Computers and Concrete, *Vol. 14, No. 4 (2014) 479-501*
DOI: http://dx.doi.org/10.12989/cac.2014.14.4.479

Shear strength of RC beams. Precision, accuracy, safety and simplicity using genetic programming

Antoni Cladera[*1], Juan L. Pérez-Ordóñez[2a] and Fernando Martínez-Abella[3b]

[1]*Department of Physics, University of the Balearic Islands, Ctra. Valldemossa km. 7.5, E-07122 Palma, Spain*
[2]*School of Building Engineering and Technical Architecture, University of A Coruña, Campus de Elviña, E-15192 La Coruña, Spain*
[3]*Department of Construction Technology, University of A Coruña, Campus de Elviña, E-15192 La Coruña, Spain*

(*Received May 7, 2014, Revised July 26, 2014, Accepted October 1, 2014*)

Abstract. This paper presents the improvement of the EC-2 and EHE-08 shear strength formulations for concrete beams with shear reinforcement. The employed method is based on the genetic programming (GP) technique, which is configured to generate symbolic regression from a set of experimental data by considering the interactions among precision, accuracy, safety and simplicity. The size effect and the influence of the amount of shear reinforcement are examined. To develop and verify the models, 257 experimental tests on concrete beams from the literature are used. Three expressions of considerable simplicity, which significantly improve the shear strength prediction with respect to the formulations of the different studied codes, are proposed.

Keywords: artificial intelligence; genetic programming; reinforced concrete; shear strength; beams; stirrups; concrete codes

1. Introduction

Since the beginning of the 20th century when Mörsh and Ritter postulated the earliest truss models, substantial progress in the analytical solution of shear problems in reinforced concrete beams has been achieved. Numerous highly sophisticated tools (Bairán and Marí 2007; Navarro Gregori *et al*. 2007; Vecchio and Collins 1986) consider 3D effects and the interactions among torsion, bending and shear but may not be used as direct designing methods. The majority of rational methods for shear require considerable simplification to make them suitable for implementation in codes of practice. As Regan indicated, the problem for simpler models is the need to neglect secondary factors. Secondary issues in one case may constitute primary issues in another case; thus, careful confirmation is always required (Regan 1993).

In 1960, Siess suggested that reinforced concrete design and all aspects of our building codes, including aspects that involve theories, are empirical in nature; that is, they are based on

∗Corresponding author, Associate Professor, E-mail: antoni.cladera@uib.es
[a]Associate professor, E-mail: jlperez@udc.es
[b]Professor, E-mail: fmartinez@udc.es

http://www.techno-press.org/?journal=cac&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

experiments, experience and the knowledge gained from research and practice (Siess 1960). He emphasized that this finding is also valid for "certain assumed theories". All theories are based on certain assumptions; their use is only justified to the extent that these assumptions are correct. The theories that are employed in connection with reinforced concrete are nearly always based on assumptions about the properties of the materials or the properties of the structure, which are not even approximately correct for reinforced concrete. This would certainly be valid for the "accepted straight line theory" and the "theory of elastic frames", as applied to reinforced concrete members. The use of these theories in connection with the design of reinforced concrete members or structures has been justified over a period of years by experience and experiments (Siess 1960).

More than fifty years after Siess' paper and after many more years of improvements in the understanding of shear behavior, the most "rational" shear design provisions in building codes regarding shear strength include assumptions that simplify and neglect specific factors. Although all theories can and should be improved over time, the cost-benefit ratios of any improvement should be carefully assessed (Collins 1998).

Some shear provisions in current codes of practices are purely empirical; for example, ACI equations or the shear strengths of members without stirrups in Eurocode 2 (EC-2). Other methods are rationally based, i.e., the shear strength of members with stirrups in EC-2, which are based on a truss model, and numerous other methods can be referred to as "semi-rational methods" with different degrees of rational and empirical derivate parameters. In some shear provisions, the fitting of the equations to empirical data has been directly conducted when designing equations. However, in more rationally based shear provisions, the fitting was conducted using the basic behavior at the material level; for example, for aggregate interlock or the residual tensile strength of concrete.

When these empirical parameters are obtained, the dilemma between precision, accuracy, safety and simplicity is introduced. Fig. 1 shows the graphical meaning of accuracy and precision. Accuracy is the similarity between a real value and a measurement, whereas precision is the degree to which repeated measurements yield the same results.

Although a code provision may be more or less accurate and precise, it must be safe. Safety is probably the most important condition, and all design procedures contain a certain number of conservative simplifying assumptions. For various combinations of parameters, these simplifying assumptions may produce conservative predictions. A well-formulated procedure will rarely result in a non-conservative prediction (Collins 2001). In a semi-probabilistic method, safety is generally considered as a minimum value for a given percentile; this notion is well known to engineers. Generally, the 5th percentile of the data of the predicted shear strength ratio must be greater than 1.00 for a given database. However, other reasonable values may be considered, as subsequently discussed.

Although a universal measure for simplicity does not exist, a formulation may be considered simple or user-friendly. It is internationally accepted that the preparation of user-friendly and practical codes requires uniformly structured contents, the consistent use of technical terms and the systematic and uniform implementation of key principles (Sigrist 2012). A given formulation may be highly complex but part of a user-friendly code. In this paper, complexity or simplicity will be related to the need for iterations to obtain the code prediction and to the length of the required calculations. Some methods enable direct calculations for designing and verifying a RC member. Other methods only enable direct calculations for designing a RC structure and iterative calculations for verifying a RC structure, as subsequently discussed.

In this paper, a genetic programming algorithm is employed as a tool to facilitate the

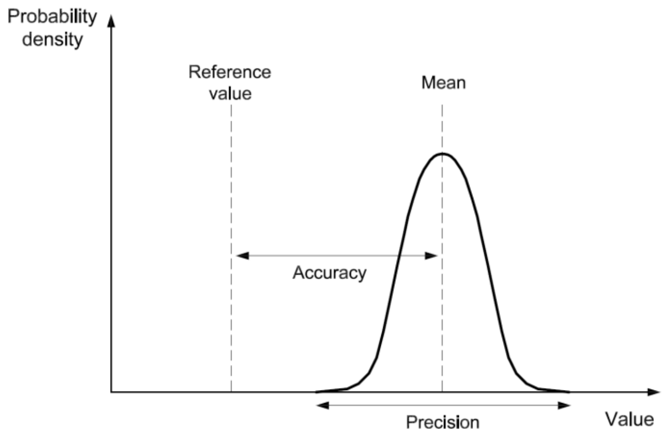


Fig. 1 Accuracy vs. precision

optimization and calibration of a previous equation by considering the relationship between accuracy, precision, safety and simplicity. Although the given methodology has been applied to the optimization of the shear procedure given in the EC-2 (European Committee for Standardization (CEN) 2002) or EHE-08 (Comisión Permanente del Hormigón 2008), it can be applied to any shear provision or any structural problem, such as Young's modulus determination, the tensile concrete strength, and creep or shrinkage. Note that a number of studies have investigated the shear behavior of reinforced concrete beams using other artificial intelligence techniques (Cladera and Marí 2004a; Cladera and Marí 2004b; Gandomi *et al.* 2013; Gandomi *et al.* 2014; Jung and Kim 2008; Keskin and Arslan 2013). However, they have generally focused on optimal precision and accuracy without a special methodology that considers safety or simplicity. Other researchers have recently employed a different genetic programming approach to solve a number of construction engineering problems (Da Silva and Štemberk 2013; Park *et al.* 2013; Tsai 2013; Tsai and Pan 2013; Tsai 2011).

The methodology that will be employed in this paper was initially employed for the shear strength of members without shear reinforcement (Pérez *et al.* 2010; Pérez *et al.* 2012). The innovation of the developed algorithm resided in its capacity to achieve simple and safe expressions from the original EC-2 equations for elements without stirrups, which improved the adjustment prior to a series of detected faults: the size effect, the low amount of longitudinal reinforcement and the bending moment-shear force interaction. The developed equation GP-4 (Eq. (1)) (Pérez *et al.* 2010) will be employed as a basis for the concrete contribution of the shear strength of RC members with stirrups:

$$\mathrm{V_u} = 0.114 \cdot \left(1 + \left(\frac{1600}{d}\right)^{0.42}\right) (100\rho_l)^{0.37} \cdot f_c^{\frac{1}{3}} \cdot \left(\frac{V\cdot d}{M}\right)^{0.21} \cdot b_w \cdot d \qquad (1)$$

where d is the effective depth of the cross-section ($\left(\frac{1600}{d}\right)^{0.42} \leq 5.0$), b_w is the width of the web, ρ_l is the reinforcement ratio for longitudinal reinforcement ($\rho_l \leq 0.08$), f_c is the compressive strength of concrete ($f_c \leq 90$ MPa), V is the shear force and M is the concomitant bending moment at the critical section ($\frac{V\cdot d}{M} \leq 1$).

The expression that will be developed for the shear strength of the elements with stirrups will be based on a variable angle truss model with a concrete contribution. Note that the objective of this study is to develop a methodology that enables the modification of previous equations based on knowledge extracted from an experimental database and to identify the aspects of current procedures that can be improved from the obtained equations. To doubt the advantages of shear design provisions, which are based on a sound mechanical model and enable not only direct design but also more refined analyses, is not an objective of this study (Collins *et al.* 2008; Marí *et al.* 2014).

Three expressions with different complexities, from a fixed angle tress model with a concrete contribution to a variable angle truss model with concrete contribution, have been obtained. The angle of the compression struts will depend on the amount of shear reinforcement, the shear force, the bending moment and the amount of longitudinal reinforcement. The three proposed equations offer better predictions of the shear strength of reinforced concrete beams compared with current international codes of practice, even for a set of beams that had not been previously employed by the GP algorithm.

1.1 Shear design procedures for RC members with shear reinforcement

In this paper, six shear design procedures will be employed to compare their predictions with the experimental results of reinforced concrete beams with stirrups. These procedures consist of the formulations given in EC-2 (European Committee for Standardization (CEN) 2002), equation 11-5 of ACI318-08 (ACI Committee 318 2008), the general method given in the Spanish Code EHE-08 (Comisión Permanente del Hormigón 2008), and three levels of approximations (LoA) given in Model Code 2010 (Féderation International du Beton 2012) for elements with stirrups. Table 1 summarizes the different shear provisions with all explicit safety factors removed. For the complete definitions of all involved symbols and parameters, refer to the corresponding code of practice.

Table 1 Summary of the shear design formulations in this paper

Shear procedure	Equation/Variables		
EC-2	$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} \cdot cot\theta \leq V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd} / (\cot\theta + \tan\theta)$		
	θ	$1 \leq cot\theta \leq 2.5$	
	ν_1	0.6 ($f_{ck} \leq 60$ MPa) 0.9-f_{ck}/200 > 0.5 (f_{ck} > 60 MPa)	
	α_{cw}	1 for non-prestressed structures	
ACI 318-08 Eq. 11-5	$V_n = V_c + V_s$; $V_c = \left(0.16\sqrt{f_c'} + 17\rho_l \frac{V_u \cdot d}{M_u}\right) b_w d \ngtr 0.29\sqrt{f_c'} b_w d$;		
	$V_s = \frac{A_v \cdot f_{yt}}{s} d \ngtr 0.66\sqrt{f_c'} b_w d$		
	ρ_l	Geometric amount of the longitudinal tensile reinforcement	$\rho_l = \frac{A_S}{b_w \cdot d}$
	$V_u \cdot d/M_u$	M_u occurs simultaneously with V_u at section considered.	$\frac{V_u \cdot d}{M_u} \ngtr 1.0$
	f'_c	Compressive strength (N/mm^2). Values of f_c greater than 69 MPa shall be permitted in computing V_c for beams with minimum web reinforcement	

Table 1 Continued

Shear procedure	***Equation/Variables***		
EHE-08 General method	$V_{Rd}=V_{Rd,c}+V_{Rd,s}\le V_{Rd,max}=\alpha f_{1cd}\, b_w d\,\frac{\cot\theta}{1+\cot^2\theta}$		
	$V_{Rd,c}=0.15\cdot k\cdot(100\cdot\rho_l\cdot f_{ck})^{1/3}\cdot b_w\cdot d$		$V_{Rd,s}=\frac{A_{sw}}{s}zf_{ywd}\cdot cot\theta$
	k	$k=1+\sqrt{\frac{200}{d}}\le 2.00$	
	ρ_l	Geometric amount of the longitudinal tensile reinforcement	$\rho_l=\frac{A_s}{b_w\cdot d}\not> 0.02$
	f_{ck}	Characteristic compressive strength (N/mm^2)	$f_{ck}\le 100MPa$
	θ	$\theta=29^o+7000\varepsilon_x$	
	ε_x	Axial strain at midheight of the member for the general method	$\varepsilon_x=\frac{M/0.9d+V}{2E_sA_s}$
	f_{1cd}	$0.6f_{cd}$ ($f_{ck}\le 60$ MPa) $(0.9-f_{ck}/200)f_{cd}>0.5f_{cd}$ ($f_{ck}>60$ MPa)	
	α	1 for non-prestressed structures	
MC-2010 Level I – Level II	$V_{Rd}=V_{Rd,s}\not< V_{Rd,c\ without\ stirrups}\le V_{Rd,max}=k_cb_wz\, f_{cd}\sin\theta\cos\theta$		
	$V_{Rd,s}=\frac{A_{sw}}{s}zf_{ywd}\cdot cot\theta$		$V_{Rd,c}=k_v\sqrt{f_{ck}}\, b_w\cdot z$
	k_v	Level I approximation: $k_v=\frac{180}{1000+1.25z}$ Level II approximation: $k_v=\frac{0.4}{1+1500\varepsilon_x}\frac{1300}{1000+k_{dg}z}$	
	k_{dg}	For concrete strengths greater than 70 MPa, d_g shall be taken as zero.	$k_{dg}=\frac{32}{d_g+16}\ge 0.75$
	θ_{min}	*Level I:* 25° prestressed/compressed; 30° reinforced; 40° axial tension *Level II:* $\theta_{min}=20^o+10000\varepsilon_x$	
	k_c	Strength reduction factor.	$k_c=k_\varepsilon\eta_{f_c}$
	K_ε	Factor that considers the strain in the web	$Lev.I$: $k_\varepsilon=0.55$ $Lev.II$: $k_\varepsilon=\frac{1}{1.2+55\varepsilon_1}\le 0.65$
	ε_1	$\varepsilon_1=\varepsilon_x+(\varepsilon_x+0.002)cot^2\theta$	
	ε_x	Axial strain at midheight of the member	$\varepsilon_x=\frac{M/0.9d+V}{2E_sA_s}$
	η_{fc}	$\eta_{fc}=\left(\frac{30}{f_{ck}}\right)^{1/3}\le 1.0$	
	f_{ck}	Characteristic compressive strength (N/mm^2). Limited for $V_{Rd,c}$ calculation.	$f_{ck}\le 64MPa$
MC-2010 Level III	$V_{Rd}=V_{Rd,c}+V_{Rd,s}\le V_{Rd,max}(\theta_{min})=k_cb_wz\,\nu_1f_{cd}\sin\theta_{min}\cos\theta_{min}$		
	$V_{Rd,s}=\frac{A_{sw}}{s}zf_{ywd}\cdot cot\theta$		$V_{Rd,c}=k_v\sqrt{f_{ck}}\, b_w\cdot z$
	k_v	Level III: $k_v=\frac{0.4}{1+1500\varepsilon_x}\left(1-\frac{V_{Ed}}{V_{Rd,max}(\theta_{min})}\right)\ge 0$	
	θ_{min}	$\theta_{min}=20^o+10000\varepsilon_x$	
	Other variables (refer to MC2010-LoA II)		

The method proposed for the Level III approximation of the MC2010 (MC2010-LoA III) is directly based on the modified compression field theory (Vecchio and Collins 1986). Within the MCFT, the concrete contribution is predicted to be carried by aggregate interlock (Walraven 1981). Members with a minimum quantity of stirrups are predicted to fail in shear by yielding of the stirrups and/or eventual crushing of the concrete in the web. The limits for the allowable angle of principal compression θ are also based on the MCFT.

The general shear procedure in EHE-08 for elements with stirrups is based on a truss model with a variable angle of inclination of the struts and a concrete contribution (Marí and Cladera 2006). The angle is obtained by compatibility, which is based on the MCFT, but is associated with an empirical equation for the concrete contribution. This empirical term is almost identical to the strength given in EC-2 for a similar member without shear reinforcement. Equation 11-5 of ACI 318-08 is a fixed 45° truss model with a concrete contribution; this method can be considered completely empirical.

The EC-2 formulation for the elements with shear reinforcement, MC2010-LoA I and MC2010-LoA II are based on a truss model with a variable angle of inclination of the struts and without any concrete contribution. For the EC-2, the inclination of the compression struts is a design parameter that must be compressed between $1 \leq \cot\theta \leq 2.5$. For MC2010-LoA I, the minimum angle of θ for reinforced beams is 30°. This LoA is a simplified form of the Level III.

The second LoA is also based on the principles of plasticity but is modified with a strain term to better model the behaviors of heavily reinforced members (Bentz 2010).

MC2010 presents a fourth level of approximation, which enables the use of tools such as nonlinear finite element analysis or generalized stress-field approaches (Bentz 2010). This LoA will not be considered in this paper because it is not a direct designing method. Note that the MC2010 shear provision is a structured approach that includes the design, detailed analysis and elaborate structural assessment of beams in shear.

1.2 Database

Two previously published databases (Cladera and Marí 2007; Yu and Bažant 2011) have been merged to develop a new database with 272 experimental tests of slender beams with stirrups; all beams were reported as shear failures. The database by Yu and Bažant (2011) contains 234 test results. The database by Cladera and Marí (2007) contains 122 test results (38 of which are not included in the first database). After merging these two original databases, 15 tests were removed because they did not represent tests with conventional conditions. Thus, the final database used in this report contains 257 experimental results: 223 test results from the reference (Yu and Bažant 2011) and 34 test results from the reference (Cladera and Marí 2007). The main objective of this study was to qualitatively compare specific shear procedures with different tested beams rather than the development of an accurate database to justify a model. Substantial efforts have been achieved to prevent inaccuracies in the database.

The 257 tests have been published in 32 papers (Adebar and Collins 1996; Anderson and Ramirez 1989; Angelakos *et al*. 2001; Bhal 1968; Bresler and Scordelis 1963; Bresler and Scordelis 1966; Cladera and Marí 2005; Collins and Kuchma 1999; Elzanaty *et al*. 1986; Etxeberria *et al*. 2007; Frosch 2000; González-Fonteboa and Martínez-Abella 2007; Johnson and Ramirez 1989; Karayiannis and Chalioris 1999; Kong and Rangan 1998; Krefeld and Thurston 1966; Leonhardt and Walther 1962; Lubell *et al*. 2004; Mattock and Wang 1984; McGormley *et al*.

Table 2 Range of variables in the databases

	General database		Training database		Evaluation database	
	Min	Max	Min	Max	Min	Max
b_w (mm)	76	457	76	457	76	457
d (mm)	95	1890	95	1890	95	925
f_c (MPa)	12.8	125.4	12.8	125.4	15.73	120.2
ρ_l (%)	0.5	7.0	0.5	7.0	0.68	5.80
$\rho_w \cdot f_y$(MPa)	0.11	8.11	0.11	8.11	0.16	4.06
a/d	2.4	6.0	2.4	6.0	2.49	5.50
V_u (kN)	14	2239	16	2239	14	1658

1996; Mphonde and Frantz 1985; Ozcebe *et al*. 1999; Placas and Regan 1971; Rajagopalan and Ferguson 1968; Roller and Russell 1990; Sarsam and Al-Musawi 1992; Shah and Ahmad 2007; Swamy and Andriopoulos 1974; Tan, K.-H., Teng, S., Kong, F.-K.,Lu, H.-Y. 1997; Tan *et al*. 1995; Tompos and Frosch 2002; Yoon *et al*. 1996; Zararis and Papadakis 1999). The range of the variables for the different tests is presented in Table 2.

The database has been divided into two subsets: a database for training the GP algorithm with 215 test results and a database with 42 test results for evaluating the equation. The final equations are evaluated with test results that have not been used in the training process. This database has been randomly performed; however, the training database should cover the available range of input variables (b_w, d, f_c, ρ_l, $\rho_w f_y$ and a/d) (Table 2), where b_w is the width of the web of the cross-section; d is the effective depth; f_c is the compressive strength of concrete; ρ_l is the amount of longitudinal reinforcement ($\rho_l = \frac{A_l}{b_w \cdot d}$), $\rho_w f_y$ is the nominal stirrup strength ($\rho_w f_y = \frac{A_{sw}}{b_w \cdot s} f_y$), where f_y is the stirrups yielding strength, s is the stirrup spacing and A_{sw} is the cross-sectional area of shear reinforcement; and a/d is the shear span-to-depth ratio.

1.3 Verification of the shear design procedures with the experimental database

An analysis of the predictions has been performed using the different formulations for the entire database. The modus operandi suggested by Collins (Collins 2001), who considers the distribution of the values of V_{test}/V_{pred} to be asymmetric with a median value that is generally lower than the average, was employed. Table 3 presents the coefficient of variation (COV) for all tests and the COV of a dataset formed by the lower 50% of the data and their symmetrical values around the median, which results in a symmetrical distribution with an identical number of data compared with the original one ($COV_{Low\ 50\%}$). In the same manner, the ($COV_{High\ 50\%}$) is identically calculated compared with the previous value but with the highest 50% values. The distribution functions for two of the shear provisions are presented in Fig. 2. The actual values of the V_{test}/V_{pred} data have been grouped for 0.05 intervals. For example, approximately 6% of the tested beams yield a V_{test}/V_{pred} in the range [1.65, 1.70) for the EC-2 predictions (Fig. 2(a)), whereas approximately 12% of the tested beams yield a V_{test}/V_{pred} ratio in the range [1.15, 1.20) for the EHE-08 predictions (Fig. 2(b)). The normal distribution, which is given by the average and the COV of V_{test}/V_{pred}, is also shown in Fig. 3. For 50% of the tests, in which the value V_{test}/V_{pred} is lower than the median, the adjustment given by the normal distribution created considering only this 50% of the data and their symmetrical values around the median is significantly better (Fig. 2). In Fig. 3, the normal distribution functions formed with all data and for 50% of the test, in which

Table 3 Verification of the different shear design procedures

V_{test}/V_{pred}	EC-2	ACI318-08 eq. 11-5	EHE-08	MC10 Lev I	MC10 Lev II	MC10 Lev III	Eq.(6)	Eq.(7)	Eq.(8)
Average	1.76	1.23	1.17	2.23	1.62	1.21	1.10	1.13	1.11
Median	1.66	1.22	1.15	2.18	1.57	1.20	1.09	1.11	1.11
Standard Deviation	0.672	0.264	0.219	0.629	0.391	0.223	0.178	0.178	0.168
RMSE	139.3	73.2	56.4	144.9	104.6	51.7	48.4	49.2	51.7
COV (%)	38.23	21.48	18.74	28.15	24.16	18.52	16.16	15.78	15.09
$COV_{Low\ 50\%}$ (%)	27.97	19.34	14.89	24.38	17.97	16.02	13.44	13.13	13.66
$COV_{High\ 50\%}$ (%)	50.60	23.77	22.62	32.75	29.92	20.92	19.03	18.63	16.39
Minimum	0.51	0.65	0.70	0.73	0.64	0.75	0.68	0.70	0.75
$(V_{test}/V_{pred})_{1\%}$	0.71	0.70	0.75	1.03	0.99	0.79	0.74	0.77	0.80
$(V_{test}/V_{pred})_{5\%}$	0.92	0.84	0.88	1.32	1.16	0.91	0.83	0.86	0.86
Maximum	5.53	2.34	2.26	4.93	3.33	2.20	1.86	1.88	1.84
$(V_{test}/V_{pred})_{95\%}$	2.91	1.64	1.56	3.30	2.27	1.57	1.38	1.40	1.37
$(V_{test}/V_{pred})_{99\%}$	3.60	2.07	1.92	4.00	3.05	2.01	1.72	1.74	1.70

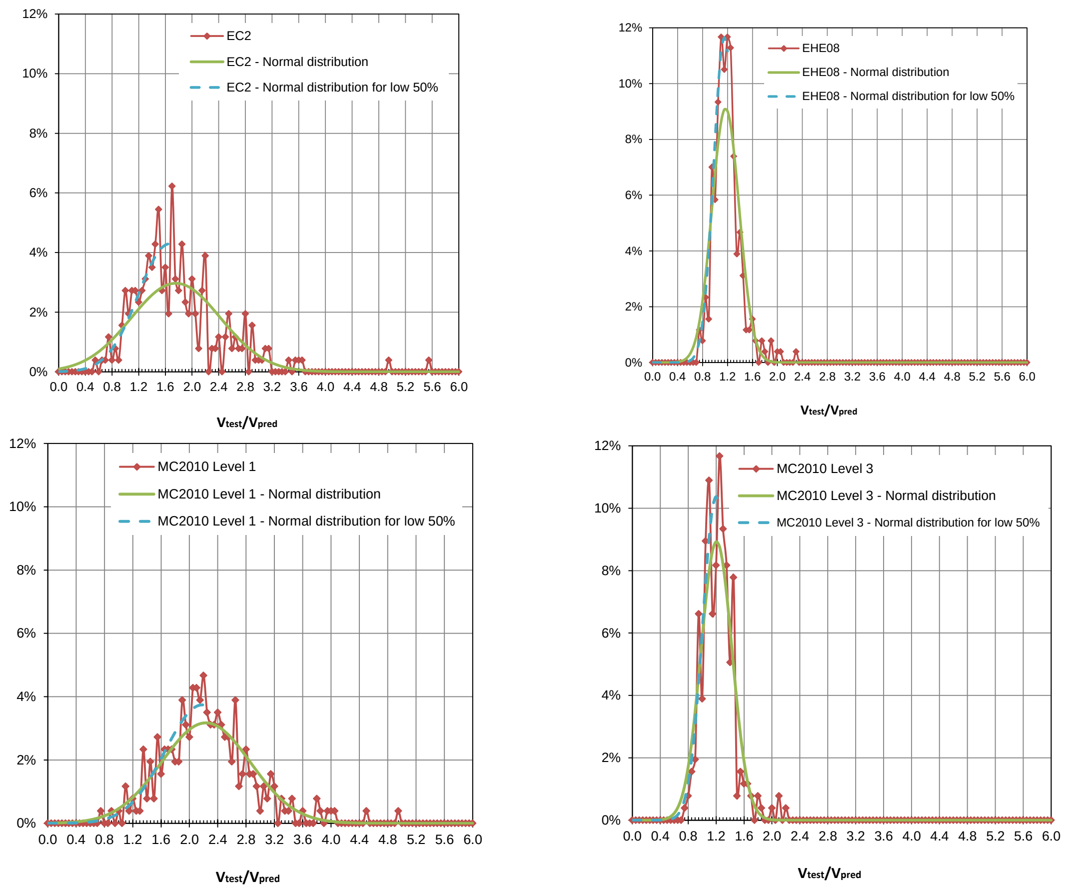


Fig. 2 Distribution functions for EC-2, EHE-08, MC2010–LoA 1 and MC2010-LoA 3

the value V_{test}/V_{pred} is lower than the median, are compared for all presented code procedures.

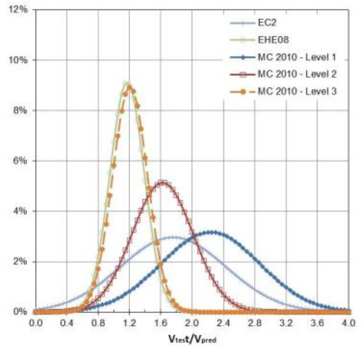
Table 3 provides a vast amount of information that is related to the verification of the evaluated concrete codes with the database; however, few aspects will be signaled. The last three columns of Table 3 refer to the equations derived by the GP algorithm; they will be discussed in section 3.

- The average of the V_{test}/V_{pred} ratio is directly related to accuracy; a value of 1.00 is highly accurate. The most accurate models among the different evaluated code procedures are EHE-08 (V_{test}/V_{pred} = 1.17) and MC2010-LoA III (V_{test}/V_{pred} = 1.21). Less accurate methods are MC2010-LoA I (V_{test}/V_{pred} = 2.23) and EC-2 (V_{test}/V_{pred} = 1.76).
- The standard deviation (SD) or the coefficient of variation (COV) is a measure of precision; the lower are the values, the higher is the precision. The most precise models are EHE-08 (SD = 0.219) and MC2010-LoA III (SD = 0.223). The least precise methods are MC2010-LoA I (SD = 0.629) and EC-2 (SD = 0.672).
- Note that the coefficients of variation for the "high data set" are significantly higher compared with the "low data set" for all evaluated shear provisions. For this reason, the use of the described technique is recommended to obtain realistic 5% percentiles.
- The 5% percentile of the V_{test}/V_{pred} ratio is a measure of safety. If a value higher than 0.85 is considered to be the appropriate level of safety (Collins 2001), all methods can be considered to be safe, with a lower 5% percentile of 0.84 for equation 11-5 of ACI Code 318-08. Despite a lower precision, MC2010-LoA I is very safe (5% percentile of 1.32) due to the lower accuracy from a safety point of view (average V_{test}/V_{pred} of 2.23).
- The relationship among accuracy, precision and safety is explained by the results of the three levels of approximation of MC2010 (Table 3 and Fig. 3). LoA III is highly accurate with high precision and safe results. When simplifying to Level II and Level I, the predictions lost precision. To predict safe results, the methods have been calibrated to reduce their accuracy (from the safe side) by always considering conservative simplifying assumptions.

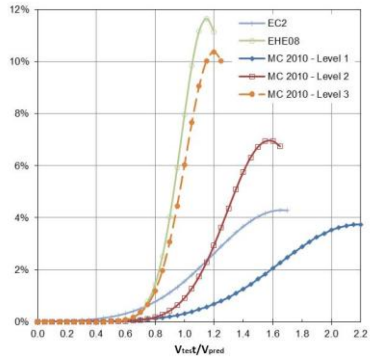
The simplicity of the different shear design procedures can be evaluated from Table 1. The provisions of MC2010-LoA III are the most complex, with iterative calculations for the concrete and the steel contributions when verifying a given RC member, followed by EHE-08, which also contains an iterative method for the steel contribution. Conversely, these two methods (MC2010-LoA III and EHE-08) are direct methods of design. EC-2, ACI318-08 and MC2010-LoA I are the simplest methods; they enable direct calculations for verification and design with few operations.

Fig. 4 shows the values V_{test}/V_{pred} for each test in terms of the effective depths of the beams. Of 257 beams, only 2 beams an effective depth greater than 1000 mm. Although a minimum amount of shear reinforcement generally produces a substantial improvement in terms of the size effect (Lubell *et al.* 2004), the size effect is not entirely eliminated by the presence of stirrups, as shown in Fig. 4. The shear stress at failure decreases as the member depth increases.

For the six evaluated shear provisions, only the method in EHE-08 considers the size effect in the concrete contribution. As shown in Fig. 4, this method considers this effect; however, it does not completely correct it. The three levels of approximation for MC2010 (refer to section 1.1 and Table 1) do not consider the size effect for elements with stirrups. For this reason, MC2010 is more conservative for small members and the value of V_{test}/V_{pred} decreases for higher beam depths. In 2011, Yu et al. (Yu and Bažant 2011) developed wrote down this phenomenon and suggested that stirrups cannot completely suppress the size effect on the shear strength of RC beams regardless of the stirrup ratio. Thus, the size effect of shear strength is mitigated in the small-size range (up about 1 m) but remains the same in the large-size range (Yu and Bažant 2011). As shown



(a) Set formed by all tests in the database



(b) Set formed by 50% of the tests in the database, where the value V_{test}/V_{pred} is lower than the median

Fig. 3 Normal distribution functions for all tests in the database

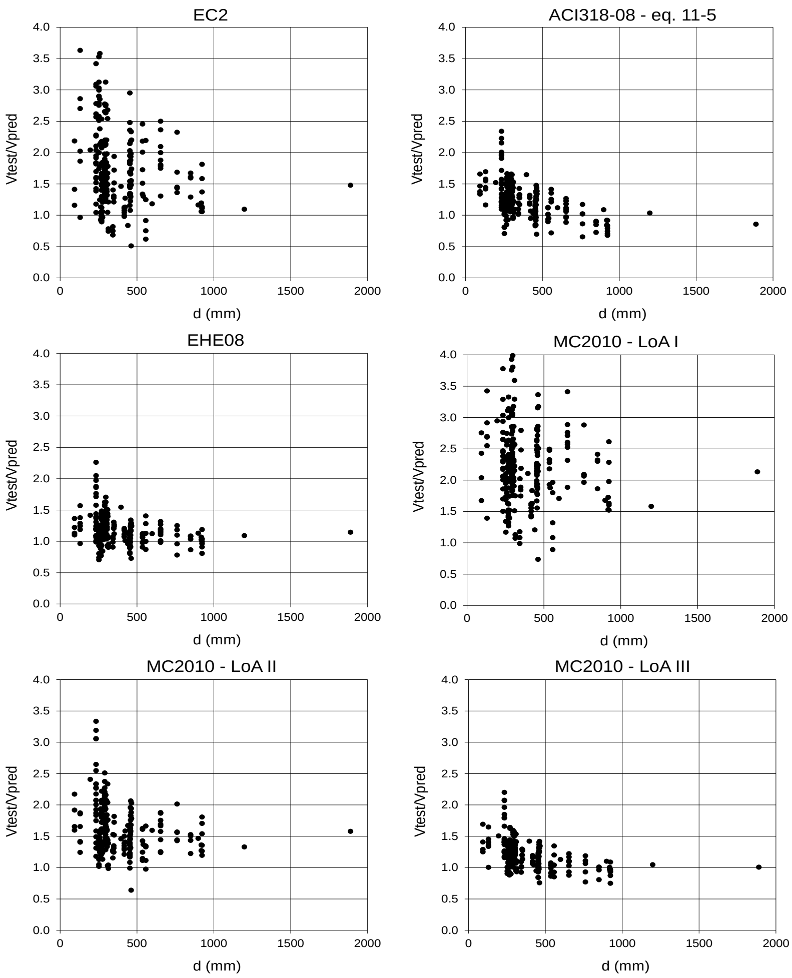


Fig. 4 Correlation between the predictions and the experimental results in terms of the depths of the beams

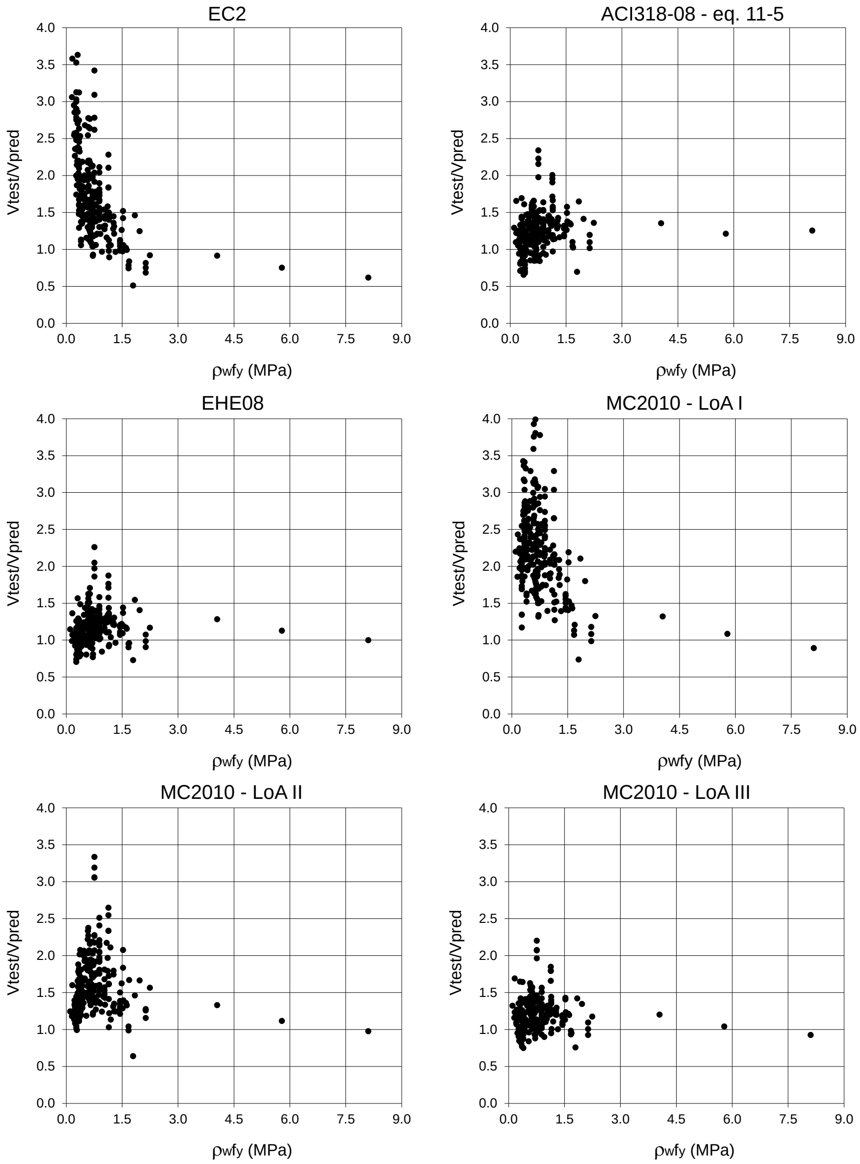


Fig. 5 Correlation between the predictions and the experimental results in terms of the nominal stirrup strength

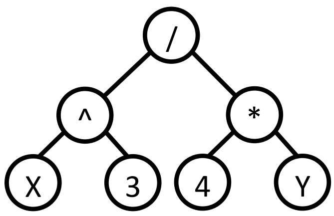


Fig. 6 Tree for the expression $(x, y) = \frac{x^3}{4y}$

in Fig. 4, the plots for the three levels of approximation of MC2010 exhibit other significant differences.

Fig. 5 shows the correlation between the predictions and the experimental results in terms of the amount of stirrups (nominal stirrup strength, expressed in MPa). Only 3 beams had nominal stirrup strengths greater than 3 MPa. For the EC-2 and MC2010-LoA I, the ratio V_{test}/V_{pred} decreases as the nominal stirrup strength increases, which produces nonconservative results (without considering the partial safety factors) for a highly shear reinforced member in the case of EC-2 predictions. EHE-08 and MC2010-LoA III generate better adjustments in the empirical tests.

1.4 Relationship between precision, accuracy and safety

Although some methods produce satisfactory results, the consideration of the size effect can be improved for different methods. Moreover, the simpler methods (EC-2, MC2010-LoA I) do not correctly consider the influence of the amount of stirrups. In the next section, a proposal to improve the EC-2 formulation to correct these effects is presented.

As previously demonstrated, accuracy, precision and safety are highly interconnected. Assuming a normal distribution for the set given by 50% lower results, Eq. (2) relates safety, $(V_{test}/V_{pred})_{5\%}$, with accuracy, $(V_{test}/V_{pred})_{\mathrm{median}}$ and precision, $\mathrm{COV}_{\mathrm{Low\ 50\%}}$ (%).

$$\left(\frac{V_{test}}{V_{pred}}\right)_{5\%} = \left(\frac{V_{test}}{V_{pred}}\right)_{median} \left(1 - 1.645 \cdot \frac{COV_{Low\ 50\%}(\%)}{100}\right) \tag{2}$$

Considering the hypothesis that it is possible to achieve a precision $\mathrm{COV}_{\mathrm{Low\ 50\%}}$ (%) equal to or less than 15% with GP techniques and that an adequate safety for shear strength can be given by $(V_{test}/V_{pred})_{5\%} = 0.85$, then $(V_{test}/V_{pred})_{\mathrm{median}} = 1.097$ from Eq. (2). Similarly, it is possible to fix any optimist scenario for the COV and any consideration about safety, which produces a different objective for the median. For example, if an adequate safety is $(V_{test}/V_{pred})_{5\%} = 1.00$, with an objective COV that is equal to or less than 15%, the objective median is 1.25.

2. Optimization procedure based on a genetic algorithm

2.1 A brief introduction to genetic programming

Genetic programming (GP) is a heuristic technique for searching solutions to user-defined problems. A typical problem in which GP shows acceptable results is the symbolic regression of data. In this case, GP obtains the relationship between the involved variables; for example, in a physical phenomenon. This relationship takes the form of mathematical equations that demonstrate how each variable influences the phenomenon.

GP is inspired by the adaptive capacity of a species to the environment and is based on an analogy with the principle of natural selection, which Darwin propounded in his theory of evolution (Darwin 1859). The origins of the GP goes back to 1992 when Koza (1992) redefined the algorithms and the structure of individuals based on the same principles of the genetic algorithms (GAs) promulgated by John Holland (1975). Koza (1992) provided solutions to problems through the program's induction and the algorithms that solve these problems.

GP is generally based on solution encoding (individuals) and the process (algorithm) that evolves solutions to obtain a better solution. The solution to a problem is referred to as the "individual", whereas the set of existing solutions is referred to as the "population". The real representation (encoding) of an individual is achieved using a structure that is referred to as a "tree", in which the mathematical equation is codified. This structure represents the operators in the non-terminal nodes (arithmetic operations and trigonometric functions) and the "leaves" or terminal nodes are represented as the constants and the variables. For example, Fig. 6 represents the following individual: $\mathrm{f}(\mathrm{x},\mathrm{y}) = \mathrm{x}\hat{}3/(4 * \mathrm{y})$. As in nature, the "operation" of "crossover" is necessary to produce new "solutions" (referred to as offspring or children). As also occurs in nature, some of the individuals experience the mutation operation; in this case, they experience it with the objective of exploring new search spaces. The obtained individuals are evaluated (fitness function). If an individual obtains a better fitness (that is, is better adapted to the environment), it will substitute one of the individuals of the current population, which constitutes a new generation (epoch). The process ends when a criterion (fitness, number of generations or other) is satisfied.

2.2 Genetic programming to improve the model of shear strength of RC members with shear reinforcement

The methodology used to improve the model of the shear strengths of RC members with shear reinforcement is similar to the model that was previously employed to improve the shear strength formulation for elements without shear reinforcement (Pérez *et al*. 2010; Pérez *et al*. 2012). However, greater emphasis is placed in this paper on solving the dilemma among precision, accuracy, safety and simplicity. The method enables improvement of a mathematical expression from knowledge extracted from experimental data. For this, the expert highlights the parts of the given equation that may be improved and establishes the constraints that must be satisfied by the new equation.

2.2.1 Fitness function

The final goal is to obtain an expression that follows the principles of precision (repeatability), accuracy, structural safety and simplicity. To achieve the third first principles (precision, accuracy and structural safety) the fitness function eq. (3) includes the l_{bias} and k_i parameters:

$$fitness(i) = \frac{\sum_{i=0}^{n} k_i * \left| l_{bias} - \frac{V_{test,i}}{V_{pred,i}} \right|}{n} + \alpha \cdot s_i \quad , l_{bias} = 1.1 \tag{3}$$

Accuracy is achieved by minimizing the value of the fitness function. Precision and structural safety are considered in the l_{bias} and k_i parameters. A general slight oversizing is achieved by setting the l_{bias} to 1.1. This value contradicts the precision that would be situated in 1.0 ("exact" value) but is necessary to ensure structural safety, as shown in section 1.4. The parameter k_i enables specific oversizing for some tests, increases the global precision and diminishes the number of unsafe results. The different penalization ranges are shown in Eq. (4). Precision is achieved by penalizing the results in the external ranges. Structural safety is achieved by an asymmetric penalty, and unsafe predictions achieve a higher value of k_i compared with predictions that are too safe, as shown in Eq. (4). In Eq. (4), the values k_i were inspired in the use of the ‘‘demerit points’’ technique, which was employed by M.P. Collins (2001) to categorize the results of different building codes. The effect of considering these k_i values was examined for the shear strength of beams without stirrups in Pérez *et al*. 2012.

$$k = \begin{cases} 10, & \frac{V_{test}}{V_{pred}} < 0.5 \\ 5, & 0.5 \le \frac{V_{test}}{V_{pred}} < 0.67 \\ 3, & 0.67 \le \frac{V_{test}}{V_{pred}} < 0.85 \\ 1, & 0.85 \le \frac{V_{test}}{V_{pred}} < 1.3 \\ 2, & 1.3 \le \frac{V_{test}}{V_{pred}} < 2 \\ 3, & \frac{V_{test}}{V_{pred}} \ge 2 \end{cases} \tag{4}$$

To ensure that the resulting expressions are as simple as possible, the parsimony level is introduced by the α parameter (Eq. 3), which multiplies the individual size s_i (number of nodes). At the same level of prediction error, large individuals (or large equations) will receive a penalty that is greater than small individuals. It can also be seen that a large value of the α parameter will yield a small tree, whereas a small α will yield a larger tree but better accuracy. The balance between the parsimony level α and accuracy has been achieved by trial and error. Two different values of parsimony were used in the initial steps of the search process, as shown in Table 6. In the final steps of the search process, a small value for the parsimony was employed because the search process was highly constrained (Table 5).

2.2.2 Search process

The initial equation for this study is shown in Eq. (5); it is derived by considering the concrete contribution optimized in a previous paper (Pérez *et al*. 2010) and a variable angle strut model for the steel contribution (from EC-2). In this case, two parts of the equation are improved (named branches). For each branch, the GP algorithm will generate an equation or a constant value. Note that this initial equation to be improved is a combination of the formulations of EC-2, EHE-08 and the GP-4 equation optimized for elements without stirrups.

$$\begin{aligned} \mathrm{V_u} = {} & 0.114 \cdot \left(1 + \left(\frac{1600}{d}\right)^{0.42}\right) (100\rho_l)^{0.37} \cdot f_c^{\frac{1}{3}} \cdot \left(\frac{V \cdot d}{M}\right)^{0.21} \cdot b_w \cdot d \cdot Branch_1 \\ & +0.9 \cdot d \cdot \rho_w f_y \cdot b_w \cdot cot(Branch_2 \cdot \frac{\pi}{180}) \end{aligned} \tag{5}$$

The equation search process has been iterative. This process was performed in nine stages. The initial stages (1 to 3) were used to determine the "shape" of each branch of the equation and the final stages (4 to 9) were used to optimize the formulas (selected in previous stages) using constants.

Table 4 Variables/equations used to create constraints in initial stages

Branch	Variables / equations
$Branch_1$	[*Cst*]
$Branch_2$	[*Cst*], [$M/\rho_l \cdot V \cdot d$], [$M/V \cdot d$], [ρ_l], [f_c], [d]
	**Cst* = constant value.

Table 5 Variables/equations used to create constraints in final stages

Branch	Variables / equations
$Branch_1$	[*Cst*], [1], [1.04], [0.85], [0.80]
$Branch_2$	$[Cst_1 + Cst_2/d + Cst_3(M/\rho V d^2)]$
	$[Cst_1 + Cst_2\ (Cste_3/d)^{Cst_4} \cdot (M/\rho V d^2)^{Cst_5}]$
	$[Cst_1 + Cst_2\ (Cst_3/d)^{Cst_4} \cdot (M/\rho V d)^{Cst_5}]$
	$\left[Cst_1 - \dfrac{Cst_2 - M/\rho V d}{d}\right]$
	$\left[Cst_1 - \dfrac{Cst_2 - (M/\rho V d)^{Cst_3}}{d^{Cte_4}}\right]$
	$\left[Cst_1 + \left(Cst_2 - (M/\rho V d)\right) \cdot (Cst_3/d^{Cst_4})\right]$
	$[Cst_1 + Cst_2(1/d)^{Cst_3}(M/\rho V d)^{Cst_4}]$
	$\left[Cst_1 + Cst_2(1/d)^{Cst_3}(M/\rho V d)^{Cst_4}\left(A_{w,MPa}\right)^{Cst_5}(f_c)^{Cst_6}(\rho_l)^{Cst_7}\right]$
	$\left[Cst_1 + Cst_2(1/d)^{Cst_3}(M/\rho V d)^{Cst_4}\left(A_{w,MPa}\right)^{Cst_5}\right]$
	**Cst* = constant value.

Table 6 Parameters that govern the genetic programming algorithm

Configuration parameters	Initial steps		Final steps
Population size	1000		1000
Maximum generation	20000		40000
Stopping criteria (epoch without improvement)	2000		3000
Algorithms	Selection: Tournament Creation: Intermediate Mutation: Subtree		
Crossover rate	90%		90%
Mutation probability:	5%		10%
Elitist strategy	Yes		
Initial tree height	3	6	6
Maximum tree height	6	9	9
Maximum mutation tree height	3	6	6
Parsimony	0	0.00001	0.00000001
Nonterminal selection rate	90%		
Terminal nodes	1, 2, 3, 4, 5, 6, 7, 8, 9, 10 -		random real numbers [-1, 1]
Nonterminal nodes	+, -, *, / Sqr, Sqrt		Sqr, Sqrt

The rules for generating the branches will be established by a constraint set. This set has been refined throughout the study. Table 4 shows the variables and equations that have been used to create the different constraints. For example, one of the first constraints is as follows: *$Branch_1$* can adopt any numeric value and *$Branch_2$* can adopt any function using the variables ρ_l, f_c, d or $M/\rho_l{\cdot}V{\cdot}d$. Only numeric values have been accepted in *$Branch_1$* because this term multiplies the shear strength that a similar reinforced concrete beam without shear reinforcement would have, refer to Eq. (1); this term was modified with a constant value rather than a completely new term that is dependent on different parameters. The parameters and the combination of parameters for *Branch*$_2$ (Table 4) have been selected due to the previous experience of the authors in shear strength models.

The authors have developed the software that was specifically employed to conduct their research on shear strength. Note that this software uses the library of genetic programming created by Dorado *et al*. (2002).

After a review of the equations obtained in initial stages, the possible "shapes" for each branch were selected. Table 5 lists the most representative “shapes” of the equations. Note that the equations and constants have evolved to obtain the final equations in section 2.2.3.

The parameters for the GP settings (refer to Table 6) vary in the initial stages, in which the branches are equations, compared with the following stages, in which only the constants were allowed to be changed.

A total of 50 sets of defined constraints are distributed in 9 stages. For each constraint, the PG algorithm was run a minimum of 150 times. It was performed an average of 9000 generations for each execution. In the initial stages, the maximum generation was achieved numerous times due to the variability in the creation of the equations.

2.2.3 Final results

At the end of this procedure, three final expressions were obtained:

$$\mathrm{V_u} = 0.1186 \cdot \left(1 + \left(\frac{1600}{d}\right)^{0.42}\right) (100\rho_l)^{0.37} \cdot f_c^{\frac{1}{3}} \cdot \left(\frac{V \cdot d}{M}\right)^{0.21} \cdot b_w \cdot d + 0.9 \cdot d \cdot \rho_w f_y \cdot b_w \cdot 1{,}2 \quad (6)$$

$$\mathrm{V_u} = 0.114 \cdot \left(1 + \left(\frac{1600}{d}\right)^{0.42}\right) (100\rho_l)^{0.37} \cdot f_c^{\frac{1}{3}} \cdot \left(\frac{V \cdot d}{M}\right)^{0.21} \cdot b_w \cdot d$$

$$+0.9 \cdot d \cdot \rho_w f_y \cdot b_w \cdot cot(36.436 + 0.05 \cdot (M/\rho V d)^{0.875}) \quad (7)$$

$$\mathrm{V_u} = 0.0912 \cdot \left(1 + \left(\frac{1600}{d}\right)^{0.42}\right) (100\rho_l)^{0.37} \cdot f_c^{\frac{1}{3}} \cdot \left(\frac{V \cdot d}{M}\right)^{0.21} \cdot b_w \cdot d$$

$$+0.9 \cdot d \cdot \rho_w f_y \cdot b_w \cdot cot\left(22.438 + 0.177 \cdot (M/\rho V d)^{0.875} \left(\rho_w f_y\right)^{1{,}0}\right) \quad (8)$$

These three equations should be analyzed. Eq. (6) represents a fixed angle truss model with a concrete contribution. The angle of the compression strut with the longitudinal axis of the beam is 39.81° (cotθ = 1.2). Eqs. (7)-(8) represent a variable angle truss model with a concrete contribution. In Eq. (7), the angle of the compression struts is only dependent on the parameter *M/ρVd*. In Eq. (8) the effect of the amount of transversal reinforcement is also included in the angle determination. Therefore, the determination of the angle of the compression struts improves from Eq. (6) to Eq. (7) or Eq. (8). The concrete contribution diminishes from Eq. (6) to Eq. (8). For a GP technique, it is not possible to differentiate the concrete contribution and the steel contribution because the only available experimental result is the shear strength (the addition of the two terms). For this reason, the methodology used to perform this study is especially important: in a previous study the shear strength of beams without stirrups was analyzed (Pérez *et al*. 2010); and in this paper the generalization for beams with stirrups has been conducted. In anycase, the GP procedure reveals that the more variables we consider in the steel contribution, the less weight the GP gives to the concrete contribution to ensure safety, accuracy and precision.

Table 7 Verification of the different shear design procedures for the evaluation database (42 tests results)

V_{test}/V_{pred}	EC-2	ACI318-08 eq. 11-5	EHE-08	MC10			Eq.(6)	Eq.(7)	Eq.(8)
				Lev I	Lev II	Lev III			
Average	1.85	1.25	1.18	2.13	1.58	1.23	1.10	1.12	1.11
Median	1.66	1.22	1.15	2.18	1.57	1.20	1.06	1.09	1.09
Standard Deviation	0.82	0.27	0.22	0.54	0.36	0.22	0.17	0.17	0.164
RMSE	92.0	85.5	69.9	128.1	102.3	59.5	58.2	51.9	62.3
COV (%)	44.56	21.84	18.75	25.43	22.78	18.02	15.66	14.76	14.71
Minimum	0.68	0.68	0.80	0.98	1.13	0.75	0.82	0.85	0.80
Maximum	4.92	2.00	1.87	3.29	2.65	1.85	1.63	1.63	1.56

3. Comparison of results

3.1 Global comparison

The comparison of the performance of the three newly developed equations with the considered code formulations is given in Table 3. Note that the three equations significantly correlate with the empirical results compared with any of the considered codes. It must be highlighted:

- The three proposed equations are more accurate than any of the considered codes for the entire database. The average of V_{test}/V_{pred} for the three equations is approximately 1.10.
- The three proposed equations are more precise than any of the considered code formulations because they present the minimum standard deviation and coefficient of variation. This finding is remarkable for the 50% low data set, which is less than 15% for the three resulting expressions.
- The three proposed equations are safe because the 5% percentile of the three methods are near the target value of 0.85, as discussed in section 1.4.
- The three proposed equations are simple and easy to use; Eq. (6) is simpler and Eq. (8) is more complex. Using Eq. (7), superior results are obtained for a direct calculation method, without the need for iterations.

Table 3 presented the results for the entire database, whereas Table 7 only lists the results for the evaluation database. These tests have not been employed by the GP algorithm to obtain the final equations. The results for these 42 test beams are similar compared with the entire database, and the three proposed equations also provide the best correlation with the experimental results.

3.2 Comparison with different subsets of beams

The experimental correlations for different subsets of beams are presented in Table 8. The lowest COV for each group is represented in bold letters. The proposed equations present the lower values of the coefficient of variation for all subsets, with the exception of one subset.

As the effective depth of the element increases, the average of the ratio V_{test}/V_{pred} for the different code predictions decreases. For instance, as the predictions according to EC-2 decreases, V_{test}/V_{pred} ranges from 1.95 for beams under 300 mm of effective depth to 1.58 for beams over 600 mm of effective depth. For the Level III approximation of the MC2010, V_{test}/V_{pred} decreases from

1.30 to 1.00. For the proposed equations, a certain reduction in the average values is obtained when the depths of the beams are increased; however, this reduction is clearly lower than for the other predictions and all the average values result on the side of safety (Table 8 and Fig. 7).

When increasing the amount of shear reinforcement ρ_w, V_{test}/V_{pred} decreases for the calculations according to different code formulations, especially for EC-2; it ranges from 2.02 for lightly reinforced beams ($\rho_w f_y \leq 0.70$ MPa) to 0.95 for highly shear-reinforced beams ($\rho_w f_y \geq 1.50$ MPa). Note that the predictions have been obtained by removing the partial safety factors of steel and concrete. If they were employed, no unsafe results would be obtained as an average value for any of the evaluated subsets. The response of the proposed equations is much more stable as a function of the nominal stirrup strength (Table 8 and Fig. 8).

Table 8 Verification of the different shear design procedures for subsets of beams

V_{test}/V_{pred}			**EC-2**		**ACI318-08 Eq. 11-5**		**EHE-08**		**MC10 Lev I**		**MC10 Lev II**		**MC10 Lev III**		**Eq. (6)**		**Eq. (7)**		**Eq. (8)**	
Criterion		#	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV
All		257	1.76	38.2	1.23	21.48	1.17	18.7	2.23	28.1	1.62	24.2	1.21	18.5	1.10	16.16	1.13	15.78	1.11	**15.09**
d (mm)	< 300	134	1.95	39.5	1.33	19.6	1.23	19.9	2.34	28.9	1.75	24.7	1.30	17.2	1.13	17.5	1.15	**16.9**	1.14	**16.9**
	300-600	94	1.54	31.8	1.17	17.1	1.11	14.9	2.09	27.4	1.46	20.3	1.13	14.9	1.08	**14.4**	1.10	**14.4**	1.08	**14.4**
	> 600	29	1.58	25.5	0.94	17.7	1.07	12.2	2.20	21.8	1.51	14.3	1.00	**11.8**	1.05	12.4	1.09	12.1	1.05	12.1
f_c (MPa)	< 40	114	1.71	45.0	1.22	19.2	1.12	15.9	2.14	27.0	1.54	20.3	1.19	17.2	1.08	**14.6**	1.11	14.7	1.09	14.7
	40-70	80	1.81	32.6	1.20	19.4	1.15	16.8	2.19	25.6	1.52	20.8	1.18	15.1	1.09	14.2	1.11	**13.5**	1.10	**13.5**
	> 70	63	1.78	32.5	1.29	26.2	1.28	21.8	2.46	30.4	1.88	25.6	1.27	22.9	1.16	19.6	1.17	**19.3**	1.16	**19.3**
ρ_l (%)	< 1	7	1.34	16.7	1.04	26.1	1.17	13.3	1.93	16.7	1.77	16.9	1.24	21.8	1.14	12.9	1.24	**11.8**	1.23	**11.8**
	1-3	193	1.78	37.8	1.19	21.3	1.14	18.3	2.22	26.4	1.60	24.6	1.19	18.5	1.09	16.1	1.12	**15.7**	1.10	**15.7**
	> 3	57	1.72	40.2	1.38	17.5	1.27	18.3	2.31	33.5	1.66	23.5	1.27	17.6	1.13	16.5	1.14	**16.4**	1.13	**16.4**
$\rho_w f_y$ (MPa)	< 0.70	144	2.02	34.2	1.16	20.3	1.13	16.3	2.37	22.8	1.57	19.0	1.18	16.7	1.07	13.7	1.11	**13.6**	1.09	**13.6**
	0.70-1.50	94	1.52	29.5	1.33	21.3	1.24	20.2	2.20	29.5	1.76	26.4	1.27	20.0	1.16	18.1	1.17	**17.9**	1.14	**17.9**
	> 1.50	19	0.95	30.2	1.25	18.0	1.13	18.7	1.37	30.2	1.29	26.1	1.12	17.2	1.09	16.2	1.08	**16.1**	1.11	**16.1**

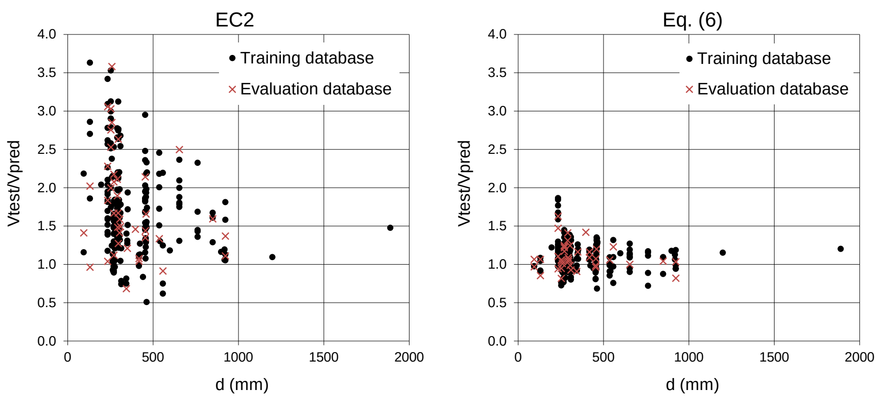


Fig. 7 Correlation between the predictions of the proposed equations and the experimental results in terms of the depth of the beams, compared with the EC-2 predictions

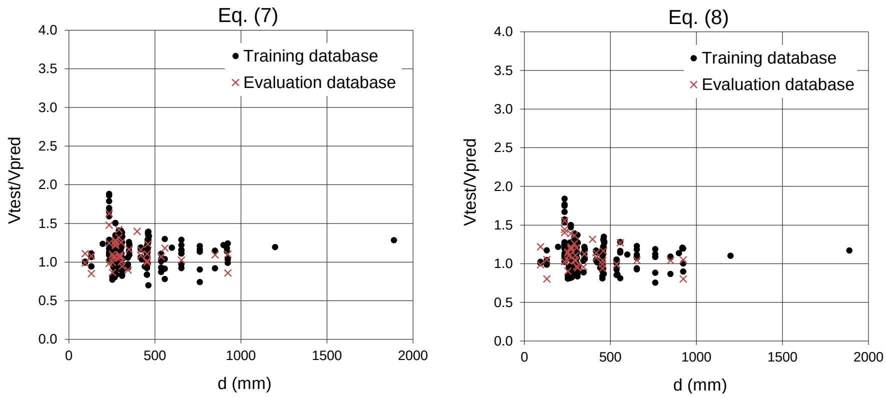


Fig. 7 Continued

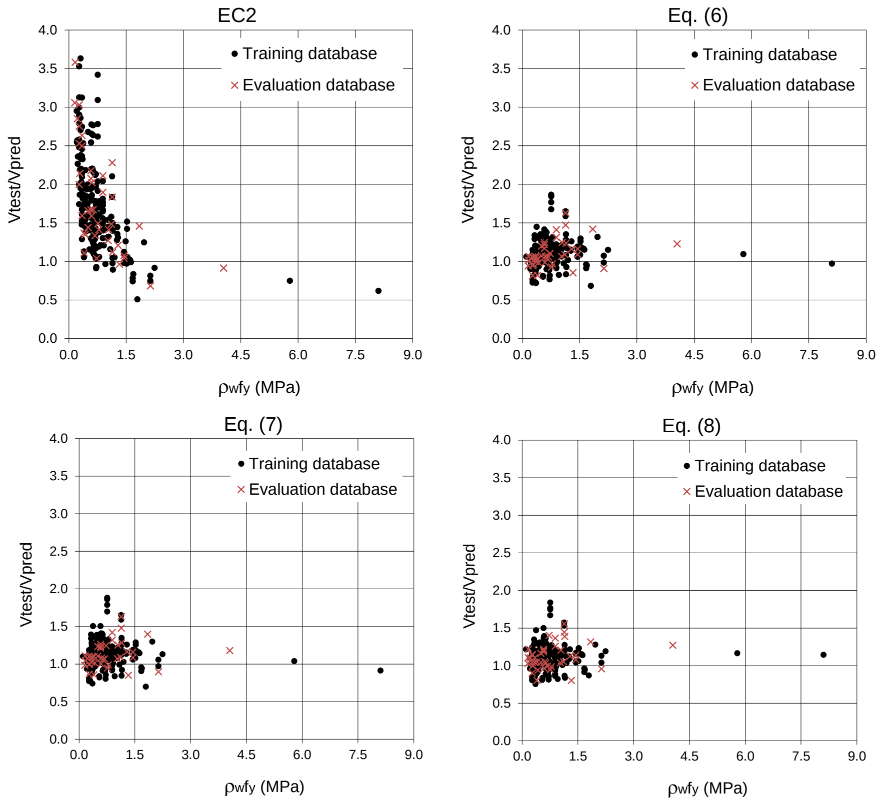


Fig. 8 Correlation between the predictions of the proposed equations and the experimental results in terms of the amount of shear reinforcement and expressed as the nominal stirrup strength, compared with EC-2 predictions

4. Conclusions

A GP algorithm that is valid for the adjustment of existing expressions has been applied to the shear formulation for concrete members with shear reinforcement. The GP algorithm surpasses the mere adjustment of numerical values within an equation; it considers the required precision, accuracy and safety for a design expression. The algorithm is capable of achieving a simple expression that begins as a previous equation. In this case, a mix between the EC-2 and EHE-08 formulations and a concrete contribution was developed using a similar technique. Using a classical GP technique, it would be possible to obtain better precision and accuracy; however, a classical GP technique does not consider safety and simplicity, which are key points in this paper.

Three expressions have been obtained: Eq. (6) is the simplest expression, and Eq. (8) is a slightly more complex equation. Eq. (6) represents a fixed angle truss model with a concrete contribution, and Eqs. (7)-(8) represent a variable angle truss model with a concrete contribution. The three proposed equations yield better predictions of the shear strength of reinforced concrete beams compared with the current international codes of practice, even for a set of beams that has not been used by the GP algorithm.

Although the given methodology has been applied to the optimization of the prediction of the shear strength of beams with stirrups, it can also be applied to any other structural problem, such as Young's modulus determination, tensile concrete strength, and creep or shrinkage.

Acknowledgments

This study was partially supported by the Spanish Ministry of Science and Innovation (Ref. BIA2007-60197 and Ref BIA2010-21551) and grants from the Ministry of Economy and Industry (Consellería de Economía e Industria) of the Xunta de Galicia (Ref. 08TMT005CT and Ref. 10TMT034E).

References

ACI Committee 318 (2008), *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary*, American Concrete Institute, Farmington Hills, MI, USA.

Adebar, P. and Collins, M.P. (1996), "Shear strength of members without transverse reinforcement", *Can. J. Civ. Eng.*, **23**(1), 30-41.

Anderson, N.S. and Ramirez, J.A. (1989), "Detailing of stirrup reinforcement", *ACI Struct. J.*, **86**(5), 507-515.

Angelakos, D., Bentz, E.C. and Collins, M.P. (2001), "Effect of concrete strength and minimum stirrups on shear strength of large members", *ACI Struct. J.*, **98**(3), 290-300.

Bairán, J.M. and Marí, A.R. (2007), "Multiaxial-coupled analysis of RC cross-sections subjected to combined forces", *Eng. Struct.*, **29**(8), 1722-1738.

Bentz, E. (2010), "MC2010: Shear strength of beams and implications of the new approaches", *Fib bulletin 57: Shear and punching shear in RC and FRC elements*, Salò (Italy).

Bhal, N.S. (1968), *Uber den Einfluss der Balkenhöhe auf Schubtragfähigkeit von einfeldrigen Stahlbetonbalken mit und ohne Schubbewehrung*, Über Den Einfluss Der Balkenhöhe Auf Schubtragfähigkeit Von Einfeldrigen Stahlbetonbalken Mit Und Ohne Schubbewehrung.

Bresler, B. and Scordelis, A.C. (1966), "Shear strength of reinforced concrete beams - Series III", *SESM Report*, (2).

Bresler, B. and Scordelis, A.C. (1963), "Shear strength of reinforced concrete beams", *ACI Journal*, **60**(1), 51-74.

Cladera, A. and Marí, A.R. (2007), "Shear strength in the new Eurocode 2. A step forward?", *Struct. Concr.*, **8**(2), 57-66.

Cladera, A. and Marí, A.R. (2005), "Experimental study on high-strength concrete beams failing in shear", *Eng. Struct.*, **27**(10), 1519-1527.
Cladera, A. and Marí, A.R. (2004a), "Shear design procedure for reinforced normal and high-strength concrete beams using artificial neural networks. Part I: Beams without stirrups", *Eng. Struct.*, **26**(7), 917-926.
Cladera, A. and Marí, A.R. (2004b), "Shear design procedure for reinforced normal and high-strength concrete beams using artificial neural networks. Part II: Beams with stirrups", *Eng. Struct.*, **26**(7), 927-936.
Collins, M.P. (1998), "Procedures for calculating the shear response of reinforced concrete elements: A Discussion", *J. Struct. Eng.*, **124**(12), 1485-1488.
Collins, M.P. (2001), *Evaluation of Shear Design Procedures for Concrete Structures*, Report Prepared for the CSA Technical Committee on Reinforced Concrete Design, Ottawa, Canada.
Collins, M.P., Mitchell, D. and Bentz, E.C. (2008), "Shear design of concrete structures", *Struct. Eng.*, **86**(10), 32-39.
Collins, M.P. and Kuchma, D. (1999), "How safe are our large, lightly reinforced concrete beams, slabs, and footings?", *ACI Struct. J.*, **96**(4), 482-490.
Comisión Permanente del Hormigón (2008), *EHE-2008. Instrucción De Hormigón Estructural*, Ministerio de Fomento, Madrid, España.
Da Silva, W.R.L. and Štemberk, P. (2013), "Genetic-fuzzy approach to model concrete shrinkage", *Comput. Concr.*, **12**(2), 109-129.
Darwin, C. (1859), *On the Origin of Species by Means of Natural Selection*, Cambridge University Press, Cambridge, UK.
Dorado, J., Rabuñal, J.R., Rivero, D., Santos, A. and Pazos, A. (2002), "Automatic recurrent ANN rule extraction with genetic programming", *In Neural Networks, IJCNN'02, Proceedings of the 2002 International Joint Conference*, **2**, 1552-1557.
Elzanaty, A.H., Nilson, A.H. and Slate, F.O. (1986), "Shear capacity of reinforced concrete beams using high-strength concrete", *J. Am.Concrete Inst.*, **83**(2), 290-296.
Etxeberria, M., Marí, A.R. and Vázquez, E. (2007), "Recycled aggregate concrete as structural material", *Mater. Struct.*, **40**(5), 529-541.
European Committee for Standardization (CEN). (2002), *Eurocode 2: Design of Concrete Structures: Part 1: General Rules and Rules for Buildings*, European Committee for Standardization.
Féderation International du Beton. (2012), *Model Code 2010, Final Draft*, Féderation International du Beton,
Frosch, R.J. (2000), "Behavior of large-scale reinforced concrete beams with minimum shear reinforcement", *ACI Struct. J.*, **97**(6), 814-820.
Gandomi, A.H., Yun, G.J. and Alavi, A.H. (2013), "An evolutionary approach for modeling of shear strength of RC deep beams", *Mater. Struct.*, **46**(12), 2109-2119.
Gandomi, A.H., Mohammadzadeh, S.D., Pérez-Ordóñez, J.L. and Alavi, A.H. (2014), "Linear genetic programming for shear strength prediction of reinforced concrete beams without stirrups", *Applied Soft Comp.*, 19, 112-120.
González-Fonteboa, B. and Martínez-Abella, F. (2007), "Shear strength of recycled concrete beams", *Constr. Build. Mater.*, **21**(4), 887-893.
Holland, J.H. (1975), *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor, MI (USA).
Johnson, M.K. and Ramirez, J.A. (1989), "Minimum shear reinforcement in beams with higher strength concrete", *ACI Struct. J.*, **86**(4), 376-382.
Jung, S. and Kim, K.S. (2008), "Knowledge-based prediction of shear strength of concrete beams without shear reinforcement", *Eng. Struct.*, **30**(6), 1515-1525.
Karayiannis, C.G., and Chalioris, C.E. (1999), "Experimental investigation of the influence of stirrups on the shear failure mechanism of reinforced concrete beams", *Proceedings of the 13th Hellenic Conference on Concrete*, **1**, 133-141.

Keskin, R.S.O. and Arslan, G. (2013), "Predicting diagonal cracking strength of RC slender beams without stirrups using ANNs", *Comput. Concr.*, **12**(5), 697-715.

Kong, P.Y.L. and Rangan, B.V. (1998), "Shear strength of high-performance concrete beams", *ACI Struct. J.*, **95**(6), 677-688.

Koza, J.R. (1992), *Genetic Programming: On the Programming of Computers by Means of Natural Selection*, MIT Press Cambridge, USA.

Krefeld, W.J. and Thurston, C.W. (1966), "Studies of the shear and diagonal tension strength of simply supported reinforced concrete beams", *ACI Journal*, **63**(4), 451-476.

Leonhardt, F. and Walther, R. (1962), "Schubversuche an Einfeldrigen Stahlbeton-Balken mit und ohne Schubbewehrung zur Ermittlung der Schubtragfhigkeit und der Oberen Schubspannungsgrenze", *Heft 151, Deutcher Ausschuss Fr Stahlbeton*, 66.

Lubell, A., Sherwood, T., Bentz, E. and Collins, M. (2004), "Safe shear design of large, wide beams", *Concr. Int.*, **26**(1), 66-78.

Marí, A. R. and Cladera, A. (2006), "Cálculo a cortante de vigas de hormigón armado y pretensado: una propuesta de cálculo normativo", *Hormigón y Acero*, **242**, 51-63.

Marí, A., Cladera, A., Oller, E. and Bairán, J. (2014), "Shear design of FRP reinforced concrete beams without transverse reinforcement", *Compos Part B: Eng*, **57**, 228-241.

Mattock, A.H. and Wang, Z. (1984), "Shear strength of reinforced concrete members subject to high axial compressive stress", *Journal of the American Concrete Institute*, **81**(3), 287-298.

McGormley, J.C., Cleary, D.B. and Ramirez, J.A. (1996), "The performance of epoxy-coated shear reinforcement", *ACI Struct. J.*, **93**(5), 531-537.

Mphonde, A.G. and Frantz, G.C. (1985), "Shear tests of high- and low-strength concrete beams with stirrups", *High-Strength Concrete*, 179-196.

Navarro Gregori, J., Miguel Sosa, P., Fernández Prada, M.A. and Filippou, F.C. (2007), "A 3D numerical model for reinforced and prestressed concrete elements subjected to combined axial, bending, shear and torsion loading", *Eng. Struct.*, **29**(12), 3404-3419.

Ozcebe, G., Ersoy, U. and Tankut, T. (1999), "Evaluation of minimum shear reinforcement requirements for higher strength concrete", *ACI Struct. J.*, **96**(3), 361-368.

Park, W.J., Noguchi, T. and Lee, H.S. (2013), "Genetic algorithm in mix proportion design of recycled aggregate concrete", *Comput. Concr.*, **11**(3), 183-199.

Pérez, J.L., Cladera, A., Rabuñal, J.R. and Martínez-Abella, F. (2012), "Optimization of existing equations using a new Genetic Programming algorithm: Application to the shear strength of reinforced concrete beams", *Adv. Eng. Software*, **50**(1), 82-96.

Pérez, J.L., Cladera, A., Rabuñal, J.R. and Abella, F.M. (2010), "Optimal adjustment of EC-2 shear formulation for concrete elements without web reinforcement using Genetic Programming", *Eng. Struct.*, **32**(11), 3452-3466.

Placas, A. and Regan, P.E. (1971), "Shear failure of reinforced concrete beams", *J Am. Concrete Inst*, **68**(10), 763-773.

Rajagopalan, K. and Ferguson, P.M. (1968), "Exploratory shear tests emphasizing percentage of longitudinal steel", *ACI Journal*, **65**(8), 634-638.

Regan, P. (1993), "Research on shear: a benefit to humanity or a waste of time? ", *Struct. Eng.*, **71**, 337-337.

Roller, J.J. and Russell, H.G. (1990), "Shear strength of high-strength concrete beams with web reinforcement", *ACI Struct. J.*, **87**(2), 191-198.

Sarsam, K.F. and Al-Musawi, J.M.S. (1992), "Shear design of high- and normal strength concrete beams with web reinforcement", *ACI Struct. J.*, **89**(6), 658-664.

Shah, A. and Ahmad, S. (2007), "An experimental investigation into shear capacity of high strength concrete beams", *Asian Journal of Civil Engineering (Building and Housing)*, **8**(5), 549-562.

Siess, C.P. (1960), "Research, building codes, and engineering practice", *J. Am. Concrete Inst.*, **56**(11), 1105-1122.

Sigrist, V. (2012), "The future of standardization - Guidelines for the development of user-friendly design codes", *Proceedings.56.BetonTage, Neu-Ulm, BFT International*, **78**(2), 166-167.

Swamy, R.N. and Andriopoulos, A.D. (1974), "Contribution of aggregate interlock and dowel forces to the shear resistance of reinforced beams with web reinforcement", *Shear in Reinforced Concrete*, 129-166.
Tan, K.H., Teng, S., Kong, F.K. and Lu, H.Y. (1997), "Main tension steel in high strength concrete deep and short beams", *ACI Struct. J.*, **94**(6), 752-768.
Tan, K., Kong, F., Teng, S. and Guan, L. (1995), "High-strength concrete deep beams with effective span and shear span variations", *ACI Struct. J.*, **92**(4), 395-405.
Tompos, E.J. and Frosch, R.J. (2002), "Influence of beam size, longitudinal reinforcement, and stirrup effectiveness on concrete shear strength", *ACI Struct. J.*, **99**(5), 559-567.
Tsai, H.C. (2013), "Polynomial modeling of confined compressive strength and strain of circular concrete columns", *Comput. Concr.*, **11**(6), 603-620.
Tsai, H.C. and Pan, C.P. (2013), "Improving analytical models of circular concrete columns with genetic programming polynomials", *Genet. Program. Evol. M.*, **14**(2), 221-243.
Tsai, H.C. (2011), "Using weighted genetic programming to program squat wall strengths and tune associated formulas", *Eng. Appl. Artif. Intel.*, **24**(3), 526-533.
Vecchio, F.J. and Collins, M.P. (1986), "The modified compression-field theory for reinforced concrete elements subjected to shear", *ACI J.*, **83**(2), 219-231.
Walraven, J.C. (1981), "Fundamental analysis of aggregate interlock", *ASCE J. Struct. Div.*, **107**(11), 2245-2270.
Yoon, Y., Cook, W.D. and Mitchell, D. (1996), "Minimum shear reinforcement in normal, medium, and high-strength concrete beams", *ACI Struct. J.*, **93**(5), 576-584.
Yu, Q. and Bažant, Z.P. (2011), "Can stirrups suppress size effect on shear strength of RC beams?", *J. Struct. Eng.*, **137**(5), 607-617.
Zararis, P.D. and Papadakis, G. (1999), "Influence of the arrangement of reinforcement on the shear strength of RC beams", *Proceedings of the 13th Hellenic Conference on Concrete*, **1**, 110-119.

CC