Computers and Concrete, *Vol. 13, No. 1 (2014) 135-165*
DOI: http://dx.doi.org/10.12989/cac.2014.13.1.135

Effective compressive strength of strut in CFRP-strengthened reinforced concrete deep beams following ACI 318-11

Mohammad Panjehpour*[1], Abang Abdullah Abang Ali[1], Yen Lei Voo[2] and Farah Nora Aznieta[1]

[1]*Housing Research Centre, Faculty of Engineering, Universiti Putra Malaysia, 43400, Serdang, Selangor, Malaysia*
[2]*Director, DURA Technology Sdn Bhd, Ipoh, Malaysia*

(*Received May 15, 2013, Revised August 5, 2013, Accepted August 31, 2013*)

Abstract. Strut-and-tie model (STM) has been recommended by many codes and standards as a rational model for discontinuity regions in structural members. STM has been adopted in ACI building code for analysis of reinforced concrete (RC) deep beams since 2002. However, STM recommended by ACI 318-11 is only applicable for analysis of ordinary RC deep beams. This paper aims to develop the STM for CFRP strengthened RC deep beams through the strut effectiveness factor recommended by ACI 318-11. Two sets of RC deep beams were cast and tested in this research. Each set consisted of six simply-supported specimens loaded in four-point bending. The first set had no CFRP strengthening while the second was strengthened by means of CFRP sheets using two-side wet lay-up system. Each set consisted of six RC deep beams with shear span to effective depth ratio of 0.75, 1.00, 1.25, 1.50, 1.75, and 2.00.The value of strut effectiveness factor recommended by ACI 318-11 is modified using a proposed empirical relationship in this research. The empirical relationship is established based on shear span to effective depth ratio.

Keywords: strut effectiveness factor; shear failure; D-region; CFRP-strengthening; deep beam

Symbols

a: shear span of deep beam (mm)
d: effective depth of deep beam (mm)
$P_{u\text{-}ordinary}$: ultimate shear strength of ordinary RC deep beam from the test (kN)
$P_{u\text{-}CFRP}$: ultimate shear strength of CFRP strengthened RC deep beam from the test (kN)
I: increase ratio; $(P_{u\text{-}CFRP}-P_{u\text{-}ordinary})/P_{u\text{-}ordinary}$
f_c' : specified concrete compressive strength (MPa)
f_{ce} : effective compressive strength of strut (MPa)
υ : strut effectiveness factor

1. Introduction

The behaviour of reinforced concrete (RC) deep beam has been the topic of interest among

*Corresponding author, Ph.D., E-mail: mdpanjehpour2006@yahoo.com

http://www.techno-press.org/?journal=cac&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

researchers since the last two decades. The increasing applications of RC deep beam in offshore structures, tall building, and foundation have made deep beam an important member of construction industry (Azam and Soudki 2012, Campione and Minafò 2012, Kim *et al*. 2011, Lee *et al*. 2011, Mohammadhassani *et al*. 2011, Mohammadhassani *et al*. 2012, Pal and Deswal 2011, Zhang and Tan 2010). There are many codes and standards which define deep beam according to the value of shear span-depth ratio (AASHTO 2012, ACI 2011, AS3600 2009, CAN/CSA-S6-06 2006, CSA-A23.3-04 2005, DIN 2001, Eurocode2 2005, NZS 2006). In this research, definition of deep beam based on ACI 381-11 is the major concern. The regions of beam with concentrated loads within twice the member depth from the support are considered as deep beam in ACI 318-11. However, ACI 318-11 emphasises that the forgoing region must be loaded on one face and supported on the opposite face (ACI 2011). This is because the strut is developed between the loads and supports. Hence, the strut and its effectiveness factor are the main focus of this study.

Apart from the wide application of RC deep beam in building and construction, there is still an issue needed to be addressed regarding the ultimate shear strength of RC deep beam. According to the prior research addition of web reinforcement beyond the minimum amount in RC deep beam provides only a marginal strength increase (Islam *et al*. 2005). It reflects the importance of external strengthening of RC deep beam using advance material.

Strengthening and repair of concrete structures with carbon fibre reinforced polymer (CFRP) has become a focus among researchers since the last decade (Abdelouahed 2006, Dias and Barros 2011, Ha *et al*. 2013, Krour *et al*. 2013, Panjehpour *et al*. 2011; Panjehpour *et al*. 2012). The properties of having high tensile strength and low weight, being corrosion-resistant, and bringing ease of installation make CFRP a competitive material against the steel (Al-Safy *et al*. 2013, Al-Zubaidy *et al*. 2012). For strengthening purpose, some researchers believe that steel material will be replaced by CFRP, for its unrivalled properties in the future mass production (Ueda *et al*. 2012). Application of CFRP in different forms of sheet, plate, and bar in RC beams has grown in importance in light of groundbreaking research conducted on this topic from different aspects (Ha *et al*. 2008, R. He *et al*. 2013, Sayed Ahmad *et al*. 2011, Sharbatdar *et al*. 2011, Tounsi and Benyoucef 2007, Tounsi *et al*. 2009).

The application of CFRP for strengthening of RC beams is on the increase (Ahmed and Sobuz 2011, Anil *et al*. 2012, Bulut *et al*. 2011, Hawileh *et al*. 2011, Jnaid and Aboutaha 2013, Panda *et al*. 2013, Shrestha *et al*. 2013). Hence several attempts have hitherto been made to propose a method for the prediction of ultimate strength of CFRP strengthened RC beams (Benyoucef *et al*. 2007, Rizzo and De Lorenzis 2009). Nonetheless, no research has yet been proposed a method for prediction of shear strength of CFRP strengthened RC deep beams. Therefore, a rational method is needed to predict the shear strength of CFRP strengthened RC deep beam for design purposes.

Strut-and tie model (STM) has been recommended as a rational approach for analysis of discontinuity-region (D-region) in many codes and standards (AASHTO 2012, ACI 2011, AS3600 2009, CAN/CSA-S6-06 2006, DIN 2001, Eurocode2 2005). The behaviour of strut in D-region is the main focus of researchers regarding the STM (Amini *et al*. 2013, Li and Tran 2012). Numerous studies have been conducted on STM from various conceivable angles (Bruggi 2009, He and Liu 2010, Khalifa 2012, Kwak and Noh 2006, Panjehpour *et al*. 2012, Panjehpour *et al*. 2012, Perera and Vique 2009, Tjhin and Kuchma 2007, Zhang and Tan 2007). ACI 318-11 in the section R11.7.2 has recommended the STM for design of deep beam regardless of how it is loaded and supported (ACI 2011). However, the ACI building code had recommended a simple equation for analysis and design of deep beam in the 1999 and earlier codes. It implies the applicability of STM for design and analysis of deep beam with respect to nonlinear behaviour of deep beam.

This study lays emphasis on the behaviour of strut among three main parts of STM; strut, tie, and node. Strut crossed by cracks inclined to the axis of the strut are weakened by the cracks (Wight and Macgregor 2009). The compressive strength of strut is reduced due to softening behaviour of concrete while RC deep beam is under the loading. The amount of reduction of strut compressive strength; along its centreline; is presented by strut effectiveness factor which is the main concern of this research. The recommended strut effectiveness factor by ACI 318-11 is defined as $0.85ß_s$. The struts located such that the width of the midsection of the strut is larger than the width at the nodes is considered as bottle-shaped strut in ACI 318-11. The value of $ß_s$ for bottle-shaped strut is recommended as 0.75 (ACI 2011). Crucially, the bottle-shaped strut is considered in this study for the experimental simple supported RC deep beams.

Recent developments in CFRP strengthening of RC structural members have heightened the need for rational methods and models for design purposes of the forgoing members. So far, no research has been conducted to propose a rational method or model for prediction of shear strength of CFRP strengthened RC deep beams for design purposes. Therefore, the main purpose of this study is to develop the STM for prediction of shear strength of RC deep beams strengthened with CFRP sheet through the strut effectiveness factor recommended by ACI 318-11. Hence, the experimental programme in this research aims to assess the effect of CFRP strengthening of RC deep beam on the value of strut effectiveness factor in STM. The scope of current research is confined to the CFRP strengthened RC deep beams using one layer of unidirectional CFRP sheet on two sides of beams.

2. Experimental program

In this experimental programme, the key variable was shear span to effective depth ratio (*a/d*) ranging from 0.75 to 2.00. The experimental specimens consisted of two groups of RC deep beams. The former group comprises six ordinary RC deep beams which have *a/d* of 0.75, 1.00, 1.25, 1.50, 1.75, and 2.00. The latter group comprises six RC deep beams with the same properties of former group which were strengthened by means of CFRP sheets with two-side wet lay-up system. The CFRP sheets fully covered the shear span of each beam on both sides. The two-side strengthening system was selected in this research among the three common methods of CFRP strengthening of beams: two-side, U-jackets, and Full wrapping. This is because the two-side strengthening is more common and applicable in construction industry for its ease of installation. However, two-side installation system does not provide the anchorage for CFRP sheet, unlike U-jacket and full wrapping installation systems. Therefore, this experimental programme aims to investigate and assess the effect of CFRP sheet strengthening without anchorage on the behaviour of inclined RC strut in deep beams.

2.1 Details of deep beams

A total of 12 RC deep beams with a rectangular cross section were designed and manufactured in this research. Dimension of all RC deep beams were as follows: a width of 140 mm, overall depth of 350 mm, and longitudinal length of 1840 mm as illustrated in Fig. 1. Nine deformed steel bars with a diameter of 16 mm were placed in three layers at the bottom of the deep beams as the flexural reinforcement. The steel bars were welded to 10 mm thick steel plates at both ends of the RC deep beams to provide an adequate anchorage capacity. Two anchorage steel plates covered the width of RC deep beams fully at both ends in height of 120 mm as shown in Fig. 1.

Web reinforcement consisted of vertical and horizontal deformed bars; in form of mesh; with spacing of 100 mm were provided according to the requirement of ACI 318-11 in section 11.7.4.2. This reinforcement satisfied the required minimum amount of web reinforcement recommended by ACI 318-11 (ACI 2011). Additional steel reinforcements were placed under the load plates and atop the support plates to prevent any premature local failure. The transverse reinforcement is not shown in Fig. 1 to avoid disordering.

2.2 Strengthening materials and methods

The RC deep beams were cast using single supply of ready-mixed concrete. The tensile strength of the reinforcing steel bars (T16) from the ultimate tensile strength test according to ASTM-E8 was 440 *MPa* while the cylindrical compressive and splitting tensile strengths of concrete were 37.02 *MPa* and 3.31 *MPa* respectively. The properties of CFRP sheets and epoxy used to bond the CFRP sheets on the concrete surface are listed in Table 1.The CFRP sheets and epoxy used in this study were provided by Sika Company with product data sheet of sikadur-330 and sikaWrap-230. One layer of unidirectional woven carbon fibre fabric having thickness of 0.111 mm/ply with two-part epoxy impregnation resin were utilised to strengthen the RC deep beams

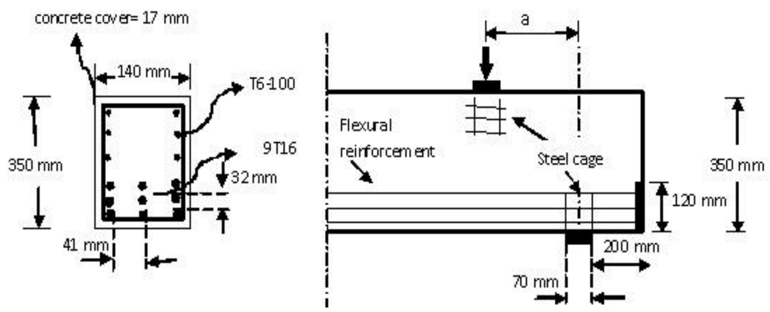

Fig. 1 Beam cross section and reinforcement details for the half of beam length

Fig. 2 Experimental test rig

Table 1 Typical properties of CFRP sheets and epoxy

Materials	Tensile strength (*MPa*)	Tensile modulus of elasticity (*GPa*)	Elongation at break	Bond strength (*MPa*)	Thickness (*mm/ply*)
CFRP sheet	3900	230	1.5% (7days at +23℃)	-	0.111
Epoxy resin	30	4.5	0.9% (7days at +23℃)	>4	-

with wet lay-up system. The surface of concrete was rubbed using cup brush to remove all debris and prepare a suitable surface for installing CFRP sheets. Then a layer of epoxy resin was coated to provide bond in interface of CFRP sheet and concrete. After installing CFRP sheet, a pre-heated ribbed roller was used to impregnate the sheets in the adhesion layer. The strengthening with CFRP sheet was only carried out on the surface of beams between the load plate and the support plate only to cover the shear span of RC deep beams. The curing time for CFRP strengthening was at least two days following the manufacturer recommendation at ambient temperature. The support plates and load plates which were 70 mm in width and 10 mm in thickness fully covered the bottom and top of the beam. The deep beams were tested at 28 days after casting.

2.3 Test procedure and instrumentation

Two groups of RC deep beams were tested till failure in a four-point bending set up as shown in Fig. 2. The applied load was uniformly increased till failure with an increment of 50 kN using a hydraulic actuator with a capacity of 5000 kN. The cracks widths of ordinary RC deep beams were measured using portable microscope in each step of loading until 90% of failure load. Two digital LVDTs were used at both sides of beams to measure the mid-span deflections.

3. Experimental results and discussion

The behaviour of RC deep beams is evaluated through the behaviour of D-region in this section. The non-linear behaviour of D-region is directly affected by shape of strut as strut represents concrete compression stress fields. The strut in this research is bottle-shaped among three common shapes of prismatic, bottle-shaped, and compression fan. This is because of using simple supports for RC deep beams in this study. The bottle-shaped strut varies in cross section along its length. This is because the concrete stress fields are wider at mid length of strut than at the ends.

This section of paper investigates and assesses the relationship between ultimate shear strength of RC deep beams with shear span-depth ratio using the experimental results and STM recommended by ACI 318-11. Consequently, an empirical relationship is recommended to modify the value of the strut effectiveness factor of CFRP strengthened RC deep beams with respect to ACI 318-11. The partial rupture of CFRP sheet was mainly dominated by the failure mode of the two-side CFRP strengthened deep beams as shown in Fig. 3(b). The failure of ordinary and CFRP strengthened RC deep beams with a/d = 0.75 is illustrated in Figs. 3(a), (b).

(a) Ordinary beam (b) CFRP strengthened beam
Fig. 3 Typical failure of strut in RC deep beams with *a/d*=0.75

3.1 Theoretical method of ACI 318-11 regarding STM

Using alternative STM calculation through CAST design program results the shear capacity of the ordinary RC deep beams with *a/d* ranging in 0.75-2.00 as presented in Table 2 (Kuchma and Tjhin 2001). Once inclined cracking commences to propagate, an RC deep beam with shear reinforcement behaves as a truss. The concrete between two alternative inclined cracks and shear reinforcement act as strut and tie in the truss respectively. Fig. 4 illustrates the STM for ordinary RC deep beam with *a/d* = 0.75 as a sample of six ordinary RC deep beams which were tested under a four-point bending set up in this research.

Three components of STM: strut, tie, and node are indicated for RC deep beam using various colours in Fig. 4. The stabiliser is used to establish the truss modelling in STM for RC deep beam. It is used as STM element to avoid ill-conditioned structure stiffness matrix in the truss analysis. In addition, the stabiliser is used to create a stable STM, and it is not included in dimensioning STM nodes. The force transferred by stabilizer element is zero, and it is used in truss model design to avoid mechanism occurrence.

The strut effectiveness factor recommended by ACI 318-11 shows that effective compressive strength of inclined strut is approximately 37% lower than cylindrical compressive strength of concrete. The strength reduction of strut is owing to the softening behaviour of concrete. However, various codes and standards give different values for strut effectiveness factor. The value of strut effectiveness factor recommended by ACI 318-11 is considered in this paper. The Eq. (1) presents the effective compressive strength of strut.

$$\mathrm{f_{ce}} = \upsilon\, \mathrm{f_c'} \tag{1}$$

Thus, in this study,

$$\mathrm{f_{ce}} = 0.63 \times 37.02 = 23.20\, MPa$$

The stress field of inclined strut is shown in red in Fig. 4. It means that among three main parts of STM, the strut has reached to its effective compressive strength (23.20 MPa) earlier than the other parts, and failed. Therefore, failure of RC deep beam occurred due to the failure of strut.

Fig. 5 illustrates the ultimate shear strength of two groups which are ordinary and CFRP strengthened RC deep beams from the experiment. According to Fig. 5, as the shear span to effective depth of RC deep beams increases, the ultimate shear strength increases. The details of Fig. 5 reveal that the shear strength of CFRP strengthened RC deep beams increases with the increase of *a/d*. Nevertheless, the slope of regression line fitted to the data indicates that the increase of shear strength of CFRP strengthened deep beams is faster than those of ordinary deep beams. In other words, the effect of CFRP strengthening on the ultimate shear strength of deep beams moderately increases as the shear span to effective depth ratio increases. The secondary struts play an important role to sustain the compressive load applied to the main strut for those RC beams having *a/d* greater than the value of 2. However, some negligible secondary struts occur in D-region even for those RC deep beams having *a/d* lower than the value of 2. As the CFRP sheets cover all these struts after the strengthening, it maybe said that the imposed load is transferred to more secondary struts with increase of *a/d*. Consequently, more parts of CFRP sheets contribute to sustain the tensile strain regarding the secondary struts. Therefore, the contribution of CFRP sheets for strengthening RC deep beams with high *a/d* is more than those of having low *a/d*.

Looking from another point of view, CFRP sheet is not fully active before CFRP strengthened deep beam reaches the ultimate shear strength of ordinary deep beam under the applied load. This is because the tensile stress perpendicular to strut centreline is sustained by reinforced concrete before the RC deep beam reaches its failure load. Meanwhile, only part of tensile stress is transferred to CFRP sheet from cracked parts of concrete. Therefore, maybe CFRP sheet is fully active whilst CFRP strengthened RC deep beam is loaded beyond the ultimate shear strength of ordinary RC deep beams. Accordingly, when the ultimate shear strength of ordinary RC deep beam

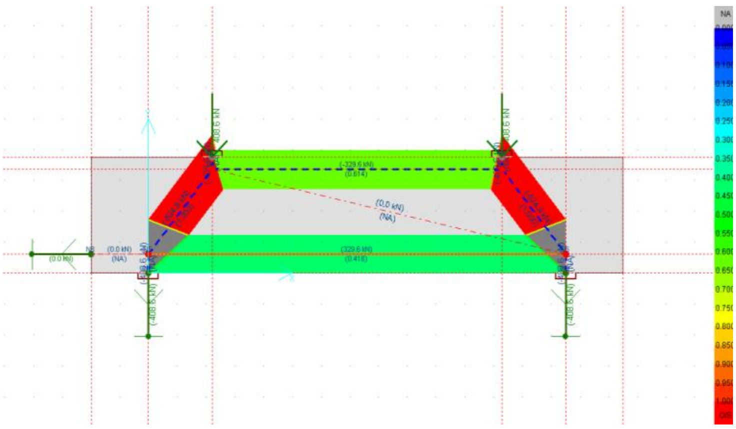

Fig. 4 STM from ACI 318-11 method for RC deep beam with *a/d*=0.75 using CAST design program

Table 2 Ultimate shear strength of ordinary and CFRP strengthened RC deep beams from the test

a/d	$P_{u\text{-ordinary}}$ *(kN)*	$P_{u\text{-FRP}}$ *(kN)*	I
0.75	756.95	905.31	0.19
1.00	709.01	857.89	0.21
1.25	604.08	740.02	0.22
1.50	555.91	691.04	0.24
1.75	403.02	510.01	0.27
2.00	360.02	468.05	0.30

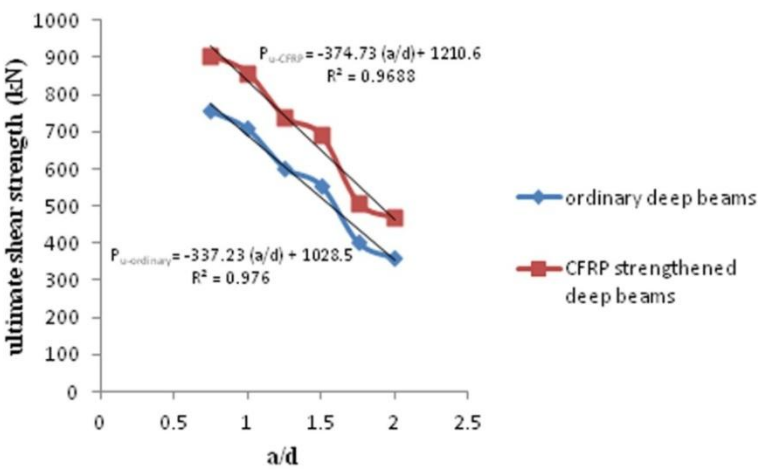

Fig. 5 Shear strength of ordinary and CFRP strengthened RC deep beams from the test

is high, CFRP sheet transfers the high shear load in its full active status, and vice versa. Thus, CFRP sheet sustains greater applied force in RC deep beams with high value of *a/d* compared to those with low *a/d*. In short, CFRP sheet proportionally contributes more in RC deep beams with high shear span to effective depth ratio compared to those of lower ratios, as indicated in Table 2.

The CFRP strengthening of RC deep beams leads to strengthening D-region and to RC strut subsequently. The value of strut effectiveness factor is increased by CFRP strengthening of strut. This paper aims to propose the value of modified strut effectiveness factor for CFRP strengthened RC deep beams with a/d ranging in 0.75-2.00. This issue is discussed in the next section of the paper.

3.2 Effective strength of strut

Struts crossed by cracks inclined to the axis of the strut are weakened by the cracks. The type of strut is bottle-shaped in this experiment. The bottle-shaped strut in this study contains the reinforcement crossing the potential splitting cracks. This strut tends to split longitudinally, but the opening of splitting cracks is restrained by the web reinforcement. The web reinforcement allows strut to carry additional load after propagation of cracks in strut (Wight and Macgregor 2009). Fig. 6 illustrates the idealised STM with compressive and tension loads regarding this experiment. However, the load directions are not as simple as illustration in Fig. 6 after installation of CFRP sheet.

The complexities of load direction in strut as well as the strain incompatibility cause difficulty in the analysis of strut behaviour. However, the CFRP strengthening of D-region in RC deep beams exacerbate this issue as CFRP sheet restrains crack propagation. Installation of CFRP sheet on the surface of strut illustrated in Fig. 6 affects the transverse strains in strut. The RC strut is partly strengthened by CFRP sheet in RC deep beams. Since the strut is not fully wrapped by CFRP sheet, it is impossible to simulate the strut behaviour with CFRP wrapped cylindrical concrete specimens. Hence, the experimental investigation is requisite to address the issue of behaviour of the CFRP strengthened RC strut. This study aims to investigate and assess the behaviour of strut in CFRP strengthened RC deep beams using an experimental test. The effective compressive strength of strut is evaluated through strut effectiveness factor. Thus, an empirical equation is integral to predict the value of strut effectiveness factor in CFRP strengthened RC deep beams.

The crushing strength of the concrete in a strut is referred to the effective strength as shown in Eq. (1).Various research have given different values of the strut effectiveness factor (Schlaich and Schafer 1991) and (M.Rogowsky and G.MacGregor 1986) through (Ramirez and E.Breen September-October 1991). The key factors affecting strut effective strength are concrete strength , load duration, tensile strain transverse to the strut, and cracked struts (Wight and Macgregor 2009).

The grade of concrete is in the range of ordinary concrete in this experimental study. Nonetheless, the effect of CFRP strengthening on the tensile strains and crack developments is investigated using the experimental results.ACI building code has recommended STM for analysis of RC deep beams since 2002. The load duration effects are accounted for in the ACI code by modifying the strut effectiveness factor using β_s as shown in Eq. (2).

$$\upsilon = 0.85\beta_s \tag{2}$$

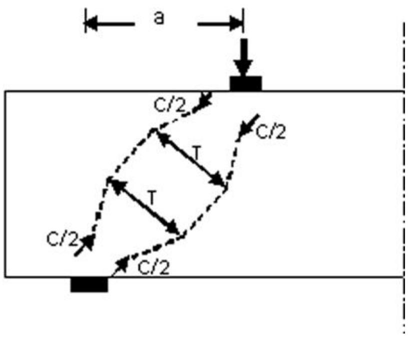

Fig. 6 Idealised bottle-shaped strut with compressive and tension forces

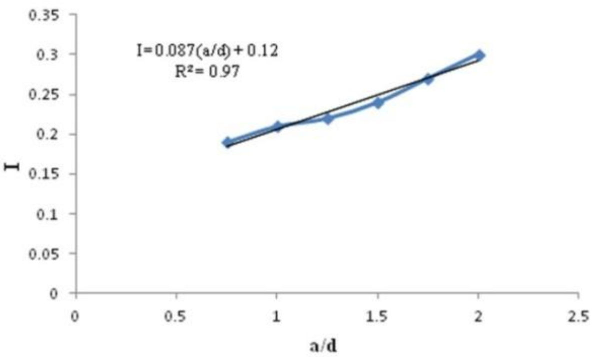

Fig. 7 Empirical relationship between I and *a*/*d*

ACI 318-11 has recommended the value of $ß_s$ for reinforced strut as 0.75 in the section A3.3 (ACI, 2011). The strut effectiveness factor recommended by ACI 318-11 is simpler and more applicable than that of recommended by AASHTO LRFD. Therefore, the effective compressive strength of strut proposed by ACI 318-11 is considered in this paper. Fig. 7 illustrates an empirical relationship between a/d and increase ratio of ultimate shear strength of RC deep beams by CFRP strengthening. Thus, the strut effectiveness factor recommended by ACI 318-11 is modified using the aforementioned empirical relationship as shown below.

$$\upsilon_{CFRP} = 0.85\beta_s (I+1) \tag{3}$$

$$I = 0.087(a/d) + 0.12$$

4. Conclusions

This paper has reviewed the latest research conducted on the CFRP strengthening of RC deep beams. This research aims to modify the value of strut effectiveness factor for the strut in CFRP strengthened RC deep beams with respect to ACI 318-11. Hence, an empirical relationship is proposed to modify the value of strut effectiveness factor recommended by ACI 318-11 for those of RC deep beam strengthened by CFRP sheet. Based on this research, the following conclusions can be drawn:

- The value of strut effectiveness factor is predicted through an empirical relationship proposed in this research.
- The strut-and-tie model can be used to predict the ultimate shear strength of CFRP strengthened RC deep beams through the proposed strut effectiveness factor in this research.
- CFRP strengthening of RC deep beams increases the value of strut effectiveness factor from 19.60% to 30.02% for a/d from 0.75 to 2.00 respectively.
- The effect of CFRP strengthening on the ultimate shear strength of RC deep beams increases moderately as the shear span to effective depth ratio increases. It may be due to the effects of secondary struts.
- CFRP sheet is not fully active before CFRP strengthened RC deep beam reaches the ultimate shear strength of ordinary RC deep beam under the applied load. CFRP sheet is fully active whilst CFRP strengthened RC deep beam is loaded beyond the ultimate shear strength of ordinary RC deep beams.

The scope of this research is confined to the RC deep beams strengthened with one layer of CFRP sheet on two sides. It would be interesting to assess the effect of size of CFRP strengthened RC deep beams on the behaviour of strut in further research

Acknowledgments

The authors acknowledge the support from the Housing Research Centre (HRC), Universiti Putra Malaysia as well as Dura Company for facilitating the experimental work. The author[1] is deeply indebted to Taw Ly Wen from UPM for her comments on the written English.

References

AASHTO (2012), *LRFD, bridge design specifications, customary U.S. units: 2012 interim revisions* (6 ed.). Washington: American Association of State Highway and Transportation Officials.

Abdelouahed, T. (2006), Improved theoretical solution for interfacial stresses in concrete beams strengthened with FRP plate, *Int. J. Solids Struct.*, **43**(14-15), 4154-4174.

ACI (2011), Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary, Section 11.7, (pp. 183-184).

Ahmed, E. and Sobuz, H.R. (2011), "Experimental study on long-term behaviour of CFRP strengthened RC Beams under sustained load", *Struct. Eng. Mech.*, **40**(1).

Al-Safy, R., Al-Mahaidi, R. and Simon, G.P. (2013), "A study of the use of high functionality-based resin for bonding between CFRP and concrete under harsh environmental conditions", *Compos. Struct.*, **95**(0), 295-306.

Al-Zubaidy, H., Al-Mahaidi, R. and Zhao, X.L. (2012), "Experimental investigation of bond characteristics between CFRP fabrics and steel plate joints under impact tensile loads", *Compos. Struct.*, **94**(2), 510-518.

Amini, H., Najafian and Vollum, R.L. (2013), "Automated nonlinear design of reinforced concrete D regions", *Struct. Eng. Mech.*, **46**(1).

Anil, O., Bulut, N. and Ayhan, M. (2012), "Strain distribution between CFRP strip and concrete at strengthened RC Beam against shear", *Struct. Eng. Mech.*, **41**(4).

AS3600 (2009), Australian standard for Concrete structures: Standard Association of Australia, North sydney, pp. 198.

Azam, R. and Soudki, K. (2012), "Structural performance of shear-critical RC deep beams with corroded longitudinal steel reinforcement", *Cement Concrete Compos.*, **34**(8), 946-957.

Benyoucef, S., Tounsi, A. and Adda Bedia, E.A. and Meftah, S.A. (2007), "Creep and shrinkage effect on adhesive stresses in RC beams strengthened with composite laminates", *Compos. Sci. Tech.*, **67**(6), 933-942.

Bruggi, M. (2009), "Generating strut-and-tie patterns for reinforced concrete structures using topology optimization", *Comput. Struct.*, **87**(23-24), 1483-1495.

Bulut, N., Anil, Ö. and Belgin, Ç.M. (2011), "Nonlinear finite element analysis of RC beams strengthened with CFRP strip against shear", *Comput. Concr.*, **8**(6).

Campione, G. and Minafò, G. (2012), "Behaviour of concrete deep beams with openings and low shear span-to-depth ratio", *Eng. Struct.*, **41**(0), 294-306.

CAN/CSA-S6-06 (2006), *Canadian highway bridge design code and S6.1-06 commentary on CAN/CSA-S6-06*, *Canadian Highway Bridge Design Code*: Association canadienne de normalisation.

CSA-A23.3-04 (2005), *Technical Committee on Reinforced Concrete Design*. *A23.3-04 Design of Concrete Structures*: Canadian Standards Association.

Dias, S.J.E. and Barros, J.A.O. (2011), "Shear strengthening of RC T-section beams with low strength concrete using NSM CFRP laminates", *Cement Concrete Compos.*, **33**(2), 334-345.

DIN (2001), *Building and Civil Engineering Standards Committee*. *Plain*, *Reinforced and Prestressed Concrete Structures*, *Part 1: Design and Construction (DIN 1045-1)*. Berlin, Germany: Deutsches Institut für Normung (DIN-Normen).

Eurocode2. (2005), *EN 1992-2:2005*, *Design of concrete structures - Part 2: Concrete bridges - Design and detailing rules*.

Ha, G.J., Cho, C.G., Kang, H.W. and Feo, L. (2013), "Seismic improvement of RC beam–column joints using hexagonal CFRP bars combined with CFRP sheets", *Compos. Struct.*, **95**(0), 464-470.

Ha, G.J., Kim, Y.Y. and Cho, C.G. (2008), "Groove and embedding techniques using CFRP trapezoidal bars for strengthening of concrete structures", *Eng. Struct.*, **30**(4), 1067-1078.

Hawileh, R.A., Abdalla, J.A., Tanarslan, M.H. and Naser, M.Z. (2011), "Modeling of nonlinear cyclic response of shear-deficient RC T-beams strengthened with side bonded CFRP fabric strips", *Comput. Concr.*, **8**(2).

He, R., Grelle, S., Sneed, L.H. and Belarbi, A. (2013), "Rapid repair of a severely damaged RC column having fractured bars using externally bonded CFRP", *Compos*. *Struct*., **101**(0), 225-242.

He, Z.Q. and Liu, Z. (2010), "Optimal three-dimensional strut-and-tie models for anchorage diaphragms in externally prestressed bridges", *Eng*. *Struct*., **32**(8), 2057-2064.

Islam, M.R., Mansur, M.A. and Maalej, M. (2005), "Shear strengthening of RC deep beams using externally bonded FRP systems", *Cement Concrete Compos*., **27**, 413-420.

Jnaid, F. and Aboutaha, R. (2013), "Review of design parameters for FRP-RC members detailed according to ACI 440.1R-06", *Comput*. *Concr*., **11**(2).

Khalifa, E.S. (2012), Macro-mechanical strut and tie model for analysis of fibrous high-strength concrete corbels, *Ain Shams Eng. J.*, **3**(4), 359–365

Kim, H.S., Lee, M.S. and Shin, Y. S. (2011), "Structural behaviors of deep RC beams under combined axial and bending force", *Procedia Engineering*, **14**(0), 2212-2218.

Krour, B., Bernard, F. and Tounsi, A. (2013), "Fibers orientation optimization for concrete beam strengthened with a CFRP bonded plate: A coupled analytical–numerical investigation", *Eng*. *Struct*., **56**(0), 218-227.

Kuchma, D.A. and Tjhin, T.N. (2001), *CAST* (*Computer Aided Strut-and-Tie*) *Design Tool* Paper presented at the Structures 2001 : A Structural Engineering Odyssey.

Kwak, H.G. and Noh, S.H. (2006), "Determination of strut-and-tie models using evolutionary structural optimization", *Eng*. *Struct*., **28**(10), 1440-1449.

Lee, H.K., Cheong, S.H., Ha, S.K. and Lee, C.G. (2011), "Behavior and performance of RC T-section deep beams externally strengthened in shear with CFRP sheets", *Compos*. *Struct*., **93**(2), 911-922.

Li, B. and Tran, C.T.N. (2012), "Determination of inclination of strut and shear strength using variable angle truss model for shear-critical RC beams", *Struct*. *Eng*. *Mech*., **41**(4).

M.Rogowsky, D. and G.MacGregor, J. (1986), "Design of deep reinforced concrete continuous beams", *Concrete Int: Des*. *Construct*., **8**(8), 49-58.

Mohammadhassani, M., Jumaat, M.Z., Ashour, A. and Jameel, M. (2011), "Failure modes and serviceability of high strength self compacting concrete deep beams", *Eng*. *Fail*. *Anal*., **18**(8), 2272-2281.

Mohammadhassani, M., Jumaat, M.Z. and Jameel, M. (2012), "Experimental investigation to compare the modulus of rupture in high strength self compacting concrete deep beams and high strength concrete normal beams", *Constr*. *Build*. *Mater*., **30**(0), 265-273.

NZS (2006), *Concrete Design Committee P 3101 for the Standards Council*. *Concrete Structures Standard: Part 1-The Design of Concrete Structures (NZS 3101-1)*, Wellington: Standards New Zealand.

Pal, M. and Deswal, S. (2011), "Support vector regression based shear strength modelling of deep beams", *Comput*. *Strcut*., **89**(13-14), 1430-1439.

Panda, K.C., Bhattacharyya, S.K. and Barai, S.V. (2013), "Shear strengthening effect by bonded GFRP strips and transverse steel on RC T-beams", *Struct*. *Eng*. *Mech*., **47**(1).

Panjehpour, M., Ali, A.A.A., anwar, M.P., Aznieta, F.N. and Voo, Y. L. (2012), "An overview of strut-and-tie model and its common challenges", *Int. J. Eng. Res. in Africa*, **8**, 37-45.

Panjehpour, M. and Ali, A.A.A., Voo, Y.L. and Aznieta, F.N. (2012), STRUT ELABORATION IN STRUT-AND-TIE MODEL. *ConstructII*, *2*.

Panjehpour, M., Farzadnia, N., Anwar, M.P. and Ali1, A.A.A. (2011), "FRP sheet contribution in common repair techniques of concrete structures with emphasis on concrete columns", *Int. J. Sustain. Constr. Eng. Technol.*, **2**(2), 54-61.

Panjehpour, M., Rashid, R.S.M., Aznieta, F.N. and Ali, A.A.A. (2012), A review for characterisation of FRP composite in the concrete structures, Paper presented at the 11th International Conference on Concrete Engineering and Technology 2012 (CONCET).

Perera, R. and Vique, J. (2009), "Strut-and-tie modelling of reinforced concrete beams using genetic algorithms optimization", *Constr*. *Build*. *Mater*., **23**(8), 2914-2925.

Ramirez, J. and E.Breen, J. (September-October 1991), "Evaluation of a modified truss-model approach for beams in shear", *ACI Sttuct*. *J*., **88**(5), 562-571.

Rizzo, A. and De Lorenzis, L. (2009), "Behavior and capacity of RC beams strengthened in shear with NSM

FRP reinforcement", *Constr*. *Build*. *Mater*., **23**(4), 1555-1567.
Sayed Ahmad, F., Foret, G. and Le Roy, R. (2011), "Bond between carbon fibre-reinforced polymer (CFRP) bars and ultra high performance fibre reinforced concrete (UHPFRC): Experimental study", *Construct*. *Build*. *Mater*., **25**(2), 479-485.
Schlaich, J. and Schafer, K. (1991), "Design and detailing of structural concrete using strut-and-tie models", *Struct*. *Eng*., **69**(6), 13.
Sharbatdar, M.K., Saatcioglu, M. and Benmokrane, B. (2011), "Seismic flexural behavior of concrete connections reinforced with CFRP bars and grids", *Compos*. *Struct*., **93**(10), 2439-2449.
Shrestha, R., Smith, S.T. and Samali, B. (2013), "Finite element modelling of FRP-strengthened RC beam-column connections with ANSYS", *Comput*. *Concr*., **11**(1).
Tjhin, T.N. and Kuchma, D.A. (2007), "Integrated analysis and design tool for the strut-and-tie method", *Eng*. *Struct*., **29**(11), 3042-3052.
Tounsi, A. and Benyoucef, S. (2007), "Interfacial stresses in externally FRP-plated concrete beams", *Int*. *J*. *Adhesion Adhesives*, **27**(3), 207-215.
Tounsi, A., Hassaine Daouadji, T., Benyoucef, S. and Adda bedia, E.A. (2009), "Interfacial stresses in FRP-plated RC beams: Effect of adherend shear deformations", *Int*. *J*. *Adhesion Adhesives*, **29**(4), 343-351.
Ueda, M., Miyake, S., Hasegawa, H. and Hirano, Y. (2012), "Instantaneous mechanical fastening of quasi-isotropic CFRP laminates by a self-piercing rivet", *Compos*. *Struct*., **94**(11), 3388-3393.
Wight, J.K. and Macgregor, J.G. (2009), Reinforced Concrete Mechanics and Desig*n*, United States: Pearson Prentice Hall.
Zhang, N. and Tan, K.H. (2007), "Direct strut-and-tie model for single span and continuous deep beams", *Eng*. *Struct*., **29**(11), 2987-3001.
Zhang, N. and Tan, K.H. (2010), "Effects of support settlement on continuous deep beams and STM modeling", *Eng*. *Struct*., **32**(2), 361-372.

CC