Computers and Concrete, *Vol. 15, No. 2 (2015) 279-304*
DOI: http://dx.doi.org/10.12989/cac.2015.15.2.279

Stochastic finite element based reliability analysis of steel fiber reinforced concrete (SFRC) corbels

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(*Received July 11, 2014, Revised November 18, 2014, Accepted November 25 2014*)

Abstract. In this study, reliability analyses of steel fiber reinforced concrete (SFRC) corbels based on stochastic finite element were performed for the first time in literature. Prior to stochastic finite element analysis, an experimental database of 84 sfrc corbels was gathered from literature. These sfrc corbels were modeled by a special finite element program. Results of experimental studies and finite element analysis were compared and found to be very close to each other. Furthermore experimental crack patterns of corbel were compared with finite element crack patterns and were observed to be quite similar. After verification of the finite element models, stochastic finite element analyses were implemented by a specialized finite element module. As a result of stochastic finite element analysis, appropriate probability distribution functions (PDF's) were proposed. Finally, coefficient of variation, bias and strength reduction (resistance) factors were proposed for sfrc corbels as a consequence of stochastic based reliability analysis.

Keywords: steel fiber reinforced concrete corbel; nonlinear finite element analysis; stochastic analysis; reliability analysis; statistical parameters

1. Introduction

Corbels are structural elements primarily used in reinforced concrete and precast structures. The main function of corbels is to transfer vertical and horizontal loads to the members to which they are connected. Corbels can be the overhanging portion of beam with a small span length. Their shear span to effective depth ratio a/d is less than unity (Ersoy *et al*. 2006).

Use of steel fiber in reinforced concrete corbels provides considerable advantages. Previous studies related with corbels have concluded that reinforced concrete corbels which include only steel fibers as secondary reinforcement have almost the same load carrying capacity with ones in which horizontal stirrups are used against shear failure. Use of steel fiber facilitates the fabrication of corbels because of easier placement of it as compared to placement of horizontal stirrups. Steel fibers allow corbels to experience large deflections after achieving ultimate load without a dramatic loss in load carrying capacity or demonstrating a sudden and brittle failure (Fattuhi and

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http://www.techno-press.org/?journal=cac&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

Hughes 1989).

The orientation of steel fiber affects the mechanical properties of concrete significantly which leads to uncertainties in various mechanical properties of sfrc. Thus more conservative decisions should be made about sfrc corbels by considering these uncertainties. Moreover, there is lack of information about statistical parameters (coefficient of variation and bias factor) and resistance factor of sfrc corbels in literature. Thus the aim of this study is to improve the theoretical and practical background about statistical parameters (coefficient of variation and bias factor) and resistance factors of sfrc corbels via stochastic finite element based reliability analyses. In summary, the outcomes of this study will contribute to analysis and design of sfrc corbels.

2. Previous experimental studies about SFRC corbels

A series of experimental studies have been carried out on normal strength steel fiber reinforced corbels by Fattuhi and Hughes (1987-1994). Fattuhi and Hughes investigated effects of steel fiber on load carrying capacity of corbels whose test configuration is shown in Fig. 1. They changed various parameters (tensile and compressive strength of concrete, steel fiber volume fraction, shear span, fiber aspect ratio, effective depth, reinforcement ratio) and observed the mechanical response of sfrc corbels. Fattuhi (1994) also investigated the mechanical behavior of trapezoidal normal strength sfrc corbels.

Campione *et al*. (2007) studied the flexural behavior of fibrous reinforced corbels experimentally and suggested simple analytical expressions for bearing capacity by considering the shear contribution due to steel reinforcements and fibers. Campione (2009) carried out two experimental studies about sfrc corbels. In one of them performance of sfrc corbels under the combined effect of vertical and horizontal loads was investigated, while in the other one studies about flexural response of sfrc corbels were implemented.

On the other hand, mechanical behavior and performance of high strength sfrc corbels were investigated in different researches. High strength steel fiber reinforced corbels in trapezoidal form were experimented by Muhammad (1998) under monotonic and cyclic loading. Yang *et al*. (2012) investigated the influence of steel fibers on the serviceability of reinforced concrete corbels.

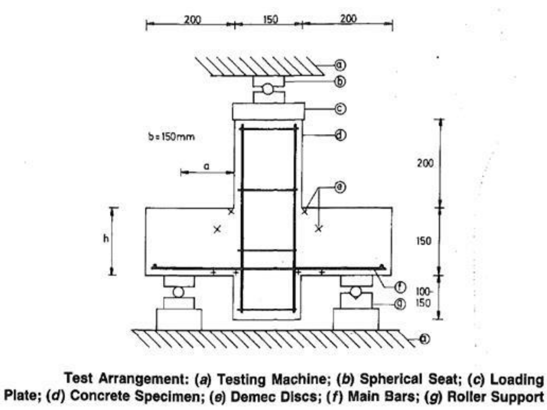


Fig. 1 Test configuration of corbels experimented by Fattuhi and Hughes (Fattuhi and Hughes 1989-1994)

3. Nonlinear finite element analysis of SFRC corbels

3.1 Theoretical basis

Finite element modeling of sfrc corbels considered in the study was performed by an effective and reliable tool for nonlinear analysis of reinforced concrete structures, called as ATENA. Three dimensional combined fracture-plastic material model which was proposed by Cervenka and Papanikolaou (2008) used in the software. Rankine failure criterion, which is one of the most suitable models for brittle and quasi-brittle materials, is used as failure criterion. The criterion is combined with classical orthotropic smeared crack formulation and crack band model to model the fracture behavior. The tensile behavior is assumed to be linear and elastic up to ultimate tensile strength. However, the behavior after ultimate tensile strength is represented by exponential softening rule (Fig. 2). During FE analysis, stresses, strains are evaluated by the consideration of material directions (one of them is normal and the other is parallel to the crack direction). Material directions are determined according to crack model (fixed or rotated) selected. In rotated crack model, material directions are principal directions corresponding to present current state. However, material directions are fixed to principal directions corresponding to crack initiation in the case of fixed crack model. The crack opening is obtained by the multiplication of total fracturing strain including the present increment of the strain and characteristic length (L_t), which is size of the element measured from the projection of the element on the crack direction.

Menetrey and Willam failure surface is employed for plasticity model of concrete crushing. The behavior of concrete from initial linear part to ultimate compressive strength and after ultimate compressive strength is directed by the hardening/softening law which is based on experimental observations of Van Mier (Fig. 3). The law is controlled by strains up to compressive strength (ascending curve); whereas the control is replaced by displacements to minimize mesh size effects during finite element solution after compressive strength (descending curve). The displacement is obtained by the use of equivalent plastic strain $\varepsilon_{eq}^{\;p}$ and L_c (Fig. 3) along the descending curve. This parameter is the projected element size on the direction of minimum principal stresses (Cervenka *et al*. (2013)).

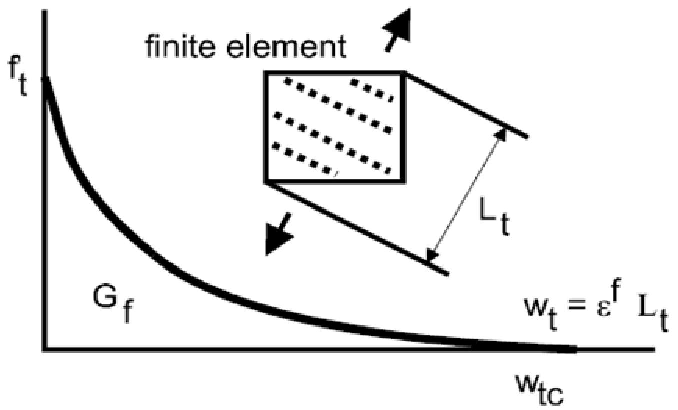


Fig. 2 Tensile exponential model used in ATENA (Cervenka and Pukl 2003)

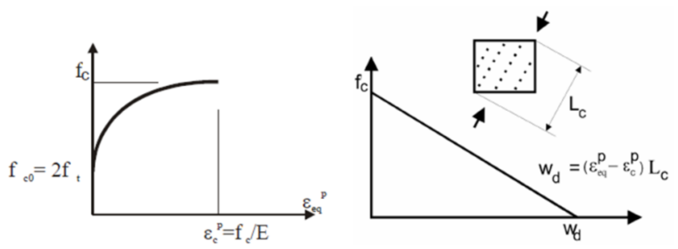


Fig. 3 Compressive hardening/softening model used in ATENA (Cervenka and Pukl 2003)

3.2 Finite element modeling of corbels

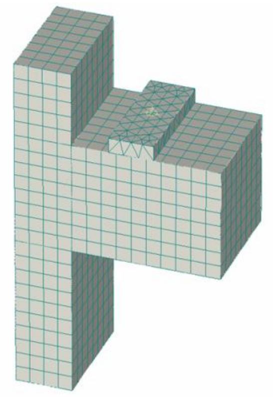
In this paper, 84 steel fiber reinforced concrete corbels which had been experimented by Fattuhi and Hughes (1989-1994) were modeled by ATENA 3D. Required material parameters for FE modeling were taken from related articles of Fattuhi and Hughes (1989-1994) whose experimental data are listed in Table A.1. Configuration of the FE model is given in Fig. 4.

Elastic perfectly plastic (bilinear) model was used for reinforcement modeling. Steel plates were modeled by the use of three dimensional elastic isotropic material model to prevent premature failure of the corbels. Effective concrete material model provided by the software, called as "3D Nonlinear Cementitious 2" was used to model the concrete material behavior. Basic assumptions of the model are stated in Section 3.1. To reflect the effect of fibers, a small modified version of "3D Nonlinear Cementitious 2", called as "CC3DNonlinCementitious2SHCC" was used by ATENA-GiD, which is a special module of ATENA. All parameters stated to both models are same, except properties of fibers (volume fraction, diameter etc.) can be specified to latter one. Perfect connection between the reinforcement and concrete material was assumed for analyses. Smeared crack approach and fixed crack model were selected for nonlinear finite element analyses of the corbels.

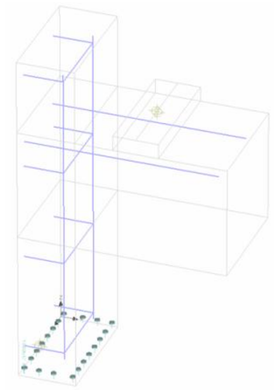
Half of the system was modeled to save time and disc space. Therefore, symmetry surface of the corbel was restrained to prevent translation and rotation along this surface. The plate on the corbel was restrained in vertical direction and also restrained in the direction along the corbel width to prevent the rigid body motion along this direction. Loading scheme of FE solution was achieved as upside down configurations. That is, initial displacement (0.1 mm) is given to the bottom surface of the column in upward direction and it was increased step by step as 0.1mm. Displacement values were recorded from bottom surface of column part and support reactions were obtained from the middle of the steel plate on the corbel (Fig. 4).

Perfect connection was assumed between steel plate and the corbel on the contact surface. On the contact surface of the plate and corbel, master element and slave element was selected by the software automatically to achieve mesh consistency. The software selects the element which has rougher meshes as master element and regulates the element type of corbel and plate accordingly. Therefore mesh compatibility was achieved by the mesh generator of the software. Newton-Raphson method was used as solution algorithm.

Various sizes (0.012, 0.015, 0.02, 0.025 and 0.03 m) were tried for meshing process. Mesh size versus load carrying capacity graph of T3 corbel is shown in Fig. 5. According to graph, there is no big difference between results of considered mesh sizes. Indeed, 4-6 elements per thickness is sufficient for bending problems. However, 8 elements per thickness, that is 0.02 m was selected as mesh size to achieve all failure modes including bending and to obtain both good accuracy and crack propagation by the consideration of time saving.



(a) Mesh Configuration



(b) Reinforcement Configuration

Fig. 4 Finite element model of a typical SFRC corbel

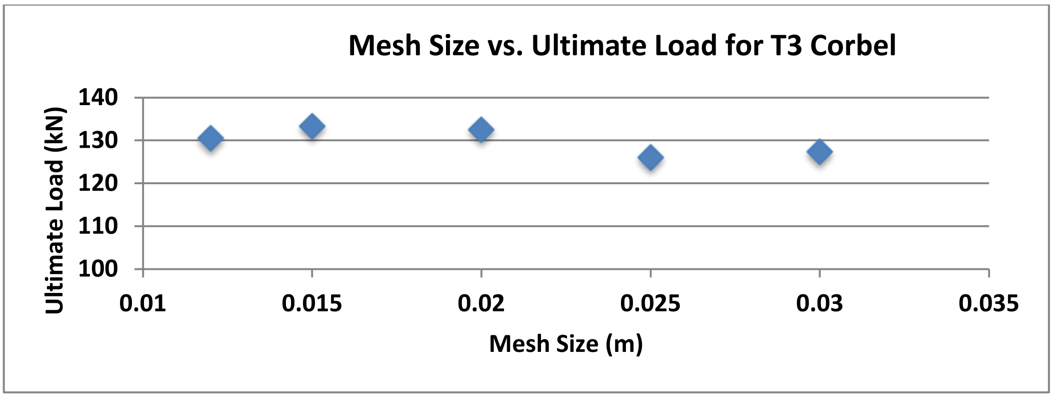


Fig. 5 Mesh size vs. Ultimate load for T3 corbel

3.3 Results of finite element analyses

According to FE results listed in Table A.1, ultimate load capacities of sfrc corbels calculated by nonlinear finite element analysis (V_{NLFEA}) were observed to very close to experimental results (V_{exp}). Mean, standard deviation, coefficient of variation and coefficient of correlation of V_{NLFEA}/V_{exp} were calculated as 1.034, 0.045, 0.044 and 0.974 respectively. These results prove the accuracy of nonlinear finite element analyses of considered corbels (Fig. 6).

Additionally, crack patterns of FE models at ultimate load were compared with experimental results which were observed to be very similar to each other. In Figs. 7(a)-(f), comparison of crack patterns between FE analyses and experimental results are shown.

Not only crack patterns, but also failure modes of corresponding corbels were observed to be almost the same for experimental and finite element results (For example, the failure mode is of flexural type for corbel 23 and of shear type for corbel 35). Maximum crack widths and crack patterns occur in same regions for both results.

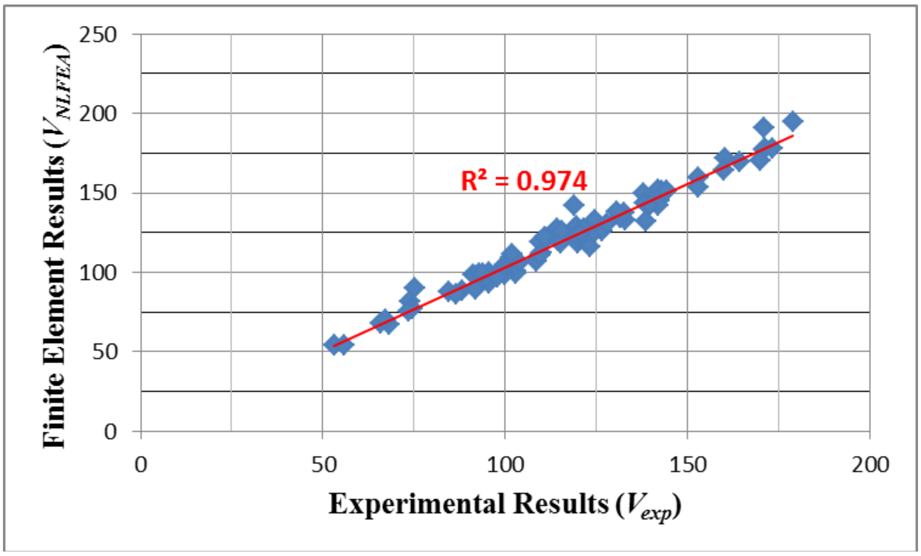
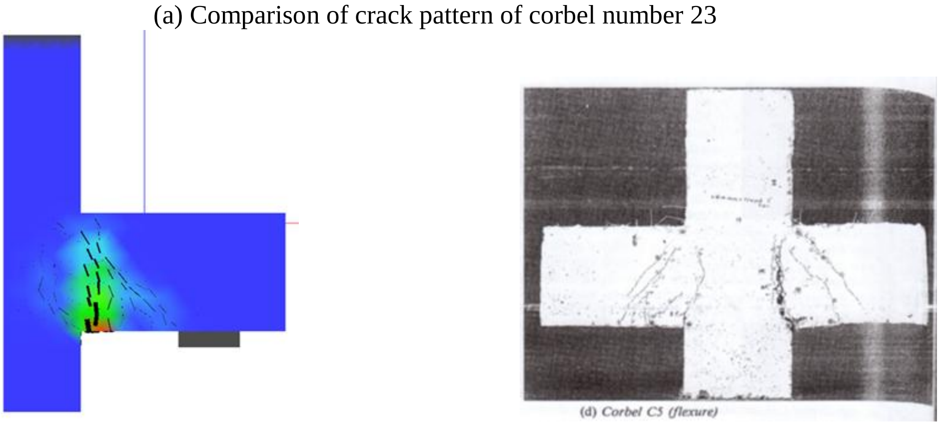


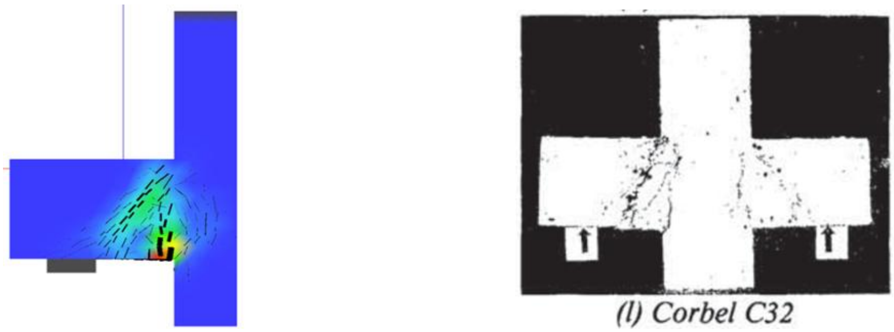
Fig. 6 Comparison of finite element and experimental for load carrying capacities

(a) Comparison of crack pattern of corbel number 23

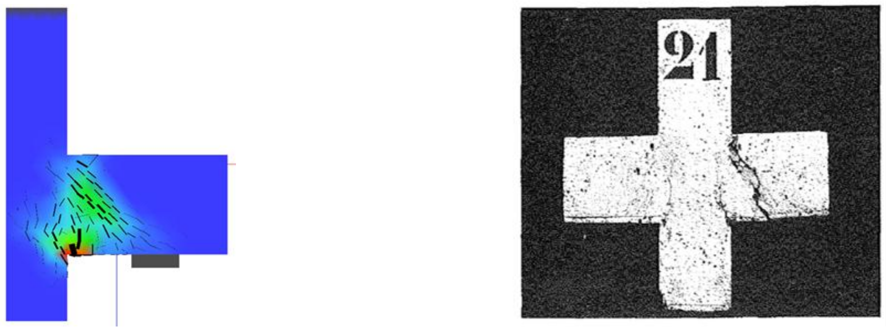


(b) Comparison of crack pattern of corbel number C5

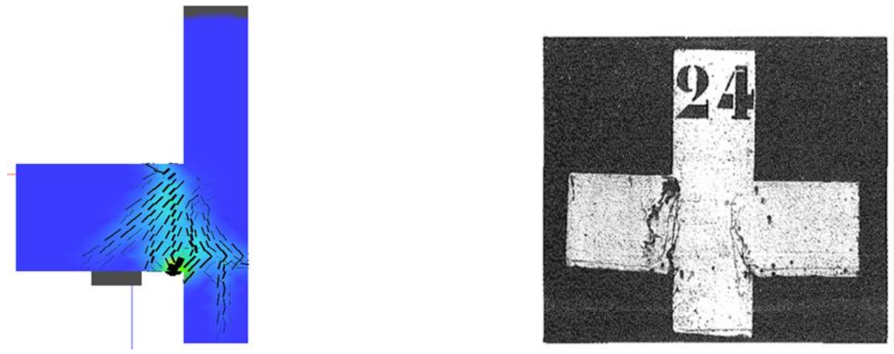
Fig. 7 Comparison of finite element and experimental results about crack patterns



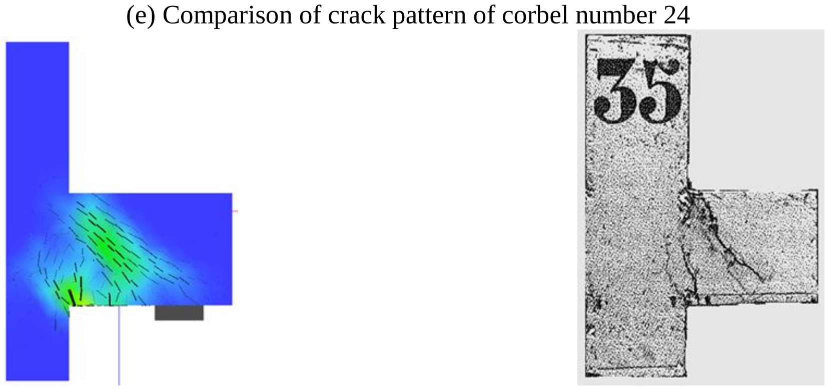
(c) Comparison of crack pattern of corbel number C32



(d) Comparison of crack pattern of corbel number 21



(e) Comparison of crack pattern of corbel number 24



(f) Comparison of crack pattern of corbel number 35

Fig. 7 Continued

4. Stochastic analysis

4.1 Inputs for stochastic analysis

As a result of nonlinear finite element analyses, it is concluded that the most important factors which influence the ultimate load carrying capacity of steel fiber reinforced concrete corbels are mechanical properties of steel fiber reinforced concrete. Therefore, uncertain parameters considered for stochastic analysis can be listed as follows;

- Tensile strength of SFRC
- Compressive strength of SFRC
- Modulus of Elasticity of SFRC
- Fracture Energy of SFRC

Mean value, coefficient of variation (COV) and probability distribution functions of related material characteristics are necessary to define the degree of uncertainty of corresponding material characteristics. These statistical and probabilistic parameters are listed in Table 1 (Mordini 2006). Related values specified by Fattuhi and Hughes for their experimental works were taken as mean values.

Besides uncertainty of sfrc properties, the correlation coefficients between each parameter are also important. For example, there is strong relationship between modulus of elasticity and compressive strength and this interdependence should be considered in stochastic analyses. Hence, material sets were prepared by taking into accounts both probabilistic parameters and relations between considered properties for repetitive calculations. In this manner, material sets which are contradictory to actual material behavior are avoided.

The correlation coefficients between considered material properties are specified in Table 2 by correlation coefficient matrix (Strauss *et al*. 2006).

Table 1 Statistical and probabilistic distribution of considered uncertain parameters (Mordini 2006)

Material Property	Probability Distribution Function	Coefficient of Variation
f_c (CompressiveStrength)	Normal	0.1
f_{ct} (Tensile Strength)	Lognormal (2 parameters)	0.12
E_c (Modulus of Elasticity)	Lognormal (2 parameters)	0.08
G_f (Fracture Energy)	Weibull min. (2 parameters)	0.25

Table 2 Correlation coefficients between considered sfrc properties (Strauss *et al*. 2006)

	E_c	f_{ct}	f_c	G_f
E_c	1	0.7	0.9	0.5
f_{ct}		1	0.8	0.9
f_c			1	0.6
G_f				1

4.2 Stochastic analysis of SFRC corbels

In this study, a special module, called as “SARA” is used for stochastic analysis of steel fiber reinforced concrete corbels. The stochastic analysis module (SARA) runs the FE program (ATENA) and sends the stochastic simulation results to the statistical module (FREET). 84 sfrc corbels which had been studied by Fattuhi and Hughes (1989-1994) were considered for stochastic analysis. Stochastic analyses were carried out according to the logic stated by Bergmeister *et al*. (2009) and Novak *et al*. (2014) For each corbel, 30 material set inputs were prepared by taking into account the uncertainties in related material characteristics (by the use of Latin Hypercube Sampling Method). These 30 samples of each corbel were analyzed separately. Consequently, 2520 analyses were carried out in total and ultimate load carrying capacity resulted from each analysis was determined and recorded.

Results of stochastic analysis (minimum, maximum and mean load carrying capacities, standard deviations) are listed in Table A.2. Also the most suitable probability distribution function (PDF’s) for each sfrc corbel according to dispersion of ultimate loads is listed in the same table. According to resulting suitable PDF results and the frequency of occurrence of PDF’s among 84 sfrc corbels, it is extracted that the most suitable probability distribution function is *“Lognormal 3. Parameter”* type of distribution (Fig. 8). Also “Beta” and “Rayleigh Negative” type of distributions can be considered to represent the uncertainty in the ultimate load carrying capacity of steel fiber reinforced concrete corbels.

Results of stochastic analysis indicate that the highest uncertainty was observed in corbel number 61 (Table A.2). Therefore, statistical outputs of this corbel (standard deviation (σ) is 7.36 kN and *COV* is 0.074) can be used in design process and reliability calculations, as it is the most risky case.

According to sensitivity analysis of SARA analysis, fracture energy and tensile strength of the concrete are the most important two factors among the considered uncertain properties. Sensitivity results of corbel T4 is displayed in Fig. 9 for tensile strength and fracture energy of the concrete. The similar situation exists in almost all corbels about sensitivity analysis.

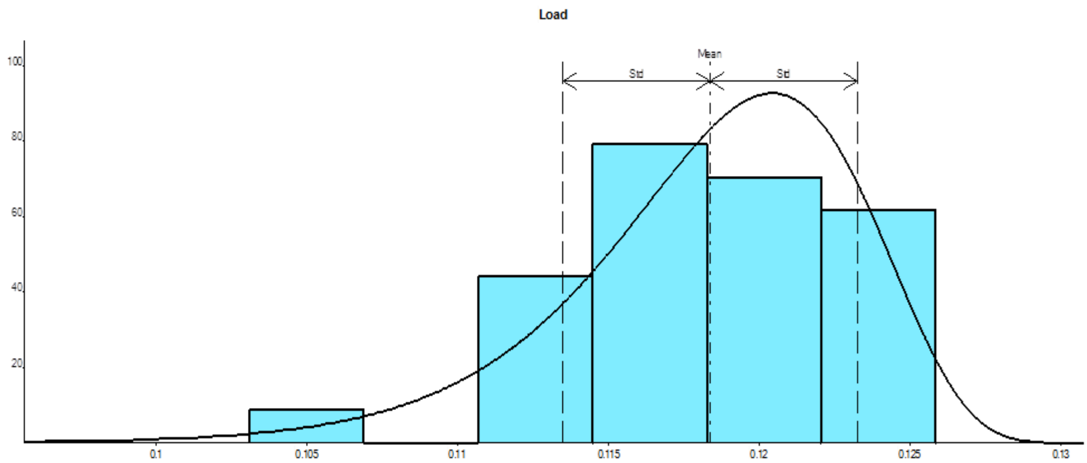


Fig. 8 Probability distribution of load carrying capacity of corbel number 45 (Lognormal 3 parameter distribution)

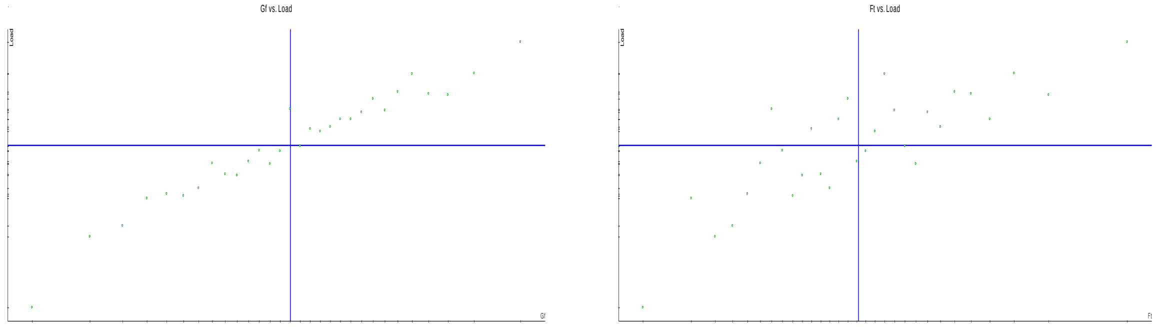


Fig. 9 Sensitivity analysis results (left for G_f and right for f_{ct}) of T3 corbel in Cartesian coordinates

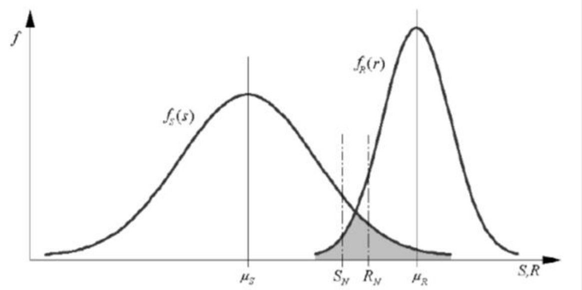


Fig. 10 Typical comparison of pdf of load and resistance (Novak *et al*. 2002)

5. Reliability analysis

Due to uncertainties in the resistance of structural elements and external loads, reliability analyses of structural elements became one of the most important aspects in design (Thomos and Trezos (2011, 2012), Bayramoglu (2012), Silva and Cremona (2014) and GuhaRay and Baidya (2014)).

Reliability analysis is based on comparison of probability distribution function of resistance and load (Fig. 10). As the area of overlap between the probability distribution function (PDF) of resistance and load gets smaller, probability of failure of the structural system decreases. Logically, this area depends on the statistical properties and shape of the curves.

The most common reliability measure is reliability index. By the assumption of normal probability distribution function for both load and resistance, reliability index of a structural system can be calculated by the following equation (Nowak and Collins 2000):

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{{\sigma_R}^2 + {\sigma_S}^2}} \tag{1}$$

In this equation, μ_R and μ_S represent mean value of resistance and total load; σ_R and σ_S represent standard deviation of resistance and total load, respectively.

6. Determination of statistical parameters of SFRC corbel by reliability analysis

Since there is no design code on steel fiber reinforced concrete corbels, there is a lack of information about determination of their statistical parameters (bias factors, coefficient of variation, etc.). Moreover, absence of resistance factor (strength reduction factor) for sfrc corbels leads to difficulties about determination of design load carrying capacities (design resistance) to achieve required safety levels during design process. To overcome these shortcomings specified above, a method was proposed to determine bias factor (λ) and resistance factor (ϕ) for sfrc corbel. The procedure is given as follows (also stated as a flowchart in Fig. 11);

- Select a conservative target reliability index compatible with several design codes.
- Calculate the nominal dead load and nominal live load by the use of Eq. (1) according to the ratio of dead load to sum of dead load and live load (*D/(D+L)*) is equal to 0.5. Since corbels can be considered as a kind of beam and common load ratios in beams are between 0.3 and 0.7 (Szerszen and Nowak 2003), 0.5 was considered to achieve a general approach.
- Calculate the nominal resistance of the corbel by the use of design formula which is commonly used in the literature considering several selected resistance factors. Design formula which is stated in ASCE 7-98 (1998) standard was considered for this study;

$$1.2D + 1.6L \leq \phi R \tag{2}$$

In this formula, *D* and *L* represent nominal dead load and nominal live load, respectively. *R* is nominal resistance and ϕ is strength reduction factor.

- Extract the mean resistance and coefficient of variation values of the corbels from results of stochastic finite element analyses of them.
- Calculate the bias factor of resistance by dividing mean resistance to calculated nominal resistance for each corbel.
- Propose the most appropriate and conservative bias factor and resistance factor by comparison of calculated bias factors from the procedure and bias factors from literature.

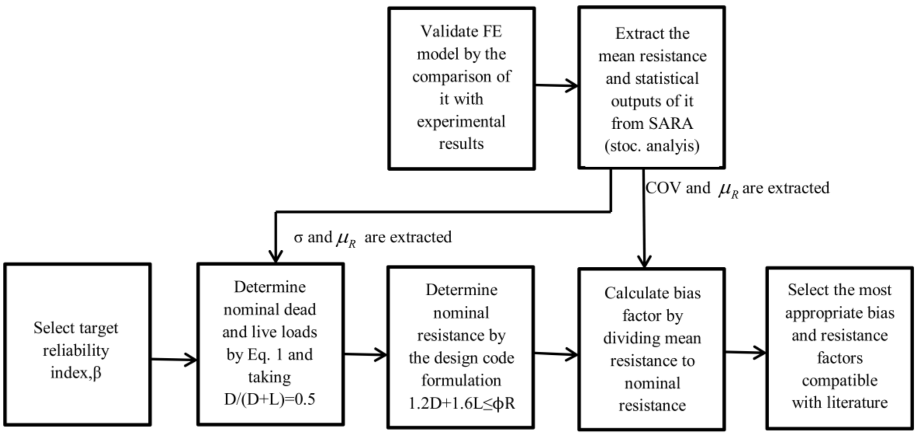


Fig. 11 Flowchart of the procedure implemented in the study

Target reliability index was selected as 4.7 (equal to 10^{-6} about probability of failure value) for the procedure. If the reliability indexes used in most of the design codes (Eurocode, ACI, ASCE design codes, etc.) are handled, this value can be considered a conservative and a reliable value. Statistical values for dead load and live load were selected according to common values used in the literature. Since corbels are generally used in precast structures, bias factor and coefficient of variation values of dead load was taken as 1.03 and 0.08, respectively, while 1.00 and 0.18 were used as bias factor and coefficient of variation value, respectively for live load (Ellingwood *et al*. 1980).

Nominal resistances of corbels were calculated according to Eq. (2) considering the possible values for resistance factors (ϕ = 0.9, 0.85, 0.8, 0.75).

6.1 Numerical Implementation

Calculated bias factors obtained as a result of the proposed procedure above are listed in Table A.3 according to the considered resistance factors. The numerical implementation of the proposed method is illustrated for corbel number 46 given as follows;

Statistical parameters of corbel 46 extracted from stochastic nonlinear finite element analysis are: μ_R = 76.57kN, σ = 5.2kN and COV = 0.068.

Rearranging Eq. (1) by taking into account the coefficient of variation (*COV*) and bias factor equation;

$$COV = \frac{\sigma}{\mu} \tag{3}$$

$$\lambda = \frac{MEAN}{NOMINAL} \tag{4}$$

$$\beta = \frac{\mu_R - (\lambda_D * D + \lambda_L * L)}{\sqrt{{\sigma_R}^2 + (\lambda_D * D * COV_D + \lambda_L * L * COV_L)^2}} \tag{5}$$

Since *D*/(*D*+*L*) = 0.5 is assumed, if *D*+*L* is considered equal to *T* (Total Load), *D* and *L* are both equal to 0.5*T*. Therefore, Eq. (5) can be rewritten by substituting the related values;

$$4.7 = \frac{76.57 - (1.03 * 0.5T + 1 * 0.5T)}{\sqrt{5.2^2 + (1.03 * 0.5T * 0.08 + 1 * 0.5T * 0.18)^2}} \tag{6}$$

From Eq. (6), T was found as 40.86 kN, D and L are both equal to 20.43 kN. Substituting these calculated load values into Eq. (2), nominal resistance can be obtained according to selected resistance factors (for example ϕ = 0.85);

$$R = \frac{1.2 * 20.43 + 1.6 * 20.43}{0.85} \tag{7}$$

R is calculated as 67.3 kN according to Eq. (7), from now on bias factor can be calculated as;

$$\lambda = \frac{76.57}{67.3} = 1.14 \tag{8}$$

6.2 Determination of nominal load carrying capacities of SFRC corbels

It was emphasized in Part 6 that the bias factor is determined by the comparison of bias factor calculated in this study and bias factors resulted from previous literature works. Truss model and flexural model which were proposed by Fattuhi (1994) to determine the ultimate load carrying capacity of sfrc corbels were taken as reference to achieve the comparison. Load carrying capacities of all of the corbels analyzed in this study were calculated according to proposed models by Fattuhi. Capacities of corbels which failed in flexural mode were calculated according to flexural model, while capacities of corbels whose failure mode was any mode, except flexural mode were calculated according to truss model. Calculated resistance values (V_{MODEL}) and bias factors ($V_{\exp}/V_{MODEL}$) are listed in Table A.1.

6.2.1 Flexural model

Fattuhi (1994) proposed the following model for load carrying capacity of sfrc corbels that fail in flexural mode (Fig. 12(a));

$$V_{MODEL} = \frac{f_y A_s}{a}\left(d - \frac{a_1}{2}\right) + \frac{k_0 f_{ct} b}{2a}\left(h - \frac{a_1}{\beta_1}\right)\left(h + \frac{a_1}{\beta_1} - a_1\right) \tag{9}$$

where,

$$k_0 = \frac{9.519}{(f_c)^{0.957}} \quad \text{and} \quad a_1 = \frac{f_y A_s + k_0 f_{ct} h b}{0.85 f_c b + k_0 f_{ct} \left(\frac{b}{\beta_1}\right)} \tag{10}$$

In Eqs. (9)-(10), a, d and h are shear span, effective depth and total depth of the corbel, respectively, f_y and A_s represent yield strength and cross sectional area of the main reinforcement, f_c and f_{ct} are cylindrical compressive strength and splitting tensile strength of sfrc, respectively. β_1 is selected according to cylindrical compressive strength of the concrete specified in ACI Building Code requirements (1999).

6.2.2 Truss model

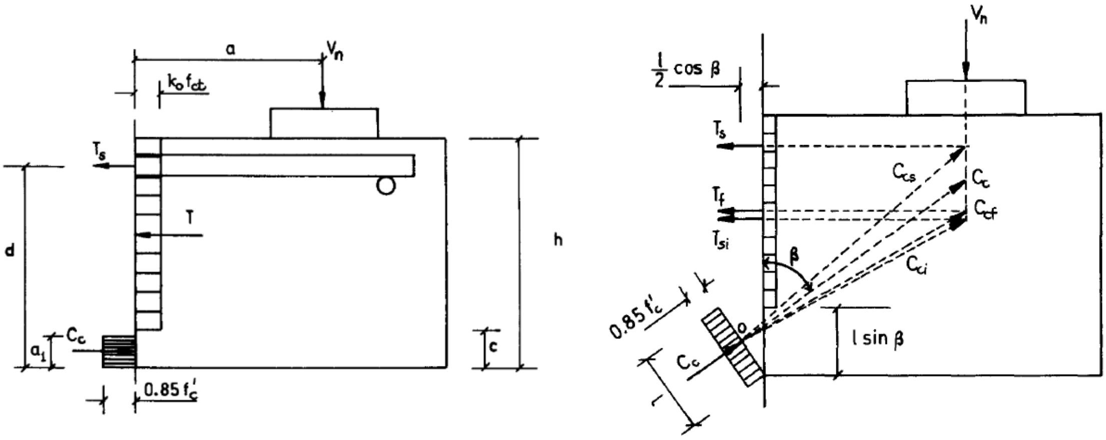
According to truss model (Fig. 12(b)) proposed by Fattuhi (1994) for sfrc corbels whose failure mode is different from flexural mode, ultimate load carrying capacity is calculated by the following equation;

$$V_{MODEL} = \frac{f_y A_s (d - (\frac{l \sin \beta}{2})) + 0.5 k_0 f_{ct} bh(h - (l \sin \beta))}{a + 0.5(l \sin \beta) \cot \beta} \quad (11)$$

where;

$$l \sin \beta = \frac{f_y A_S + k_0 f_{ct} bh}{0.85 f_c b + k_0 f_{ct} b} \quad (12)$$

and cot β is determined according to following quadratic equation;



(a) Flexural Model (b) Truss Model

Fig. 12 Proposed flexural and truss models for sfrc corbels proposed by Fattuhi (Fattuhi 1994)

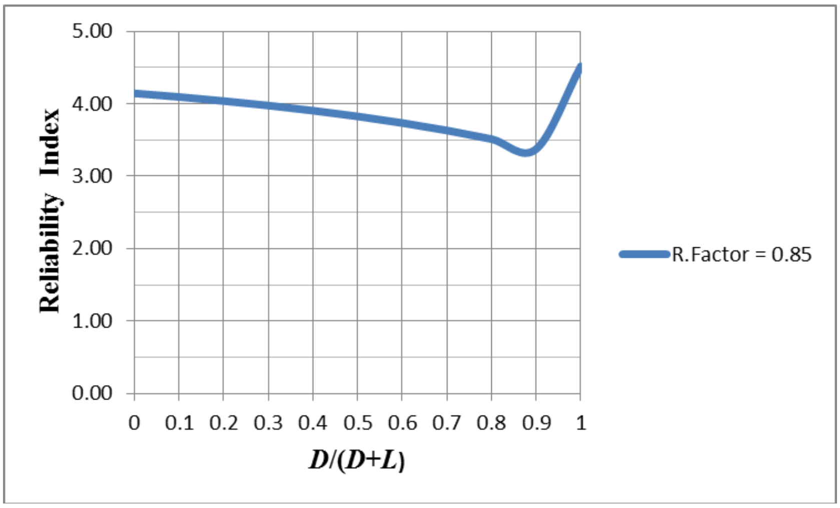


Fig. 13 Reliability Index vs. Load Ratio according to ASCE 7-98 (1.2*D*+1.6*L* or 1.4*D*)

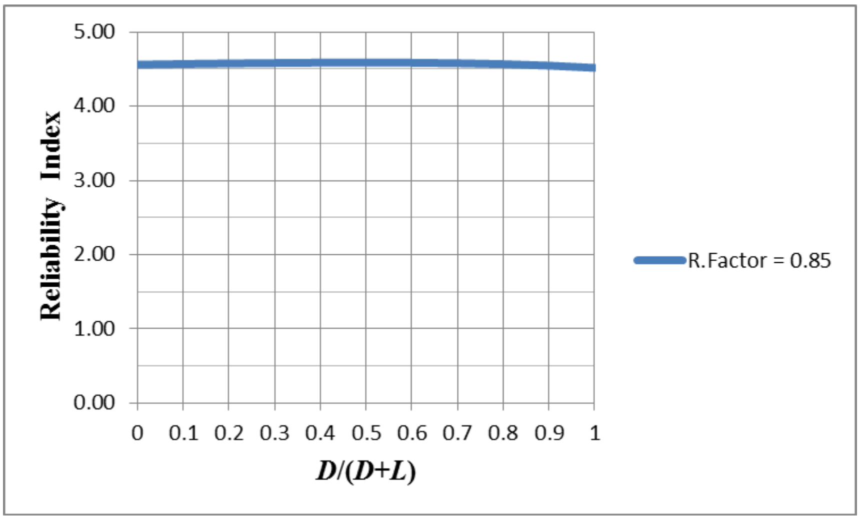


Fig. 14 Reliability Index vs. Load Ratio according to ACI 318-99 (1.4*D*+1.7*L*)

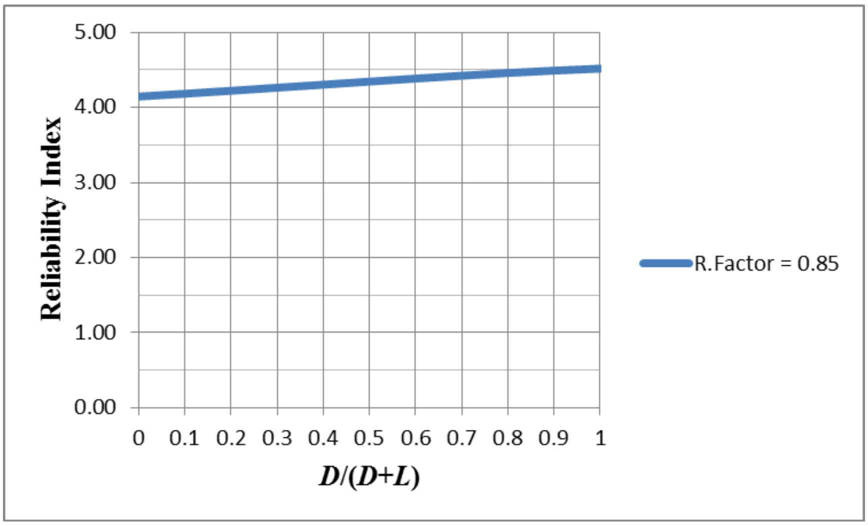


Fig. 15 Reliability Index vs. Load Ratio according to TS 500 (1.4D+1.6L)

$$0.425 f_c b(l\sin\beta)^2\cot^2\beta + 0.85 f_c ab(l\sin\beta)\cot\beta - f_y A_s\left(d - \frac{(l\sin\beta)}{2}\right) - 0.5k_0 f_{ct} bh(h-(l\sin\beta)) = 0 \tag{13}$$

and k_0 is calculated in the same way as stated Eq. (10). Note that since corbels analyzed in this study do not contain stirrup reinforcements, the stirrup related terms are removed from Eqs. (11)-(13).

6.3 Recommended satistical prameters and rsistance fctor for SFRC crbels

According to bias factors calculated from flexural and truss models (Fattuhi, 1994), mean value is 1.03 and highest value is 1.14 (Table A.1). If these values are compared with the bias factor resulted from reliability analysis (Table A.3), the most well matched point is corresponding to $\phi = 0.85$ (mean bias is 1.04 and the highest bias is 1.17). Besides, values lower than 0.9 are recommended for bias factors of sfrc structural elements given that fibrous concrete plays an important role in resisting mechanism (ACI Committee 544 1999). Additionally highest uncertainty was observed in corbel number 61 (*COV*=0.074) as a result of stochastic finite element analysis.

In conclusion, recommended values are found to be as 0.85, 1.04 and 0.074 for resistance factor, bias factor and coefficient of variation value of sfrc corbels, respectively. If reliability indexes are plotted versus $D/(D+L)$ for the load combinations used in ACI 318-99, ASCE 7-98 and TS500 (Turkish Standard) (Figs. 13-15), it can be extracted that the proposed statistical parameters and resistance factor are suitable for the application of the related design codes.

7. Conclusion

This article is a pioneer study on stochastic finite element based reliability analysis of sfrc corbels which has not been investigated in literature so far. Significant probabilistic and statistical parameters (probability distribution functions, coefficient of variation value and bias factor) and resistance factor were proposed for steel fiber reinforced concrete corbels for the first time in literature. These statistical and probabilistic parameters will provide considerable convenience in design of sfrc corbels. To obtain related statistical and probabilistic parameters, experimental studies of sfrc corbels gathered from literature were modeled by a specialized nonlinear finite element software.

Comparison of experimental and FE results of sfrc corbels showed an excellent correlation which proved the accuracy of the FE modeling process. Not only ultimate load capacity but also crack patterns of sfrc corbels were accurately captured when compared with actual test results. Afterwards, reliability analyses of sfrc corbels were carried out via a specialized stochastic analysis module of the finite element software used in this study. Based on results of stochastic finite element based reliability analyses, probabilistic, statistical parameters (probability distribution functions, coefficient of variation value and bias factor) and resistance factor were proposed considering uncertainties in sfrc mechanical properties.

As a result of stochastic FE analyses of sfrc corbels, it can be concluded that there is a close relationship between the uncertainty of load capacity and shear span to effective depth ratio of sfrc corbels. If the standard deviation values of sfrc corbels are analyzed in a general perspective, it can be seen that the smaller the shear span to effective depth ratio is, the higher the uncertainty (higher standard deviation and higher coefficient of variation) of the corbel. Also the uncertainty of short span corbels increases when effective depth becomes smaller, even if a/d ratio is high. Thus, analysis of sfrc corbels with short spans is found to be more critical and during design process, a designer should be more careful. As a result of stochastic FE analyses, "*Lognormal 3 parameter"* type distribution was recommended as probability distribution function (PDF) for the load carrying capacity of sfrc corbels. Lognormal type probability distribution function is one of the most suitable probability distribution functions for the properties which cannot take negative value (for example, load carrying capacity).

According to reliability analysis results and reliability index versus load ratio graphs, it was concluded that recommended statistical parameters (bias factor = 1.04 and *COV* = 0.074) and strength reduction factor (ϕ = 0.85) can be used safely for load combinations stated in most of the design codes (Eurocode, ACI, ASCE, TS 500). The proposed parameters achieve the reliability requirements of the codes (β is commonly between 3.5 and 4.7).

8. Appendix

See Table A.1, A.2 and A.3.

Acknowledgments

This research was supported by Gaziantep University Scientific Research Projects Unit. (Project Code and Name: MF.12.13 - Modeling of Inelastic Behavior of Structures by Soft Computing Techniques)

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Table A.1 Material, geometric properties of analyzed sfrc corbels and experimental, finite element and Fattuhi's models (1994) results for ultimate loads

References	Corbel Number	Shear Span a (mm)	Width b (mm)	Effec. depth d (mm)	Height h (mm)	Reinfor. Ratio A_s/bh (%)	Comp. Strength f_c (MPa)	Sp. Ten. Strength f_{ct} (MPa)	V_{exp} (kN)	V_{NLFEA} (kN)	V_{MODEL} (kN)	V_{NLFEA}/V_{exp}	V_{exp}/V_{MODEL}
Hughes and Fattuhi (1989)													
	C2	125	152	120	147.5	0.700	43.34	4.37	84.5	87.80	86.43	1.039	0.98
	C3	125	152	119	146	0.710	42.61	5.45	92.9	92.21	91.98	0.993	1.01
	C4	125	151	123	149.5	0.700	41.63	4.79	91.8	97.32	93.74	1.060	0.98
	C5	125	152	119	146	0.710	41.39	5.36	96.0	97.19	91.85	1.012	1.05
	C6	125	156	117	146.5	0.690	32.48	3.19	75.2	90.21	80.92	1.200	0.93
Fattuhi and Hughes (1989)													
	C27	52.5	153	121	148.5	0.450	38.31	4.64	125.8	130.10	130.81	1.034	0.96
	C28	89	151	124	148	0.450	45.12	6.09	88.2	88.87	90.91	1.008	0.97
	C29	125	153	130	149	0.440	45.12	6.09	65.9	67.99	67.64	1.032	0.97
	C30	52.5	154	121.5	146.5	0.700	41.63	4.79	171.0	190.60	181.17	1.115	0.94
	C31	64.5	153	118	146	1.020	46.17	5.05	179.0	194.90	179.16	1.089	1.00
	C32	125	153	118	148	1.000	38.31	4.64	110.1	112.30	101.47	1.020	1.09
Fattuhi and Hughes (1989)													
	T3	89	152	122	148	0.700	38.80	4.66	133.0	132.50	119.46	0.996	1.11
	T4	89	151	123	147	0.710	45.28	6.19	142.5	145.10	135.42	1.018	1.05
	T5	89	152	123	147	0.700	46.49	9.28	143.0	151.40	145.61	1.059	0.98
	T10	89	151	117	147	1.020	38.80	4.66	138.0	149.90	134.19	1.086	1.03
	T11	89	152	121	146	1.020	45.28	6.19	160.2	171.9	145.15	1.073	1.10
	T12	89	152	121	147	1.020	46.49	9.28	171.2	176.8	169.24	1.033	1.01

Table A.1 Continued

References	**Corbel Number**	**Shear Span** a **(mm)**	**Width** b **(mm)**	**Effec. depth** d **(mm)**	**Height** h **(mm)**	**Reinfor. Ratio** A_s/bh **(%)**	**Comp. Strength** f_c **(MPa)**	**Sp. Ten. Strength** f_{ct} **(MPa)**	V_{exp} **(kN)**	V_{NLFEA} **(kN)**	V_{MODEL} **(kN)**	V_{NLFEA}/V_{exp}	V_{exp}/V_{MODEL}
Fattuhi (1990)													
	1	80	152.5	123	149	1	33.53	5.84	153.0	159.40	149.00	1.042	1.03
	2	80	155	124	150	0.98	35.15	5.44	160.0	164.00	169.49	1.025	0.94
	3	80	152.5	126	150	0.44	34.02	4.86	91.2	98.29	96.25	1.078	0.95
	4	80	155	125	149	0.44	32.89	5.30	93.0	98.97	98.86	1.064	0.94
	5	140	155	123	149	0.98	32.81	5.46	103.0	100.90	95.86	0.980	1.07
	6	140	154.5	124	150	0.98	30.78	5.35	95.7	100.20	91.76	1.047	1.04
	7	140	153	126	150	0.44	27.38	3.89	53.3	54.13	53.70	1.016	0.99
	8	140	153	125.5	149.5	0.44	29.89	3.72	53.1	53.78	52.18	1.013	1.02
	9	80	152.5	123	149	1	27.95	5.29	152.9	153.30	143.78	1.003	1.06
	10	140	155.5	123	149	0.98	30.05	5.24	102.9	99.10	90.66	0.963	1.14
	11	140	153	126	150	0.44	29.00	3.76	56.0	53.84	51.74	0.961	1.08
	12	80	154	125	149	0.44	30.78	3.89	92.0	89.41	86.68	0.972	1.06
	13	110	154.7	123	149	0.99	27.54	5.04	111.7	121.30	110.50	1.086	1.01
	14	110	153.5	125	149	0.44	29.57	4.24	68.3	67.38	68.52	0.987	1.00
	15	110	152.5	126	150	0.44	31.59	3.92	67.2	70.21	66.99	1.045	1.00
	16	110	154.5	123.5	149.5	0.98	30.54	4.94	114.3	127.50	111.58	1.115	1.02
	18	89	154	124.5	150.5	0.99	26.41	4.98	119.0	141.70	132.41	1.191	0.90
Fattuhi (1990)													
	20	110	153	123.5	149.5	0.99	31.27	5.43	126.0	128.00	122.29	1.016	1.03
	21	110	156	122	148	0.98	29.97	4.73	118.0	123.5	109.76	1.047	1.08
	22	100	153	123	149	0.69	29.97	4.73	108.5	107.1	102.44	0.987	1.06
	23	110	153	122.5	148.5	1	27.38	5.12	126.5	125.7	119.28	0.994	1.06
	24	80	153	124	150	0.69	27.38	5.12	131.5	134.4	120.21	1.022	1.09

Table A.1 Continued

References	Corbel Number	Shear Span a (mm)	Width b (mm)	Effec. depth d (mm)	Height h (mm)	Reinfor. Ratio A_s/bh (%)	Comp. Strength f_c (MPa)	Sp. Ten. Strength f_{ct} (MPa)	V_{exp} (kN)	V_{NLFEA} (kN)	V_{MODEL} (kN)	V_{NLFEA}/V_{exp}	V_{exp}/V_{MODEL}
	27	80	153.5	123.5	149.5	0.99	34.26	6.29	171.5	178	172.37	1.038	0.99
	28	60	154	124	150	0.68	34.26	6.29	173.5	178	181.73	1.026	0.95
	29	75	151.5	122.5	148.5	0.45	30.21	4.42	100.0	98.12	99.21	0.981	1.01
	30	120	153.9	120.2	146.2	0.7	30.21	4.42	86.5	85.95	82.03	0.994	1.05
	31	135	154.5	124	150	1.19	32.89	5.50	119.5	128.7	108.78	1.077	1.10
	32	120	154	120.2	146.2	1.23	32.89	5.50	132.5	137.1	116.09	1.035	1.14
	35	135	155.1	122.5	148.5	1.48	31.35	4.91	124.5	132.7	119.86	1.066	1.04
	36	60	154.8	122	148	0.44	31.35	4.91	123.5	122.5	127.31	0.992	0.97
	37	135	153.8	123.1	149.1	1.49	32.08	5.72	140.0	143.1	132.23	1.022	1.06
	38	110	152.2	124	150	0.44	32.08	5.72	74.0	81.58	73.42	1.102	1.01
	39	110	153.5	124	150	1.2	31.35	5.64	144.5	150.90	129.20	1.044	1.12
	40	125	155.5	122.8	148.8	1.47	31.35	5.64	142.0	152.00	130.33	1.070	1.09
	44	135	153.8	122.6	148.6	1.21	28.67	4.85	109.5	118.80	103.59	1.085	1.06
	45	135	153	122.3	148.3	1.5	28.19	4.37	120.0	118.50	115.05	0.988	1.04
	46	75	154.5	92	146	0.45	28.19	4.37	74.5	76.84	81.20	1.031	0.92
	48	80	155.5	93.2	148.2	0.68	28.92	5.16	100.0	104.50	104.37	1.045	0.96
	49	80	154.1	122.1	148.2	1	30.46	5.81	164.5	169.60	147.55	1.031	1.11
Fattuhi (1994)													
	51	110	153.4	132.3	148.3	1.000	31.27	5.83	130.5	138.20	121.22	1.059	1.08
	52	110	152.2	94	150	1.000	31.27	5.83	99.0	100.20	96.11	1.012	1.03
	53	135	153.6	133.6	149.6	1.480	33.29	5.68	144.5	150.80	133.34	1.044	1.08
	54	135	151.7	93.8	149.8	1.490	33.29	5.68	101.5	109.50	94.18	1.079	1.08
	55	75	153.7	135.3	149.3	0.440	29.89	4.06	104.0	106.40	105.72	1.023	0.98
	56	75	152.9	115.8	149.8	0.440	29.89	4.06	95.5	93.44	93.94	0.978	1.02
	57	80	152.2	135.1	150.1	0.690	31.43	5.92	138.5	143.40	144.80	1.035	0.96

Table A.1 Continued

References	Corbel Number	Shear Span a (mm)	Width b (mm)	Effec. depth d (mm)	Height h (mm)	Reinfor. Ratio A_s/bh (%)	Comp. Strength f_c (MPa)	Sp. Ten. Strength f_{ct} (MPa)	V_{exp} (kN)	V_{NLFEA} (kN)	V_{MODEL} (kN)	V_{NLFEA}/V_{exp}	V_{exp}/V_{MODEL}
	58	80	152.8	113.3	148.3	0.690	31.43	5.92	121.5	127.60	124.69	1.050	0.97
	59	135	153	114	150	0.990	29.32	5.37	97.5	97.87	87.89	1.004	1.11
	60	110	152.8	112.6	148.6	1.490	29.32	5.37	142.0	142.20	129.81	1.001	1.09
	61	60	152.6	95	149	0.440	29.40	4.82	98.5	100.60	107.51	1.021	0.92
	62	135	153	114.1	150.1	1.200	29.40	4.82	109.5	110.40	97.30	1.008	1.13
	63	80	153	94	150	0.680	30.94	5.94	101.8	111.20	108.63	1.092	0.94
	64	60	152.6	92.5	147.5	1.000	30.94	5.94	170.0	170.10	172.90	1.001	0.98
	75	75	154.3	125.9	149.9	0.440	25.11	4.05	94.8	96.41	102.97	1.017	0.92
	76	75	154.5	94.8	148.8	0.440	25.11	4.05	73.5	75.66	83.72	1.029	0.88
	77	110	153.1	122.3	148.3	1.000	26.89	4.96	114.5	120.2	108.96	1.050	1.05
	78	135	153.1	121.7	147.7	1.500	26.89	4.96	120.0	122.3	114.45	1.019	1.05
	79	135	153.2	123.4	149.4	1.480	27.38	5.26	128.0	130.6	117.70	1.020	1.09
	80	110	154	122.1	148.1	1.000	27.38	5.26	120.8	124.7	110.32	1.032	1.09
	81	135	153.6	121.6	147.6	1.220	28.67	5.04	110.8	122.2	103.01	1.103	1.08
	82	110	154	92	148	1.000	28.67	5.04	98.0	96.66	91.08	0.986	1.08
	83	135	150.4	111.9	147.9	1.530	28.27	4.96	115.3	118.6	105.73	1.029	1.09
	84	135	152.4	91.7	147.7	1.510	28.27	4.96	94.0	99.29	86.68	1.056	1.08
	85	110	154.2	112.5	148.5	0.990	28.43	5.17	123.3	116.2	110.73	0.942	1.11
	86	135	153.2	113.9	149.9	1.480	28.43	5.17	115.5	125.8	109.63	1.089	1.05
	87	60	152.9	93.5	148.5	0.690	29.32	6.01	139.8	141.8	144.34	1.014	0.97
	88	80	153.1	93.1	149.1	1.000	29.32	6.01	138.8	132.2	131.39	0.952	1.06
										Mean		1.034	1.03
										Std. Dev.		0.045	0.062
										COV		0.044	0.060
										Highest Val.		1.20	1.14

Table A.2 Stochastic finite element results of sfrc corbels

Corbel No	*a* (mm)	*a/d*	Standard Dev. (σ_R) (kN)	Coefficient of Variation (*COV*) of Resistance	Stochastic Analysis Results Resistance (kN) V_{min}	V_{mean}	V_{max}	Distribution Type
C2	125	1.04	0.73	0.008	85.40	87.51	88.90	Lognormal 3 parameter
C3	125	1.05	2.33	0.025	87.43	93.17	96.69	Rayleighnegative
C4	125	1.02	2.14	0.022	91.74	96.68	99.94	Rayleighnegative
C5	125	1.05	2.63	0.027	90.96	97.49	103.10	Logistic
C6	125	1.07	1.64	0.018	85.08	89.57	91.85	Half-normal negative
C27	52.5	0.43	5.26	0.041	118.72	127.65	136.66	Beta
C28	89	0.72	4.67	0.053	79.81	88.37	98.93	Lognormal 3 parameter
C29	125	0.96	3.77	0.056	58.99	67.71	73.93	Triangular
C30	52.5	0.43	3.83	0.020	184.88	191.31	199.01	Weibull min. 3 parameter
C31	64.5	0.55	4.92	0.025	184.29	193.57	203.48	Rayleighnegative
C32	125	1.06	1.93	0.017	107.83	111.35	114.22	Beta
T3	89	0.73	4.55	0.035	122.10	130.92	137.08	Half-normal negative
T4	89	0.72	5.55	0.038	129.07	145.20	155.45	Lognormal 3 parameter
T5	89	0.72	6.77	0.045	132.73	150.86	161.79	Lognormal 3 parameter
T10	89	0.76	4	0.027	139.14	149.34	156.26	Half-normal negative
T11	89	0.74	5.72	0.033	153.95	171.08	179.52	Beta
T12	89	0.74	7.43	0.042	157.08	176.17	188.91	Weibullmax. 3 parameter
1	80	0.65	4.3	0.027	147.09	159.48	166.80	Gamma negative 3 parameter
2	80	0.65	4.8	0.029	149.55	163.01	169.06	Beta
3	80	0.63	5.93	0.061	81.98	97.42	107.61	Lognormal 3 parameter
4	80	0.64	6.08	0.063	83.00	97.27	108.44	Rayleighnegative
5	140	1.14	2.58	0.026	93.46	100.39	104.96	Logistic
6	140	1.13	2.3	0.023	93.00	99.64	103.36	Lognormal 3 parameter
7	140	1.11	2.94	0.054	46.67	54.05	59.23	Lognormal 3 parameter
8	140	1.12	2.84	0.053	46.82	53.69	58.54	Lognormal 3 parameter
9	80	0.65	3.78	0.025	143.09	152.33	160.68	Gumbelmax. EV I
10	140	1.14	2.09	0.021	94.18	98.65	102.17	Gumbel min. EV I
11	140	1.11	2.85	0.053	46.85	53.72	58.50	Weibull min. 3 parameter
12	80	0.64	4.67	0.052	79.05	90.01	98.44	Gumbel min. EV I
13	110	0.89	2.6	0.022	113.22	119.74	123.38	Beta
14	110	0.88	3.44	0.051	59.10	67.24	73.31	Weibullmax. 3 parameter
15	110	0.87	3.54	0.051	60.95	69.89	75.89	Weibull min. 2 parameter
16	110	0.89	3.4	0.027	116.54	125.87	132.76	Weibull min. 3 parameter
18	89	0.71	3.51	0.025	129.03	140.43	145.41	Gumbel min. EV I
20	110	0.89	3.52	0.028	118.60	127.72	132.11	Beta
21	110	0.90	2.83	0.023	115.71	123.21	127.39	Beta
22	100	0.81	3.78	0.035	96.06	106.65	113.47	Lognormal 3 parameter
23	110	0.90	3.35	0.027	117.73	125.97	131.44	Normal
24	80	0.65	5.39	0.040	119.10	133.83	144.90	Rayleighnegative
27	80	0.65	5.83	0.033	159.94	177.29	185.46	Rayleighnegative
28	60	0.48	7.88	0.045	157.26	176.70	193.02	Rayleighnegative
29	75	0.61	5.11	0.052	86.16	97.86	107.03	Rayleighnegative
30	120	1.00	2.73	0.032	78.92	85.76	90.68	Weibullmax. 3 parameter
31	135	1.09	3.24	0.025	119.35	128.43	133.92	Lognormal 3 parameter
32	120	1.00	3.31	0.024	128.75	136.41	142.70	Normal
35	135	1.10	4.65	0.035	121.73	131.10	141.28	Gumbel min. EV I

Table A.2 Continued

Corbel No	*a* (mm)	*a/d*	Standard Dev. (σ_R) (kN)	Coefficient of Variation (*COV*) of Resistance	Stochastic Analysis Results Resistance (kN) 134.84 $V_{\min}$	V_{mean}	$V_{\max}$	Distribution Type
36	60	0.49	6.87	0.057	105.00	121.43	134.84	Gamma negative 3 parameter
37	135	1.10	5	0.035	125.97	142.35	150.92	Gumbel min. EV I
38	110	0.89	5.5	0.068	67.98	81.46	93.33	Logistic
39	110	0.89	3.85	0.026	138.97	150.26	156.88	Gumbelmax. EV I
40	125	1.02	4.93	0.033	136.70	150.31	157.58	Rayleighnegative
44	135	1.10	3.38	0.029	109.96	117.98	122.34	Half-normal negative
45	135	1.10	4.87	0.041	103.09	118.35	125.83	Lognormal 3 parameter
46	75	0.82	5.2	0.068	64.24	76.57	86.95	Weibull min. 2 parameter
48	80	0.86	5.5	0.052	92.34	105.15	115.60	Triangular
49	80	0.66	5.17	0.031	153.79	168.56	176.73	Normal
51	110	0.83	3.48	0.025	127.41	137.80	142.67	Gumbel min. EV I
52	110	1.17	2.77	0.028	92.50	99.69	104.26	Weibull min. 2 parameter
53	135	1.01	5.39	0.036	133.25	150.41	157.60	Gamma negative 3 parameter
54	135	1.44	2.12	0.019	103.99	108.85	112.86	Rayleighnegative
55	75	0.55	5.41	0.051	92.54	106.03	114.03	Weibullmax. 3 parameter
56	75	0.65	5.6	0.060	78.64	93.31	103.01	Gamma negative 3 parameter
57	80	0.59	5.9	0.041	128.30	143.17	151.49	Beta
58	80	0.71	6.54	0.052	110.45	126.55	137.18	Beta
59	135	1.18	2.52	0.026	91.46	97.68	101.78	Lognormal 3 parameter
60	110	0.98	5.49	0.039	125.85	141.73	153.37	Lognormal 3 parameter
61	60	0.63	7.36	0.074	83.87	99.92	113.06	Triangular
62	135	1.18	2.38	0.022	102.06	109.64	113.03	Beta
63	80	0.85	6.34	0.057	96.64	111.43	123.45	Lognormal 3 parameter
64	60	0.65	6.01	0.036	151.69	168.12	179.01	Weibullmin 3 para
75	75	0.60	5.03	0.052	84.21	96.25	104.52	Weibullmax. 3 parameter
76	75	0.79	4.36	0.058	63.31	74.81	82.54	Weibull min. 2 parameter
77	110	0.90	2.93	0.024	110.50	120.19	124.08	Lognormal 3 parameter
78	135	1.11	5.12	0.042	104.33	120.88	129.74	Weibull min. 2 parameter
79	135	1.09	4.9	0.038	119.12	130.16	138.14	Lognormal 2 para
80	110	0.90	3.25	0.026	117.40	124.93	132.81	Weibullmax. 3 parameter
81	135	1.11	3.29	0.027	109.57	120.67	127.74	Student-t
82	110	1.20	2.93	0.030	88.85	96.10	100.85	Half-normal negative
83	135	1.21	4.67	0.040	105.88	116.75	124.52	Logistic
84	135	1.47	3.67	0.038	85.53	97.68	103.00	Lognormal 3 parameter
85	110	0.98	3.78	0.033	107.53	115.97	121.57	Normal
86	135	1.19	3.87	0.031	114.61	124.77	132.44	Lognormal 3 parameter
87	60	0.64	7.6	0.054	119.77	140.37	152.03	Gamma negative 3 parameter
88	80	0.86	4.06	0.031	119.81	131.67	138.19	Normal

Table A.3 Results of reliability analyses of sfrc corbels

Cor. No	Nominal Load($D/(D+L)$=0.5)	LoadCombination ($1.2D+1.6L$) (kN)	Nominal Resistance (R) (kN)				Bias Factors (λ)			
			ϕ = 0.9	ϕ = 0.85	ϕ = 0.8	ϕ = 0.75	ϕ = 0.9	ϕ = 0.85	ϕ = 0.8	ϕ = 0.75
C2	53.52	74.93	83.25	88.15	93.66	99.90	1.05	0.99	0.93	0.88
C3	56.06	78.48	87.20	92.33	98.11	104.65	1.07	1.01	0.95	0.89
C4	64.20	89.88	99.87	105.74	112.35	119.84	0.97	0.91	0.86	0.81
C5	58.49	81.89	90.98	96.34	102.36	109.18	1.07	1.01	0.95	0.89
C6	54.36	76.10	84.56	89.53	95.13	101.47	1.06	1.00	0.94	0.88
C27	74.41	104.17	115.75	122.56	130.22	138.90	1.10	1.04	0.98	0.92
C28	49.85	69.79	77.54	82.11	87.24	93.05	1.14	1.08	1.01	0.95
C29	37.84	52.98	58.86	62.32	66.22	70.63	1.15	1.09	1.02	0.96
C30	115.88	162.23	180.26	190.86	202.79	216.31	1.06	1.00	0.94	0.88
C31	116.41	162.97	181.08	191.73	203.72	217.30	1.07	1.01	0.95	0.89
C32	67.65	94.71	105.23	111.42	118.39	126.28	1.06	1.00	0.94	0.88
T3	77.44	108.42	120.46	127.55	135.52	144.55	1.09	1.03	0.97	0.91
T4	85.24	119.34	132.60	140.40	149.17	159.11	1.10	1.03	0.97	0.91
T5	87.12	121.97	135.52	143.49	152.46	162.62	1.11	1.05	0.99	0.93
T10	89.62	125.47	139.41	147.61	156.84	167.29	1.07	1.01	0.95	0.89
T11	101.46	142.04	157.83	167.11	177.56	189.39	1.08	1.02	0.96	0.90
T12	102.45	143.43	159.37	168.74	179.29	191.24	1.11	1.04	0.98	0.92
1	95.68	133.95	148.84	157.59	167.44	178.60	1.07	1.01	0.95	0.89
2	97.39	136.35	151.50	160.41	170.43	181.79	1.08	1.02	0.96	0.90
3	53.45	74.83	83.14	88.04	93.54	99.77	1.17	1.11	1.04	0.98
4	53.04	74.26	82.51	87.36	92.82	99.01	1.18	1.11	1.05	0.98
5	60.35	84.49	93.88	99.40	105.61	112.65	1.07	1.01	0.95	0.89
6	60.12	84.17	93.52	99.02	105.21	112.22	1.07	1.01	0.95	0.89
7	30.34	42.48	47.20	49.97	53.10	56.63	1.15	1.08	1.02	0.95
8	30.28	42.39	47.10	49.87	52.99	56.52	1.14	1.08	1.01	0.95
9	91.69	128.37	142.63	151.02	160.46	171.15	1.07	1.01	0.95	0.89
10	59.67	83.54	92.82	98.28	104.42	111.38	1.06	1.00	0.94	0.89
11	30.28	42.39	47.10	49.87	52.99	56.52	1.14	1.08	1.01	0.95
12	50.93	71.30	79.22	83.88	89.13	95.07	1.14	1.07	1.01	0.95
13	72.38	101.33	112.59	119.21	126.67	135.11	1.06	1.00	0.95	0.89
14	38.13	53.38	59.31	62.80	66.73	71.18	1.13	1.07	1.01	0.94
15	39.70	55.58	61.76	65.39	69.48	74.11	1.13	1.07	1.01	0.94
16	75.51	105.71	117.46	124.37	132.14	140.95	1.07	1.01	0.95	0.89
18	84.50	118.30	131.44	139.18	147.88	157.73	1.07	1.01	0.95	0.89
20	76.55	107.17	119.08	126.08	133.96	142.89	1.07	1.01	0.95	0.89
21	74.35	104.09	115.66	122.46	130.11	138.79	1.07	1.01	0.95	0.89
22	62.99	88.19	97.98	103.75	110.23	117.58	1.09	1.03	0.97	0.91
23	75.62	105.87	117.63	124.55	132.34	141.16	1.07	1.01	0.95	0.89
24	78.19	109.47	121.63	128.78	136.83	145.95	1.10	1.04	0.98	0.92
27	105.26	147.36	163.74	173.37	184.21	196.49	1.08	1.02	0.96	0.90
28	102.12	142.97	158.85	168.20	178.71	190.62	1.11	1.05	0.99	0.93
29	55.31	77.43	86.04	91.10	96.79	103.25	1.14	1.07	1.01	0.95
30	51.02	71.43	79.36	84.03	89.29	95.24	1.08	1.02	0.96	0.90
31	77.26	108.16	120.18	127.25	135.21	144.22	1.07	1.01	0.95	0.89
32	82.17	115.04	127.82	135.34	143.80	153.38	1.07	1.01	0.95	0.89
35	77.43	108.40	120.45	127.53	135.50	144.54	1.09	1.03	0.97	0.91

Table A.3 Continued

Cor. No	Nominal Load($D/(D+L)$=0.5)	LoadCombination ($1.2D+1.6L$) (kN)	Nominal Resistance (R) (kN)				Nominal Resistance (R) (kN)			
			ϕ = 0.9	ϕ = 0.85	ϕ = 0.8	ϕ = 0.75	ϕ = 0.9	ϕ = 0.85	ϕ = 0.8	ϕ = 0.75
36	67.66	94.72	105.25	111.44	118.41	126.30	1.15	1.09	1.03	0.96
37	84.13	117.78	130.87	138.57	147.23	157.04	1.09	1.03	0.97	0.91
38	43.54	60.96	67.73	71.71	76.20	81.27	1.20	1.14	1.07	1.00
39	90.34	126.48	140.53	148.80	158.10	168.63	1.07	1.01	0.95	0.89
40	89.25	124.95	138.83	147.00	156.19	166.60	1.08	1.02	0.96	0.90
44	70.59	98.83	109.81	116.27	123.53	131.77	1.07	1.01	0.96	0.90
45	69.00	96.60	107.33	113.65	120.75	128.80	1.10	1.04	0.98	0.92
46	40.86	57.20	63.56	67.30	71.51	76.27	1.20	1.14	1.07	1.00
48	59.42	83.19	92.43	97.87	103.99	110.92	1.14	1.07	1.01	0.95
49	100.50	140.70	156.33	165.53	175.88	187.60	1.08	1.02	0.96	0.90
51	82.89	116.05	128.94	136.52	145.06	154.73	1.07	1.01	0.95	0.89
52	59.73	83.62	92.91	98.38	104.53	111.50	1.07	1.01	0.95	0.89
53	88.76	124.26	138.07	146.19	155.33	165.69	1.09	1.03	0.97	0.91
54	65.98	92.37	102.64	108.67	115.47	123.16	1.06	1.00	0.94	0.88
55	60.16	84.22	93.58	99.09	105.28	112.30	1.13	1.07	1.01	0.94
56	51.36	71.90	79.89	84.59	89.88	95.87	1.17	1.10	1.04	0.97
57	83.46	116.84	129.83	137.46	146.06	155.79	1.10	1.04	0.98	0.92
58	71.65	100.31	111.46	118.01	125.39	133.75	1.14	1.07	1.01	0.95
59	58.71	82.19	91.33	96.70	102.74	109.59	1.07	1.01	0.95	0.89
60	83.11	116.35	129.28	136.89	145.44	155.14	1.10	1.04	0.97	0.91
61	51.98	72.77	80.86	85.61	90.97	97.03	1.24	1.17	1.10	1.03
62	66.27	92.78	103.09	109.15	115.97	123.70	1.06	1.00	0.95	0.89
63	62.02	86.83	96.48	102.15	108.54	115.77	1.16	1.09	1.03	0.96
64	99.23	138.92	154.36	163.44	173.65	185.23	1.09	1.03	0.97	0.91
75	54.40	76.16	84.62	89.60	95.20	101.55	1.14	1.07	1.01	0.95
76	41.44	58.02	64.46	68.25	72.52	77.35	1.16	1.10	1.03	0.97
77	72.39	101.35	112.61	119.23	126.68	135.13	1.07	1.01	0.95	0.89
78	70.26	98.36	109.29	115.72	122.96	131.15	1.11	1.04	0.98	0.92
79	76.51	107.11	119.02	126.02	133.89	142.82	1.09	1.03	0.97	0.91
80	75.06	105.08	116.76	123.63	131.36	140.11	1.07	1.01	0.95	0.89
81	72.36	101.30	112.56	119.18	126.63	135.07	1.07	1.01	0.95	0.89
82	57.31	80.23	89.15	94.39	100.29	106.98	1.08	1.02	0.96	0.90
83	68.26	95.56	106.18	112.43	119.46	127.42	1.10	1.04	0.98	0.92
84	57.43	80.40	89.34	94.59	100.50	107.20	1.09	1.03	0.97	0.91
85	68.89	96.45	107.16	113.47	120.56	128.59	1.08	1.02	0.96	0.90
86	74.34	104.08	115.64	122.44	130.10	138.77	1.08	1.02	0.96	0.90
87	78.85	110.39	122.66	129.87	137.99	147.19	1.14	1.08	1.02	0.95
88	78.48	109.87	122.08	129.26	137.34	146.50	1.08	1.02	0.96	0.90
					Mean		1.10	1.04	0.98	0.92
					Highest Value		1.24	1.17	1.10	1.03