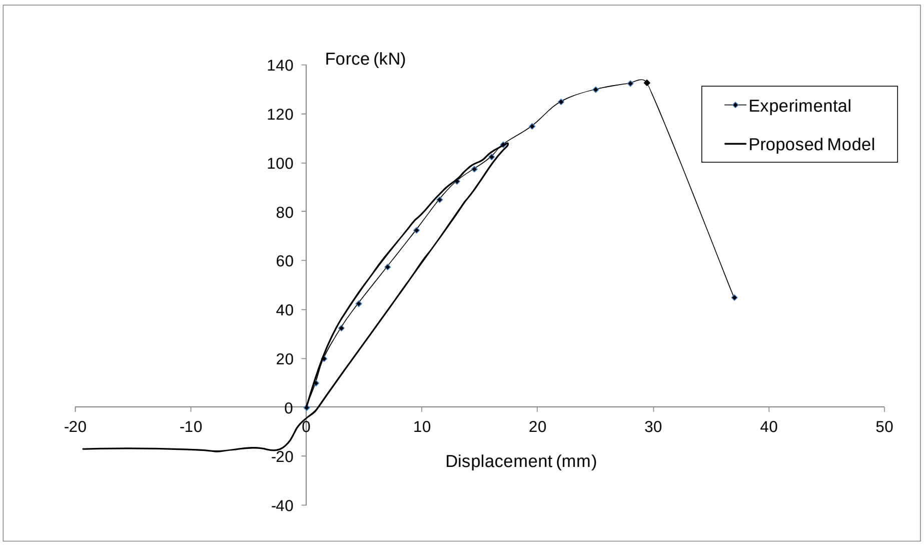


Fig. 15 Numerical and experimental results of the reinforced concrete beam

The third numerical application is about a reinforced concrete beam with unsymmetrical reinforcement. This numerical application has been originally performed by Pituba and Lacerda (2012), but only monotonic loading has been imposed to the beam in that work. The elastic parameters of the concrete are f_c=25MPa and E_c=32.3MPa. For the reinforcement has been adopted E_s= 205 GPa, yielding stress 590 MPa and ultimate stress 750 MPa. The geometric characteristics of the beam are given in Figure 14. The loading is composed by two equal forces applied on the beam.

Table 3 presents the values of the parameters used in the analysis and adopted from Pituba and Lacerda (2012), however in this work is considered the plastic strains generated by the damage model.

The structure has been discretised into 16 finite elements and the transversal sections have been divided into 15 layers where 3 layers have been used to represent the reinforcement bars according with Fig. 14. The numerical and experimental responses are illustrated in Fig. 15.

In the first loading, it is noted that the numerical results are very close to the experimental ones evidencing a good quality response in the sense that captures the history of the mechanical behaviour of the structure. In this work, the numerical analysis continues with the unloading process about 110 kN, where the beam is quite damaged in tension zone (bottom of the beam) and the reinforcement bars present evident yielding in the same zone, Pituba and Lacerda (2012).

The unloading process modelled by the damage model presents very important qualitative results. The damage model can simulate a residual displacement when the reverse loading takes place. Furthermore, it is observed that due to the asymmetric arrangement of the reinforcement, i. e., there is sufficient reinforcement at the bottom to resist the tension stresses in the first loading and insufficient reinforcement (2#6.3mm) on the upper zone to resist the tension stresses in that zone when the load is changed. In this situation, the structure experiences a damage process in tension very intense in the upper zone of the beam. Therefore, it is natural that the strength of the beam be much smaller than in the initial first loading. It can be observed that the concrete does not have strength to the applied force and only the reinforcement resists to tension stresses indicating a strong plastic strain.

5. Conclusions

In this work, a damage constitutive model taking into account the unilateral effect has been presented. The constitutive model is able to capture the damage diffuse created when previous damage processes have been activated in compression dominant regimes.

This paper has shown that the proposed damage model assists to the requirements suggested by Welemane and Comery (2002) and Pensée *et al.* (2002) for the micromechanical analysis of the unilateral effect in materials. Besides, the continuity of the stress-strain law across the tension-compression interface has been assured and the model always preserves the isotropy of the elasticity tensor.

In order to validate the proposed model in practical situations, 1D version of the model has been used. The numerical analysis has shown an efficient and practical employment without numerical problems and low computational cost. Besides, the parametric identification is performed using only uniaxial tests in concrete specimens. Therefore, the damage model could be used in estimative analyses of RC structures in practical situations, such as: numerical simulation of displacement in cracking concrete beams submitted to service loadings, estimative of ultimate load capacity of frames and beams and collapse configuration of reinforced concrete frames.

The results presented in this work encourage us to proceed in the improvement of the model to deal with more complex phenomena in future works, e.g., blocking and dissipative sliding of closed microcracks lips, non-local version of the model and a more efficient parametric identification of β_1 and β_2.

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References

Araújo, F.A. and Proença, S.P.B. (2008), "Application of a lumped dissipation model to reinforced concrete structures with the consideration of residual strains and cycles of hysteresis", *J. Mech. Mater. Struct.*, **3**(5), 1011-1031.

Bielski, J., Skrzypek, J.J. and Kuna-Cisal, H. (2006), "Implementation of a model of coupled elastic-plastic unilateral damage material to finite element code", *Int. J. Damage Mech.*, **15**(1), 5-39.

Brancherie, D. and Ibrahimbegovic, A. (2009), "Novel anisotrtopic continuum-discrete damage model capable of representing localized failure of massive structures. Part I: theoretical formulation and numerical implementation", *Eng. Comput.*, **26**(1-2), 100-127.

Brünig, M. (2004), "An anisotropic continuum damage model: theory and numerical analyses", *Lat. Am. J .Solids Struct.*, **1**(2), 185-218.

Cao, V.V. and Ronagh, H.R. (2013), "A model of damage analysis of concrete", *Adv. Concr. Constr.*, **1**(2), 187-200.

Carol, I. and Willam, K. (1996), "Spurious Energy dissipation/generation in stiffness recovery models for elastic degradation and damage", *Int. J. Solids Struct.*, **33**, 239-257.

Cauvin, A. and Testa, R.B. (1999), "Damage mechanics: basic variables in continuum theories", *Int. J. Solids Struct.*, **36**, 747-761.

Comi, C. (2001), "A non-local model with tension and compression damage mechanisms", *Eur. J. Mech. A-Solid.* **20**, 1-22

Curnier, A., He, Q. and Zysset, P. (1995), "Conewise linear elastic materials", *J. Elasticity*, **37**(1), 1-38.

Desmorat, R. (2000), "Strain localization and unilateral conditions for anisotropic induced damage model", *Proceedings of the Continuous Damage and Fracture*, Cachan.

Deudé, V., Dormieux, L., Kondo, D. and Maghous, S. (2002), "Micromechanical apprroach to nonlinear poroelasticity: application to cracked rocks", *J. Eng. Mech. - ASCE.*, **128**, 848-855.

Esposito, R. and Hendriks, M.A.N. (2014), "Computational modelling of concrete and concrete structures", *Proceedings of the EURO-C International Conference of Computational Modelling of* Concrete *and* Concrete *Structures*, London.

Fernandes, G.R., Pituba, J.J.C. and Souza Neto, E.A. (2015), "Multi-scale modelling for bending analysis of heterogeneous plates by coupling BEM and FEM", *Eng. Anal. Bound. Elem.*, **51**, 1-13. http://dx.doi.org/10.1016/j.enganabound.2014.10.005

Hervé, G., Gatuingt, F. and Ibrahimbegovic, A. (2005), "On numerical implementation of a coupled rate dependent damage-plasticity constitutive model for concrete in application to high-rate dynamics", *Eng. Comput.*, **22**(5-6), 583-604.

Ibrahimbegovic, A., Jehel, P. and Davenne, L. (2008), "Coupled damage-plasticity constitutive model and direct stress interpolation", *Comput. Mech.*, **42**(1), 1-11.

Kucerova, A., Brancherie, D., Ibrahimbegovic, A., Zeman, J. and Bittnar, Z. (2009), "Novel anisotropic continuum-discrete damage model capable of representing localized failure of massive structures: Part II: identification from tests under heterogeneous stress field", *Eng. Comput.*, **26**(1-2), 128-144.

Kupfer, H., Hilsdorf, H. K. and Rush, H. (1969), "Behavior of concrete under biaxial stresses", *ACI J.*, **66**(8), 656-666.

La Borderie, C. (1991), "Phenomenes unilateraux dans un materiau endommageable: modelisation et application a l'analyse de structures en beton", Ph. D. Dissertation, University of Paris 6, France.

Lemaitre, J. (1996), *A Course on Damage Mechanics*, Springer Verlag, Berlin, Germany.

Lemaitre, J., Desmorat, R. and Sauzay, M. (2000), "Anisotropic damage law of evolution", *Eur. J. Mech. A-Solid*, **19**(2), 187-208.

Liu, Y., Teng, S. and Soh, C. (2008), "Three-dimensional damage model for concrete. II: verification", *J. Eng. Mech. -.ASCE.*, **132**, 82-89.

Matallah, M. and La Borderie, C. (2009), "Inelasticity–damage-based model for numerical modeling of concrete cracking", *Eng. Fract. Mech.*, **76**, 1087-1108.

Mohammed, T.A. and Parvin, A. (2013), "Evaluating damage scale model of concrete materials using test data", *Adv. Concr. Constr.*, **1**(4), 289-304.

Pensée, V., Kondo, D. and Dormieux, L. (2002), "Micromechanical Analysis of Anisotropic damage in Brittle Materials", *J. Eng. Mech. - ASCE.*, **128**, 889-897.

Pichler, B. and Dormieux, L. (2009), "Instability during cohesive zone growth", *Eng. Fract. Mech.*, **76**(11), 1729-1749.

Pituba, J.J.C. (2006), "On the formulation of damage constitutive models for bimodular anistropic media". *(Ed., C.A.M. Soares) European conference on computational mechanics, 3rd Ed. Springer*, 371. DOI: 10.1007/1-4020-5370-3_371.

Pituba, J.J.C. (2010), "Validation of a damage model for the non linear analysis of reinforced concrete structures", *Acta Sci. Technol.*, **32**(3), 251-259. DOI: 10.4025/actascitechnol.v32i3.7009.

Pituba, J.J.C. and Fernandes, G.R. (2011), "An anisotropic damage model for concrete", *J. Eng. Mech. – ASCE*, **137**(9), 610-624. DOI: 10.1061/(ASCE)EM.1943-7889.0000260.

Pituba, J.J.C. and Lacerda, M.M.S. (2012), "Simplified damage models applied in the numerical analysis of reinforced concrete structures", *Rev. IBRACON Estrut. Mater.*, **1**, 26-37.

Pituba, J.J.C., Delalibera, R.G. and Rodrigues, F.S. (2012), "Numerical and statistical analysis about displacements in reinforced concrete beams using damage mechanics", *Comput Concr.*, **10**(3), 307-330.

Pituba, J.J.C. and Souza Neto, E. A. (2012), "Characterization of macroscopic mechanical behavior of concrete with mesoscopic scale finite element analysis", *(Eds., J. Eberhardsteiner, H.J. Böhm, F.G. Rammerstorfer), European Congress on Computational Methods in Applied Sciences and Engineering*, Vienna University of Technology, Viena, ISBN: 978-3-9502481-9-7.

Skarzynski, L. and Tejchman, J. (2012), "Determination of representative volume element in concrete under tensile deformation", *Comput Concr.*, **9**(1), 35-50.

Van Mier, G. M. (1997), *Fracture Process of Concrete*, CRC Press, Zurich, Switzerland.

Vecchio, F.J. and Emara, M.B. (1992), "Shear deformations in reinforced concrete frames", *ACI Struct. J.*, **89**(1), 46-56.

Welemane, H. and Cormery, F. (2002), "Some remarks on the damage unilateral effect modeling for microcracked materials", *Int. J. Damage Mech.*, **11**(1), 65-86.

Willam, K., Stankowski, T. and Sture, S. (1988), "Theory and basic concepts for modelling concrete behaviour", Contribution to the 26th CEB Plenary Session, Dubrovnik.

Zhu, Q., Kondo, D., Shao, J. and Pensee, V. (2008), "Micromechanical modelling of anisotropic damage in brittle rocks and application", *Int. J. Rock Mech. Min.*, **45**(4), 467-477.

Zhu, Q., Kondo, D. and Shao, J. (2009), "Homogenization-based analysis of anisotropic damage in brittle materials with unilateral effect and interactions between microcracks", *Int. J. Numer. Anal. Met.*, **33**(6), 749-772.

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List of Symbols

$\boldsymbol{D_T}$	Fourth-order damage tensor in tension regimes
$\boldsymbol{D_C}$	Fourth-order damage tensor in compression regimes
D_i	Scalar damage variables for 1D constitutive model (i=1 for tension regimes; i=2 for compression regimes)
$\boldsymbol{I}$	Second-order identity tensor
$\boldsymbol{A}$	Second-order tensor related to transverse isotropy symmetry
$\boldsymbol{E_T}$	Constitutive tensor in tension regimes
$\boldsymbol{E_C}$	Constitutive tensor in compression regimes
λ_0, μ_0	Lamè constants
λ_{ij}^{+}	Damage functions related to damage tensor in tension regimes
λ_{ij}^{-}	Damage functions related to damage tensor in compression regimes
μ_i	Damage functions related to shear behaviour of the concrete
$g(\varepsilon)$	Hyperplane in the strain space
$\boldsymbol{N}$	Unite vector perpendicular to hyperplane $g(\varepsilon)$
γ_i	Damage functions related to hyperplane $g(\varepsilon)$
$Y_{T,C}$	Associated variables in tension or compression regimes
A_i, B_i, Y_{0i}	Parameters of the constitutive model related to damage process for 1D constitutive model (i=1 for tension regimes; i=2 for compression regimes)
β_i	Parameters of the constitutive model related to plasticity process for 1D constitutive model (i=1 for tension regimes; i=2 for compression regimes)
f_t	Tension strength of the concrete
f_c	Compression strength of the concrete
E_S	Elasticity module of reinforcement bar
A_C	Cross section area
A_S	Reinforcement area in the cross section