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A new approach to determine the moment-curvature relationship of circular reinforced concrete columns

Naci Caglar*[1], Aydin Demir[1], Hakan Ozturk[1] and Abdulhalim Akkaya[2]

[1]Department of Civil Engineering, Engineering Faculty, Sakarya University, 54187 Sakarya, Turkey
[2]Department of Civil Engineering, Technology Faculty, Sakarya University, 54187 Sakarya, Turkey

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Abstract. To be able to understand the behavior of reinforced concrete (RC) members, cross sectional behavior should be known well. Cross sectional behavior can be best evaluated by moment-curvature relationship. On a reinforced concrete cross section moment-curvature relationship can be best determined by both experimentally or numerically with some complicated iteration methods. Making these experiments or iterations manually is very difficult and not practical. The aim of this study is to research the efficiency of Neural Networks (NN) as a more secure and robust method to obtain the moment-curvature relationship of circular RC columns. It is demonstrated that the NN based model is highly successful to determine the moment-curvature relationship of circular reinforced concrete columns.

Keywords: moment-curvature; circular RC column; neural networks; column behavior; confinement

1. Introduction

It is known that the columns are the most critical elements of any RC building during earthquakes. Failure of one of columns could lead the building to collapse. Therefore behavior of the entire structure should be known well to make earthquake resistant design. The behavior of a structure depends on behavior of its members. In order to be able to understand the behavior of reinforced concrete members, cross sectional behavior should be known well. Cross sectional behavior can be best evaluated by moment-curvature (MC) relationship.

On a reinforced concrete cross section, moment-curvature relationship can be best determined by experimental studies. For this aim, there are also some complicated numerical iteration methods. Testing all members however is not feasible and also making these numerical iterations manually is very difficult and not practical. Some spread sheet programs can be used for this purpose. In this study, the Neural Network (NN) is applied to estimate the moment-curvature relationship of circular reinforced concrete columns.

In recent years, the NN was widely applied in many engineering applications (Caglar and Garip 2013, Caglar 2009, Gunaratnam and Gero 2008, Pala 2006, Adeli and Samant 2000, Bishop 1995, Kulkarni 1994) and proved to be very promising. However in the literature there are few studies regarding NN applications of moment-curvature relationship of circular reinforced concrete

*Corresponding author, Associate Professor, E-mail: ancaglar@gmail.com

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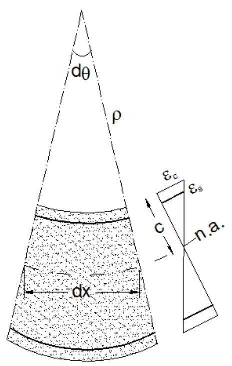
columns. It is difficult to find a study consisting of the most crucial parameters of moment-curvature relationship such as confinement, axial load level, section dimension and vertical and horizontal reinforcement. Jadid and Fairbairn (1996) studied the concept of parallel distributed processing base learning in artificial neural networks, assisting with experimental evidence to predict moment-curvature parameters that are usually accomplished solely by experimental work. However in that study input data is very limited.

Arslan (2012) studied the parameters effecting curvature ductility of RC columns by applying a pushover analysis to different statuses of the sample RC system for the same parameters and calculated the ratio variations and respective displacement (global) ductility of the frames. The relationship between obtained ductility values with the parameters, as well as the accuracy of the established model, were estimated using regression analyses (Multi-linear and Nonlinear Regression (MLR, NLR)) and 11 various Artificial Neural Networks (ANN) methods. In addition, Petschke *et al*. (2013) analyzed in their studies conventional beam cross-sections with MC diagrams considering asymmetrically reinforced cross-sections under combined effect of bending and moderate axial force.

The main objective of this study is applying the NN to determine the moment-curvature relationship of circular reinforced concrete columns which have different dimensions and/or reinforcing configurations. Sections are selected from widely-used circular reinforced concrete sections and the numerical study to produce data for NN analysis is made by a cross sectional analysis program, XTRACT. The NN-based estimates are compared with the numerical results. The results are presented in graphical form.

2. Moment-curvature

The behavior of a member subjected to bending or combined bending and axial load can be understood if the moment-curvature relationship is available. By studying this relationship one can predict the strength and the stiffness, as well as the ductility characteristics of the cross-section (Ersoy *et al*. 2008).



$$Curvature = \phi = \frac{d\theta}{dx} = \frac{d^2 y}{dx^2} = \frac{1}{\rho}$$

$$\phi = \frac{M}{EI}$$

EI: Flexural stiffness

Fig. 1 Moment-curvature relationship.

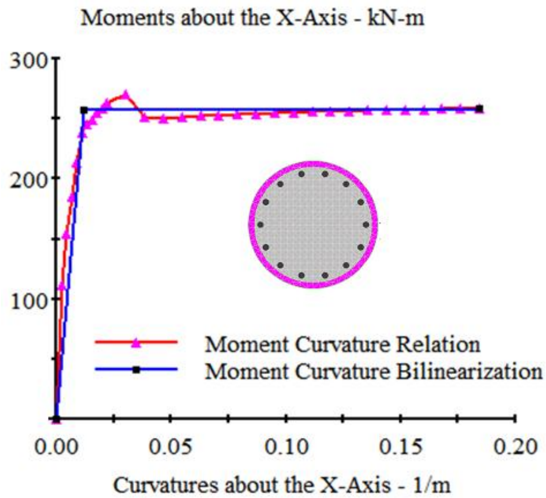


Fig. 2 MC relationship of a circular RC section obtained from XTRACT.

Curvature which is one of geometrical parameters representing deformation is defined as unit rotation angle. It is the derivative of the inclination of the tangent with respect to arc length.
Moment-curvature relationship can be obtained experimentally or analytically. Since it is not feasible to test members whenever moment-curvature relationship is needed, realistic analytical methods are of primary importance.

Moment-curvature relationship for reinforced concrete sections can be generated using equilibrium and compatibility equations and material models. With the aid of computers it is possible to use more realistic material models, and to consider strain hardening in steel and step by step cover crushing (Ersoy and Ozcebe 1998).

3. Cross sectional analysis program for moment curvature relationship

"XTRACT is a fully interactive program for the analysis of any cross section. It can generate moment curvatures, axial force-moment interactions, and moment-moment interactions for concrete, steel, prestressed and composite structural cross sections. It can handle the input of any arbitrary cross section (even with holes) made up of any material input from the available nonlinear material models." (XTRACT and User Manual)

Moment-curvature relationship of a circular reinforced concrete section derived from XTRACT and its bilinearized curve are shown in Fig. 2.

4. Neural networks

NN is a computational tool, which tries to simulate the architecture and internal operational features of the human brain and nervous system. NN architectures are formed by three or more layers and they consist of an input layer, an output layer and a lot of hidden layers in which neurons are connected to each other with modifiable weighted interconnections. The NN architecture is generally referred to as a fully interconnected feedforward multilayer perceptron. Moreover, there is a bias only connected to neurons in the hidden and output layers with modifiable weighted connections. The number of neurons in each layer might vary depending on the problem (Caglar 2009).

The most commonly used training algorithm for multi-layered feedforward networks is the back-propagation (BP) algorithm. The BP algorithm basically includes two phases. The first phase is the forward where the activations are propagated from input to output layer. The second phase is the backward where the error between the observed actual value and the desired nominal value in the output layer is propagated backwards to modify the weights and bias values. The inputs and the outputs of training and testing sets must be initialized before the training a feed work network. In the forward phase, the weighted sum of input components is calculated as;

$$\mathrm{net}_j = \sum_{i=1}^{n} w_{ij} x_i + \mathrm{bias}_j \tag{1}$$

where net_j is the weighted sum of the j^{th} neuron for the input received from the preceding layer with n neurons, w_{ij} is the weight between the j^{th} neuron and the i^{th} neuron in the preceding layer and x_i is the output of the i^{th} neuron in the preceding layer.

The output of the j^{th} neuron out_j is calculated with a sigmoid function as follows;

$$\mathrm{out}_j = f\left(\mathrm{net}_j\right) = \frac{1}{1+e^{-(\mathrm{net}_j)}} \tag{2}$$

The training of the network is achieved by adjusting the weights and it is carried out through a lot of training sets and training cycles. The training procedure is carried out to find the optimal set of weights, which would produce the right output for any input in the ideal case. Training the weights of the network is iteratively adjusted to find the relationship between the input and output patterns (Caglar 2009).

Back-propagation algorithm is the most common training algorithm for the multi-layer feed-forward neural network. The gradient descent and gradient descent with momentum back-propagation training algorithms are slow. Therefore, several adaptive training algorithms for NN have recently been discovered such as Conjugate Gradient Algorithm (CG) and Scaled Conjugate Gradient Algorithm (SCG). In this study, SCG is used to optimize the algorithm, which is all set to standard values suggested in Moller (1993).

The output of the network is compared with a desired response to produce an error. The sum of the squares error (SSE) is the performance function for feed forward networks. The process of feed forward and back-propagation continue until the intended sum of the squares error is reached. The SSE is defined as;

$$\mathrm{SSE} = \sum_{i=1}^{m} \left(T_i - \mathrm{out}_i\right)^2 \tag{3}$$

where T_i is the target outputs and out_i is output of neural network values for i^{th} output neuron, and m is the number of neurons in the output layer (Caglar 2009).

5. Numerical study

In the study, the NN based model was applied to estimate the moment-curvature relationship of circular reinforced concrete columns which have different dimensions and/or reinforcing configurations (Fig. 3). To train and test the NN model, training and testing sets were generated. For this purpose; 314 circular reinforced concrete column sections which have different geometrical properties, longitudinal and transverse reinforcing configurations and under different axial load were composed and they were designed according to both Turkish Earthquake Code (TEC 2007) and Turkish design and construction code for reinforced concrete structures (TS 500). Different axial force combinations acting on the section was selected by following requirements of above codes:

$$N_{dm} \leq 0{,}5 A_c\, f_{ck} \qquad \text{(TEC2007)} \tag{4}$$

$$N_{d} \leq 0{,}6 A_c\, f_{ck} \qquad \text{(TS500)} \tag{5}$$

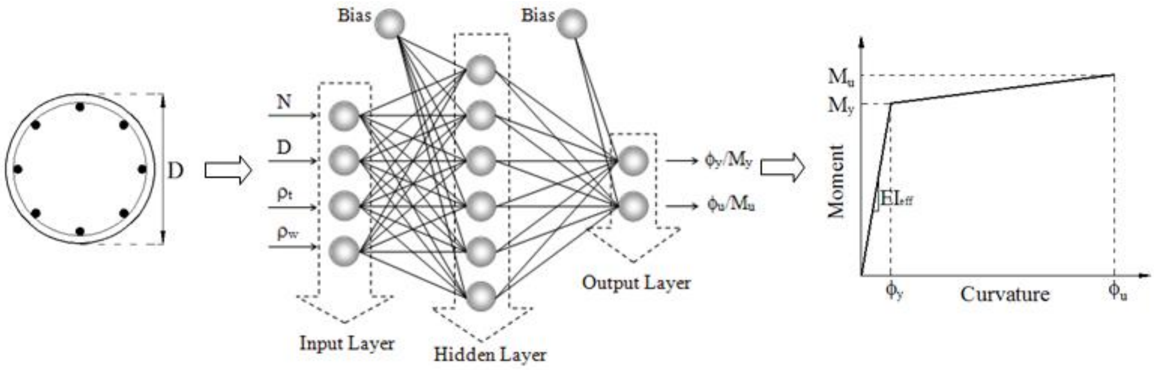


Fig. 3 General architecture of proposed NN based model

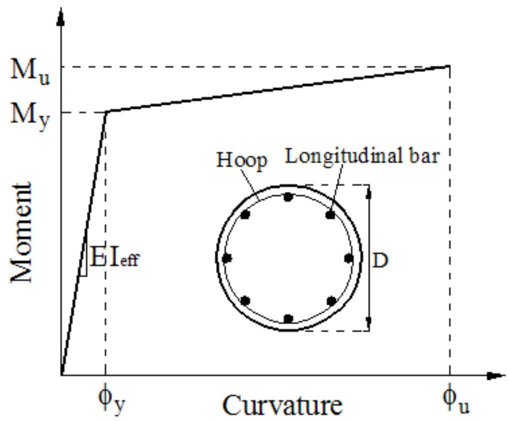


Fig. 4 The general form of a circular RC column section and bilinear MC curve

Table 1 Range of parameters in the database and normalization values

Input Parameters	Min	Max	Normalization Values
$D(m)$	0.30	3.00	3.00
$N(kN)$	0	88390	100000
$\rho_t(\%)$	0.01048	0.03840	0.040
$\rho_w(\%)$	0.007178	0.01608	0.0170
Output Parameters	**Min**	**Max**	**Normalization Values**
$M_y(kNm)$	41.17	108000	120000
$M_u(kNm)$	48.27	111000	120000
ϕ_y	0.00142	0.02047	0.0250
ϕ_u	0.01284	0.55710	0.6000

Table 2 Statistical parameters of NN formulation for moments

	M_y		M_u	
	Training Set	Testing Set	Training Set	Testing Set
SSE	0.000560	0.0000585	0.000433	0.0000839
RMS	0.243744	0.249892	0.256861	0.264096
R^2	0.999952	0.999962	0.999966	0.999951

Table 3 Statistical parameters of NN formulation for curvatures

	ϕ_y		ϕ_u	
	Training Set	Testing Set	Training Set	Testing Set
SSE	0.025752	0.003370	0.022121	0.001915
RMS	0.330605	0.395226	0.189242	0.086722
R^2	0.99774	0.998074	0.99517	0.99901

In which; N_d and N_{dm} is axial force acting on the section, A_c is area of the section and f_{ck} is compressive strength of concrete.

It is given in TEC 2007 that minimum requirement for compressive strength of concrete and maximum requirement for minimum yield strength of reinforcing steel which can be used in buildings must be 20 MPa (concrete grade is C20) and 420 MPa (reinforcing steel grade is S420) respectively. C25 and S420 are widely-used in practice. Therefore concrete compressive strength of sections used in this study is selected as 25 MPa (C25). Additionally minimum yield and rapture strength of reinforcing bars are 420 MPa and 550 MPa respectively (S420). Mander concrete model (Mander *et al*. 1988) was used for confined and unconfined concrete in compression in which unconfined concrete strain corresponding to maximum strength and concrete crushing are taken 0.002 and 0.003 respectively according to TEC 2007. For reinforcing bar stress-strain relationship widely used stress-strain graph was used given in TEC 2007. It is assumed that plane sections remain plain after bending and there is perfect bond between reinforcing steel and concrete. Moreover the shear deformation of the section is neglected.

Moment-curvature relationship of those sections was determined by using XTRACT and it is defined as a bilinear curve with 4 parameters showed in Fig. 4. Consequently the data for NN analysis were created. By using curvature parameters; yield curvature (ϕ_y) and ultimate curvature (ϕ_u) and moment parameters; yield moment (M_y) and ultimate moment (M_u), NN based models of NN_1 and NN_2 were created respectively.

Totally 314 data are randomly divided into two parts as the training and testing sets. 283 data from database are selected as training set and employed to train NN based model. 31 data, which are not used in the training process, are selected as the testing set and used to validate the generalization capability of NN based model. Inputs and outputs are normalized in the (0–1) range by using simple normalization methods and values are given in Table 1. The maximum and minimum values of inputs and outputs are also given in Table 1. The testing set is tabulated in Table 4.

In Table 1;

- D is diameter of section and *N* is axial load,
- ρ_t is longitudinal reinforcing steel ratio can be calculated as total area of longitudinal reinforcing bars is divided by area of the section.
- ρ_w is transverse (volumetric) reinforcing steel ratio taken as the (volume of transverse reinforcement)/(volume of concrete). It is also the reinforcing ratio in the two principal orthogonal directions added together.

The numbers of neurons in input and output layers are based on the geometry of the problem. But, there is no general rule for selection of the number of neurons in a hidden layer and the number of the hidden layers. Therefore they are determined by trial and error method in this study. Numbers of different NN models with various hidden neurons are trained and tested for only 3000 epochs. Each NN model is initialized with different random weights. The most appropriate NN models are chosen corresponding to performance of both training and testing sets. Hence, both of the NN models are selected as having 4 neurons in input layer, 6 neurons in hidden layer and 2 neurons in output layer to define the curvatures (ϕ_y and ϕ_u) and moments (M_y and M_u) of circular reinforced concrete sections, respectively (Figs. 5-6). The performance of the selected NN models are tabulated in terms of the sum of the squares error (SSE), the root-mean-squared (RMS) and the absolute fraction of variance (R2) for both training and testing sets (Table 2-3). All of these statistical values have proved that the proposed NN models are appropriate and they can estimate reliably the moment-curvature relationship of circular reinforced concrete columns.

A MATLAB based program with a graphical user interface (GUI) was developed to train and test the NN model (Pala *et al*. 2008). In the NN model, type of back-propagation is scaled conjugate gradient algorithm (SCGA), activation function is Sigmoidal Function, and number of epochs (learning cycle) are 7000000.

The values of parameters used in this research are summarized as follow:

- Number of input layer unit = 4
- Number of hidden layers =1
- Number of hidden layer units = 6
- Number of output layer units = 2

- Learning algorithm = scaled conjugate gradients algorithm(SCGA)
- Learning cycle = 7000000

Table 4 Testing set

#	D	N	ρ_t	ρ_w	M_y (kN-m)		M_u (kN-m)		ϕ_y (1/m)		ϕ_u (1/m)	
	(m)	(kN)	(%)	(%)	XTRACT	NN	XTRACT	NN	XTRACT	NN	XTRACT	NN
1	0.50	1200	0.01098	0.00873	321.7	255.8	323.6	296.0	0.01030	0.00969	0.13700	0.13150
2	0.50	2100	0.01434	0.00873	408.5	340.7	402.7	384.9	0.00921	0.00932	0.09411	0.10180
3	0.50	600	0.01814	0.00873	341.0	356.8	365.5	408.2	0.01001	0.01041	0.15230	0.15205
4	0.50	1250	0.02710	0.00873	497.2	538.0	517.4	582.9	0.01107	0.01064	0.11800	0.11314
5	0.50	200	0.03226	0.00873	456.2	589.3	507.6	631.9	0.01039	0.01061	0.14850	0.14648
6	0.75	0	0.01320	0.00748	719.2	848.5	860.2	964.0	0.00590	0.00567	0.14220	0.14491
7	0.75	3100	0.01670	0.00748	1416	1267	1447	1374	0.00703	0.00661	0.06942	0.06958
8	0.75	4700	0.02062	0.00748	1860	1577	1705	1662	0.00647	0.00658	0.05567	0.05564
9	0.75	600	0.02495	0.00748	1356	1452	1529	1589	0.00648	0.00676	0.09407	0.09519
10	0.75	5500	0.02972	0.00748	2016	1979	2069	2002	0.00644	0.00675	0.05062	0.04836
11	0.75	400	0.03485	0.00748	1727	1893	1950	2003	0.00671	0.00674	0.08754	0.08446
12	1.00	8000	0.01604	0.00802	3616	3438	3662	3622	0.00480	0.00455	0.04653	0.04800
13	1.00	4000	0.01941	0.00802	3356	3189	3578	3469	0.00508	0.00490	0.06295	0.06399
14	1.00	6000	0.02310	0.00802	4027	3910	4187	4167	0.00527	0.00492	0.05224	0.05302
15	1.00	6000	0.03144	0.00802	4761	4743	5004	4986	0.00535	0.00508	0.05090	0.04892
16	1.50	20000	0.01294	0.00718	11700	11660	11800	11889	0.00301	0.00311	0.02722	0.03064
17	1.50	5000	0.02096	0.00718	10700	10709	11900	11892	0.00321	0.00325	0.04513	0.04729
18	1.50	10000	0.01540	0.00718	10500	10327	11000	11092	0.00334	0.00323	0.03898	0.04260
19	1.50	0	0.02738	0.00718	11100	11138	13000	12718	0.00315	0.00315	0.05175	0.05220
20	2.00	0	0.01155	0.00838	12800	13415	15400	15695	0.00220	0.00224	0.06357	0.05954
21	2.00	8000	0.02053	0.00838	25100	25189	28400	28439	0.00239	0.00239	0.03935	0.04119
22	2.00	16000	0.01356	0.00838	23100	22999	24700	24633	0.00244	0.00240	0.03430	0.03720
23	2.00	24000	0.01572	0.00838	27900	27733	29000	29103	0.00259	0.00243	0.02789	0.02940
24	2.00	32000	0.02599	0.00838	37300	37308	39000	39279	0.00252	0.00258	0.02326	0.02100
25	2.50	52000	0.01643	0.00848	59800	59627	61800	61506	0.00191	0.00194	0.01888	0.01624
26	2.50	13000	0.02079	0.00848	49800	50006	56400	56551	0.00191	0.00192	0.03139	0.03182
27	2.50	26000	0.01084	0.00848	41400	41254	43800	43500	0.00193	0.00200	0.02805	0.02760
28	2.50	39000	0.01444	0.00848	53200	53126	55200	55183	0.00205	0.00200	0.02233	0.02100
29	3.00	80000	0.01048	0.00722	89700	89829	90700	91573	0.00149	0.00150	0.01381	0.01360
30	3.00	0	0.01369	0.00722	49800	49644	60200	60095	0.00146	0.00142	0.03633	0.03509
31	3.00	60000	0.01733	0.00722	101000	100727	104000	103607	0.00169	0.00166	0.01586	0.01751

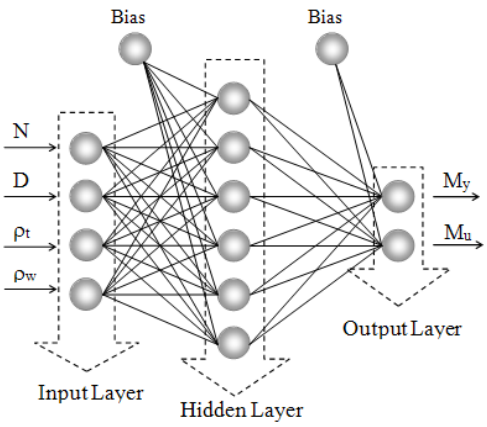


Fig. 5 Architecture of the NN model for moment

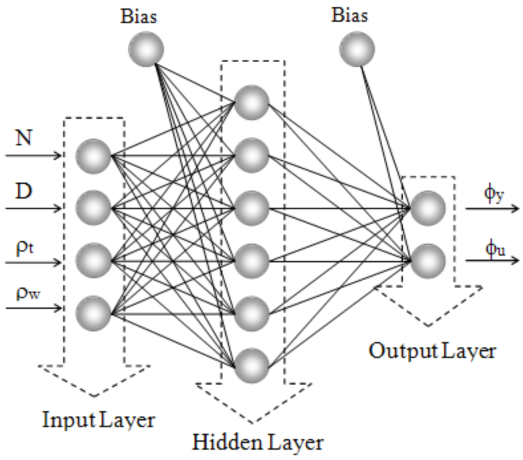


Fig. 6 Architecture of the NN model for curvature

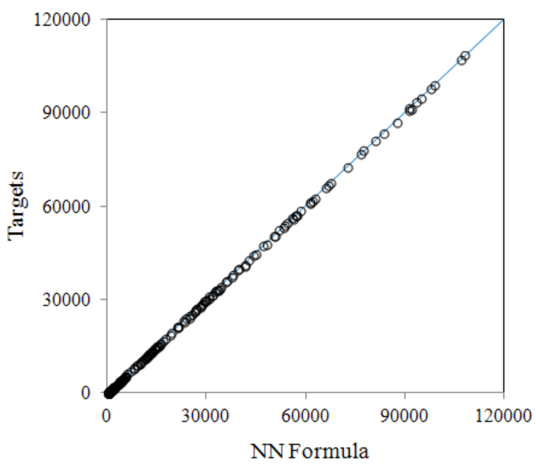


Fig. 7 Performance of training set for M_y

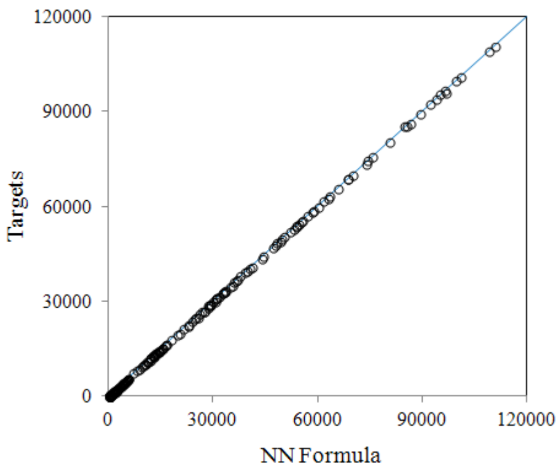


Fig. 8 Performance of training set for M_u

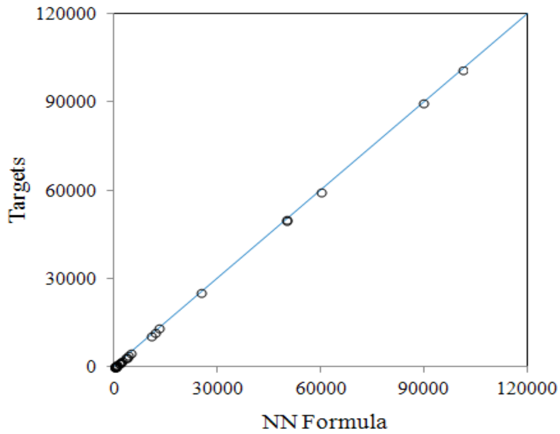


Fig. 9 Performance of testing set for M_y

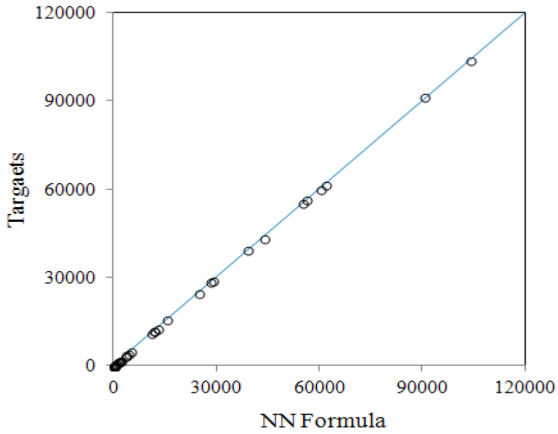


Fig. 10 Performance of testing set for M_u

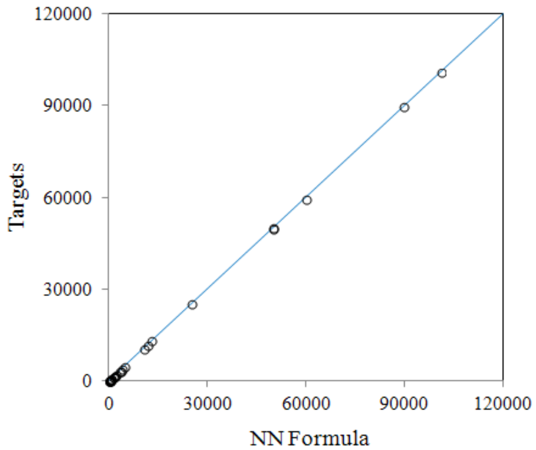


Fig. 11 Performance of training set for ϕ_y

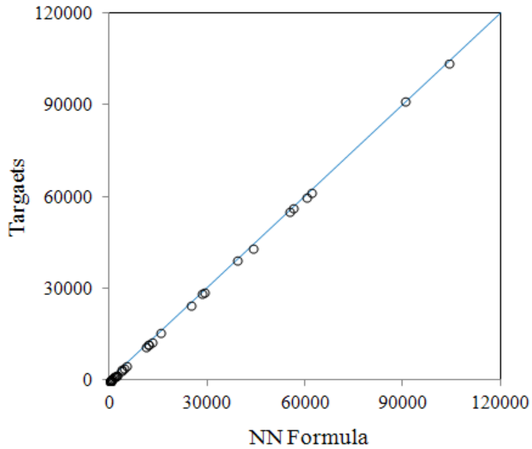


Fig. 12 Performance of training set for ϕ_u

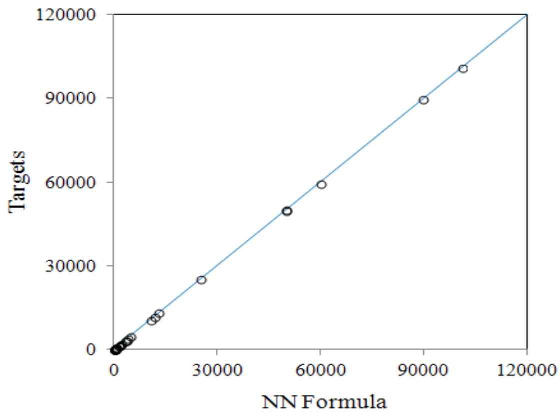


Fig. 13 Performance of testing set for ϕ_y

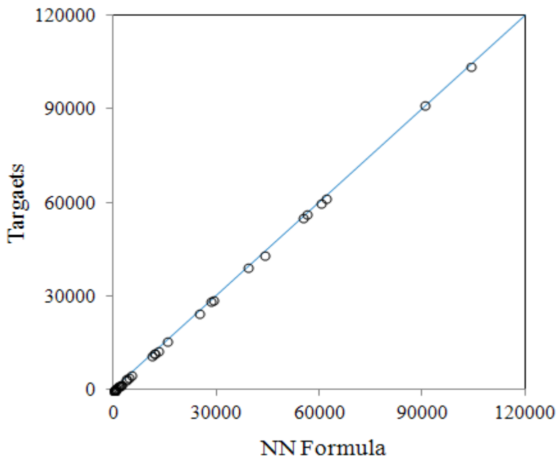


Fig. 14 Performance of testing set for ϕ_u

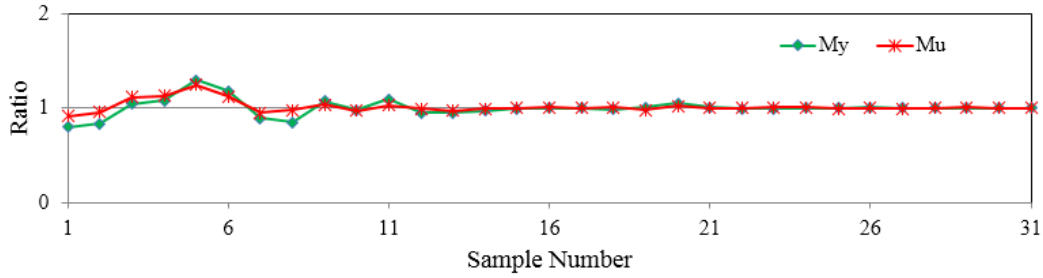


Fig. 15 The ratio of NN versus numerical study results for moments

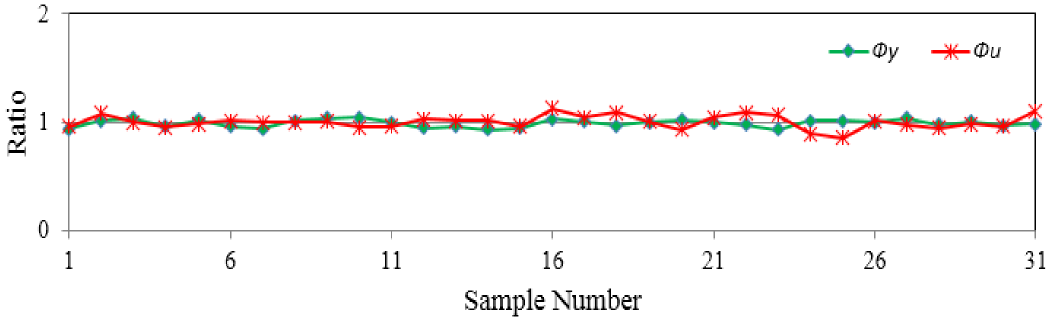


Fig. 16 The ratio of NN versus numerical study results for curvatures

The performance of the NN model showed that the correlations between targets and outputs are consistent as shown in Figs. 7-8 and 11-12 for training set and in Figs. 9-10 and 13-14 for testing set in which the closer circles to the diagonal line mean the better result. The results of Figs. 7-8 and 11-12 indicate that the NN based model is successful in learning the relationship between the input parameters and outputs. The results of testing phases in Figs. 9-10 and 13-14 show that the NN based model is capable of generalizing between input and output variables.

The results of trained NN models are compared with the results of XTRACT fully interactive program. The ratio values of NN versus XTRACT results are illustrated for testing set in Figs. 15-16. As can be seen from the Figs. 15-16, NN results agree well with the XTRACT results.

6. Explicit formulation of moment-curvature relationship of circular RC sections

The explicit formulation of moment-curvature relationship of circular reinforced concrete sections is derived by using the parameters (inputs, weights, normalization factors) of the proposed NN models. All necessary parameters are obtained from the trained NN, and the explicit expression is formed from the weights of the trained NN model. The weights and bias values in the derivations of NN based formulations are given in Tables 5-6 and Tables 7-8 for moments and curvatures, respectively. Each input is multiplied by a connection weight. In the simplest case, products and biases are simply summed, then transformed through a transfer function (logistic sigmoid) to generate a result, and finally outputs are obtained more easily. Inputs and outputs are normalized before the learning process of the NN. To get an accurate result from the proposed formula in this study, normalization values have to be considered as well. It should be noted that the proposed formulation is valid between the maximum and minimum values of the input parameters given in Table 1. The main goal is to obtain the prediction of moment-curvature relationship of circular reinforced concrete sections in a functional form in terms of (D), (N), (ρ_t) and (ρ_w). The output is the curvatures and moments of circular reinforced concrete sections and, given as follows in Eqs. (6-13) and Eqs. (14-21), respectively.

$$F_1 = 10.3169 \times N - 11.8706 \times d + 1.4520 \times \rho_t - 5.368 \times \rho_w + 1.228 \tag{6}$$

$$F_2 = -0.0009 \times N - 1.4794 \times d - 0.0423 \times \rho_t - 0.2114 \times \rho_w + 1.8831 \tag{7}$$

$$F_3 = -5.7584 \times N + 4.5034 \times d - 1.6257 \times \rho_t - 1.4087 \times \rho_w - 2.1352 \tag{8}$$

$$F_4 = -2.0794 \times N - 6.7606 \times d - 6.7698 \times \rho_t + 9.5631 \times \rho_w + 9.5495 \tag{9}$$

$$F_5 = 6.1874 \times N + 4.2503 \times d + 0.9065 \times \rho_t - 1.6198 \times \rho_w + 1.1913 \tag{10}$$

$$F_6 = 1.7571 \times N - 2.3284 \times d + 3.6728 \times \rho_t - 1.2011 \times \rho_w + 3.2583 \tag{11}$$

$$M_y = \frac{120000}{1 + e^{\left(-\frac{5.0127}{1+e^{-F1}} - \frac{16.7954}{1+e^{-F2}} - \frac{0.7761}{1+e^{-F3}} - \frac{13.2649}{1+e^{-F4}} + \frac{13.2165}{1+e^{-F5}} + \frac{5.8745}{1+e^{-F6}} + 4.9366 \right)}} \tag{12}$$

$$M_u = \frac{120000}{1 + e^{\left(-\frac{5.9316}{1+e^{-F1}} - \frac{17.4077}{1+e^{-F2}} - \frac{0.158}{1+e^{-F3}} - \frac{14.7174}{1+e^{-F4}} + \frac{12.0141}{1+e^{-F5}} + \frac{7.5828}{1+e^{-F6}} + 6.3564 \right)}} \tag{13}$$

Table 5 Weight values between input and hidden layers of NN model for moments

	Number of hidden layer neurons (1)					
	1	2	3	4	5	6
w11	10.3169	.-0.0009	-5.7584	-2.0794	6.1874	1.7571
w21	-11.8706	-1.4794	4.5034	-6.7606	4.2503	-2.3284
w31	1.4520	-0.0423	-1.6257	-6.7698	0.9065	3.6728
w41	-5.368	-0.2114	-1.4087	9.5631	-1.6198	-1.2011
Bias	1.2280	1.8831	-2.1352	9.5495	1.1913	3.2583

Table 6 Weight values between hidden and output layers of NN model for moments

	Number of hidden layer neurons (1)					
	1	2	3	4	5	6
w1	-5.0127	-16.7954	-0.7761	-13.2649	13.2165	5.8745
w2	-5.9316	-17.4077	-0.1580	-14.7174	12.0141	7.5828

Table 7 Weight values between input and hidden layers of NN model for curvatures

	Number of hidden layer neurons (1)					
	1	2	3	4	5	6
w11	-9.6334	-0.4632	-0.0604	159.4341	-2.1477	72.0605
w21	-8.8205	-2.8440	3.4833	10.8219	3.8971	68.3898
w31	-0.8340	-0.4847	0.6447	1.9643	0.9307	1.7212
w41	2.4424	-1.6197	-2.2265	-3.8263	2.8597	3.3448
Bias	-2.6515	1.1638	-0.0402	4.2580	3.6184	-8.6633

Table 8 Weight values between hidden and output layers of NN model for curvatures

	Number of hidden layer neurons (1)					
	1	2	3	4	5	6
w1	7.5286	-73.9538	19.9062	101.1282	133.9029	-112.8784
w2	89.0581	106.9607	-42.7741	-121.7444	-44.5398	109.4638

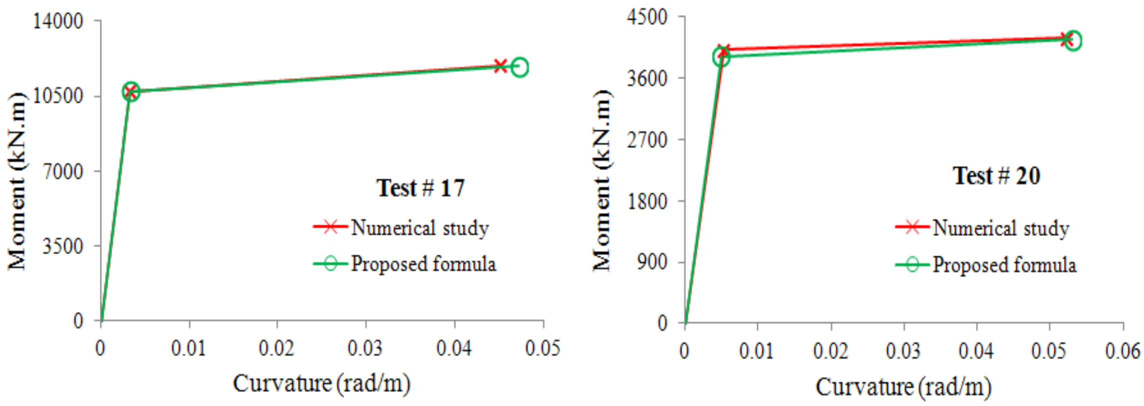


Fig. 17 Comparison of moment-curvature relationships of some sections

$$F_1 = -9.6334\times N - 8.8205\times d - 0.8340\times \rho_t + 2.4424\times \rho_w - 2.6515 \tag{14}$$

$$F_2 = -0.4632\times N - 2.844\times d - 0.4847\times \rho_t - 1.6197 + 1.1638 \tag{15}$$

$$F_3 = -0.0604\times N + 3.4833\times d + 0.6447\times \rho_t - 2.2265 - 0.0402 \tag{16}$$

$$F_4 = 159.4341\times N + 10.8219\times d + 1.9643\times \rho_t - 3.8263 + 4.2580 \tag{17}$$

$$F_5 = -2.1477\times N + 3.8971\times d + 0.9307\times \rho_t - 2.8597 + 3.6184 \tag{18}$$

$$F_6 = 72.0605\times N + 68.3898\times d + 1.7212\times \rho_t + 3.3448 - 8.6633 \tag{19}$$

$$\phi_y = \frac{0.025}{1+e^{\left(\frac{7.5286}{1+e^{-F1}}-\frac{73.9538}{1+e^{-F2}}+\frac{19.9062}{1+e^{-F3}}+\frac{101.1282}{1+e^{-F4}}+\frac{133.9029}{1+e^{-F5}}-\frac{112.8784}{1+e^{-F6}}-71.7028\right)}} \quad (20)$$

$$\phi_u = \frac{0.600}{1+e^{\left(\frac{89.0581}{1+e^{-F1}}+\frac{106.9607}{1+e^{-F2}}-\frac{42.7741}{1+e^{-F3}}-\frac{121.7444}{1+e^{-F4}}-\frac{44.5398}{1+e^{-F5}}+\frac{109.4638}{1+e^{-F6}}-9.6758\right)}} \quad (21)$$

The independent variables (D), (N), (ρ_t) and (ρ_w) are as mentioned before diameter of section, axial load, longitudinal reinforcing steel ratio and transverse (volumetric) reinforcing steel ratio, respectively. Functions F_1, F_2, F_3, F_4, F_5 and F_6 were obtained by employing these independent variables from Eqs. (1) and (2). The moments and curvatures were obtained by using functions F_1, F_2, F_3, F_4, F_5 and F_6 in Eqs. (6-13) and Eqs. (14-21), respectively. The last terms in the functions F_1, F_2, F_3, F_4, F_5 and F_6 and Eqs. (12-13) and (20-21) are bias values.

Parameter ranges should be selected to envelop the majority of the suitable range of applications. High deviation could be attributed to the fact that some of the input data used in the method were outside the input range where the formula is valid.

Moment-curvature relationship of some selected samples from testing set is illustrated for both of numerical study and proposed formula in order to demonstrate the effectiveness of proposed formula (Fig. 17). As shown in Fig. 17, the moment-curvature relationship of circular RC columns determined by NN based model shows reasonably close values to the numerical study results.

7. Conclusion

This paper deals with the validation of the formulas which are based on NN to calculate the moment-curvature relationship of reinforced concrete sections under axial force and bending moment. It is a prominent work in this field by taking into account the effect of confinement and dimension of circular reinforced concrete sections on moment-curvature relationship.

The sections designed according to TEC 2007 and TS 500 were analyzed by XTRACT to generate moment-curvature relationship and required data for NN analysis are obtained. Finally an explicit formulation of moment-curvature relationship of circular reinforced concrete sections is proposed. Results obtained from the formulation are truly competent and showed good generalization. The generalization capability of the explicit formulation obtained by using NNs is confirmed by cross sectional analysis results. The proposed explicit formulation in this study is time consuming and can predict the moment-curvature relationship of circular reinforced concrete sections in an effective manner. Moreover, the advantage of using this formulation is that it only consists of basic mathematical operations. Thus the new NN based formula can be easily employed to any programming language for determination of moment-curvature. This paper also shows how robust NNs can be, where used for the explicit formulation of various experimental works in engineering mechanics, especially for which an analytical formulation could not be obtained from the mathematical models and the results of experimental and numerical studies.

The input variables of the proposed formula should be selected from envelop of input parameter range of the study. Otherwise the determining moment-curvature relationship with the proposed NN formula using any input parameters outside the input range could cause the deviation to be increased.

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