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Predictions of curvature ductility factor of doubly reinforced concrete beams with high strength materials

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Abstract. The high strength materials have been more widely used in reinforced concrete structures because of the benefits of the mechanical and durable properties. Generally, it is known that the ductility decreases with an increase in the strength of the materials. In the design of a reinforced concrete beam, both the flexural strength and ductility need to be considered. Especially, when a reinforced concrete structure may be subjected an earthquake, the members need to have a sufficient ductility. So, each design code has specified to provide a consistent level of minimum flexural ductility in seismic design of concrete structures. Therefore, it is necessary to assess accurately the ductility of the beam sections with high strength materials in order to ensure the ductility requirement in design. In this study, the effects of concrete strength, yield strength of reinforcement steel and amount of reinforcement including compression reinforcement on the complete moment-curvature behavior and the curvature ductility factor of doubly reinforcement concrete beam sections have been evaluated and a newly prediction formula for curvature ductility factor of doubly RC beam sections has been developed considering the stress of compression reinforcement at ultimate state. Based on the numerical analysis results, the proposed predictions for the curvature ductility factor are verified by comparisons with other prediction formulas. The proposed formula offers fairly accurate and consistent predictions for curvature ductility factor of doubly reinforced concrete beam sections.

Keywords: doubly RC beam; ductility factor; high strength material; moment-curvature curve; numerical analysis; predictions

1. Introduction

Recently, the high strength materials are being more widely used in reinforced concrete structures. With the rapid advancement in concrete technology, the high strength concrete with up to 100 MPa is being used in reinforced concrete buildings. Also, in the design codes, the upper limit for yielding strength of reinforcing steel has specified up to about 600 MPa (CEN 2004, ACI 2008,Korea Concrete Institute 2007). In the design of a reinforced concrete beam, both the flexural strength and ductility need to be considered. The high strength concrete provides significantly better structural engineering properties including higher compressive and tensile strengths, higher stiffness, better durability compared with normal strength concrete (Mendis 2003).

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Using high strength materials, mainly high strength concrete, it is possible to reduce the size and weight of reinforced concrete members, but the concrete with very high compressive strength can be results in a less ductile response of the structural members. When a reinforced concrete structure may be subjected an earthquake, if the member has a sufficient ductility, a plastic hinge may be form at critical sections of the members, which can absorb more excessive energy through inelastic deformation before the resistant capacity of moment decreases significantly. Therefore, the ductility of reinforced concrete members is a desirable property where resistance to brittle failure during flexure is required to ensure structural integrity. It is especially critical to reinforced concrete members manufactured using high strength concrete that is essentially more brittle than normal strength concrete. Inseismic design of earthquake resistant concrete structureswith high strength concrete, it is necessary to possess a sufficient ductility in flexural members.

In order to provide a consistent level of minimum flexural ductility, each design code has specified in different methods. Eurocode (CEN 2004) and British Code (BSI1997) specify the limits of neutral axis of compressive stress block according to the cube strength of concrete in order to guarantee the provisions of minimum flexural ductility and American Code (ACI 2008) and Korean Code (Korea Concrete Institute 2007) specify the minimum tension steel strain at ultimate state as 0.004 or 2ε_y in order to achieve sufficient ductility. For the same limits of neutral axis, it is reported by several researchers (Ho *et al*. 2004, Bai and Au 2011) that the values of curvature ductility for the beam sections with normal strength concrete and high strength concrete are different. Also, it has been founded that the flexural ductility of a reinforced beam sections is dependent not only on an amount of reinforcement including compression reinforcement, but also the concrete compressive strength and the steel yield strength. Therefore, it is necessary to evaluate accurately the ductility of the beam sections with high strength materials in order to ensure the ductility requirement in design.

The present study attempts to develop the newly prediction formula for the curvature ductility factor of the doubly reinforced beam sections with high strength materials. A numerical analysis for full- range moment curvature of beam sections has been conducted to evaluate the effects of various structural parameters such as concrete strength, steel yield strength, tension and compression reinforcement ratios on the curvature ductility and a predicting formula for the curvature ductility factor of the doubly reinforced beam sections is proposed based on the regression analysis of the numerical results.

2. Previous research

The researches on the curvature ductility and the prediction equation of ductility factor of unconfined reinforced concrete (RC) beam sections using high strength concrete have been conducted by many researchers (Pendyala*et al.* 1996, Pam *et al.* 2001a, 1b, Kwan *et al.* 2002, 2006, Ho *et al.* 2003, 2004, Rashid and Mansur 2005,Jang *et al.* 2008, Au *et al.* 2009, Maghsoudi and Sharifi 2009, Arslan and Cihanli 2010, Bai and Au 2011, Au *et al.* 2011). Pam *et al.* (2001a) experimentally studied the flexural strength and ductility of singly reinforced normal- and high-strength concrete beams and developed a simple formula for predicting the curvature ductility of beam sections as following.

$$\mu_{\varphi} = 9.5\left(f_{cu}\right)^{-0.30}\left(\rho/\rho_b\right)^{-0.75} \quad (1)$$

where f_{cu} is the cube compressive strength of concrete (MPa), ρ is the tension steel ratio and ρ_b isthe balanced steel ratio of the beam section. The Eq. (1) is derived based on the experimental results for the beams with concrete strength of 30 ~ 100 MPa and yield strength of steel of 519 ~ 579 MPa. Ho *et al.* (2004) investigated the influence of the parameters including concrete strength on the flexural ductility of doubly reinforced concrete beam sections and proposed the maximum values of tension steel ratio and neutral axis corresponding to the minimum curvature ductility factor for the beams with the concrete strength of 40 ~ 100 MPa and the steel yield strength of 250, 460 and 600 MPa. Rashid and Mansur (2005) tested twelve reinforced concrete beams with concrete strength of 43 ~ 126 MPa, ratios of tensile and compressive reinforcement and spacing of lateral ties as the main parameters and reported the influences of the parameters on the cracking and ultimate moment capacity, the deflection and crack width, and the flexural ductility index. Jang *et al.* (2008) evaluated the ductility of reinforced high-strength concrete beams by 23 flexural tests for beam having concrete compressive strength of 40, 60 and 70 MPa and steel yield strength of 400-450 MPa.Arslan and Cihanli (2010) proposed the curvature ductility prediction equation such as Eq. (2) for unconfined singly RC beam sections based on the regression analysis of the numerical analysis results for the beams with concrete strength of 50 ~ 110 MPa, yield strength of reinforcement of 220, 420 and 530MPa, and tensile reinforcement ratio from 0.0059 ~ 0.0708.

$$\mu_\varphi = 40\left(\rho/\rho_b\right)^{-1.18}\left(f_{ck}\right)^{-0.17}\left(f_y\right)^{-0.42} \tag{2}$$

where f_{ck} is the concrete strength (MPa), f_y is the yield strength of reinforcement (MPa), ρ is the tensile reinforcement ratio, and ρ_b is the balanced steel ratio of the beam section.

Pam *et al.* (2001b) investigated the post-peak behavior and the flexural ductility of doubly reinforced high strength concrete beams and developed a simple formula for predicting the curvature ductility of beam sections as follows

$$\mu_\varphi = 10.7\left(f_{ck}\right)^{-0.45}\left[\frac{\rho-\rho'}{\rho_b}\right]^{-1.25} \times \left[1+95.2\left(f_{ck}\right)^{-1.1}\left(\rho'/\rho\right)^3\right] \tag{3}$$

where f_{ck} is the compressive strength of concrete (MPa), ρ andρ' are the tension and compression steel ratio, respectively, and ρ_b is the balanced steel ratio of the beam section without compression reinforcement. The Eq. (3) is derived by regression analysis for numerical results for the beams with concrete strength of 30 ~ 100 MPa and yield strength of steel of 460 MPa. Bai and Au (2011) investigated the influence of the parameters such as the concrete strength of 30 ~ 90 MPa, steel yield strength of 300 ~ 450 MPa, and tension and compression reinforcement ratios on the flexural ductility of doubly RC beam sections, and proposed the empirical the formula such as Eq. (4) and tables about the relation of the maximum difference of tension and compression reinforcement ratio$(\rho-\rho')_{max}$ and the minimum curvature ductility factor μ_{min} in order to facilitate flexural ductility design of RC beams.

$$\left(\rho-\rho'\right)_{max} = 9.5\left(\frac{2.88 f_{ck}}{f_y^{1.35}} + \frac{83}{f_y^{1.32}}\right) \times \mu_{min}^{-\left(0.005 f_y f_{ck} + 18 f_{ck} + 1.9 f_y + 5{,}447\right)/10^4} \tag{4}$$

where f_{ck} is the concrete compressive strength (MPa), f_y is the steel yield strength (MPa), ρ and ρ'

are the tension and the compression steel ratio, respectively. Au *et al.* (2011) evaluated the effects of steel content, yield strength and degree of prestressing on the yield curvature and ultimate curvature for reinforced and prestressed concrete sections.Lam *et al.* (2009a, 2009b) evaluated the effects of concrete strength, axial load level, confining pressure and longitudinal steel ratio the flexural ductility of reinforced concrete column using nonlinear moment-curvature analysis.

As stated above, in the case of singly RC beam section with high strength concrete, the equation for the prediction has been proposed by several researchers. In the case of doubly RC beam section with high strength concrete, the prediction formula for curvature ductility factor has rarely been proposed. Pam *et al.* (2001b) proposed the prediction formula for curvature ductility such as Eq. (3), however, this formula does not involve the effect of the yield strength of reinforcement, although the yield strength of reinforcement affects fairly on the curvature ductility of RC beams sections. Therefore, it is necessary to develop the prediction formula for curvature ductility considered the effects of the yield strength of reinforcement in order to predict actually the curvature ductility of doubly RC beam section using the high strength materials.

3. Analysis of moment-curvature relationship and curvature ductility factor

3.1 moment-curvature relationship

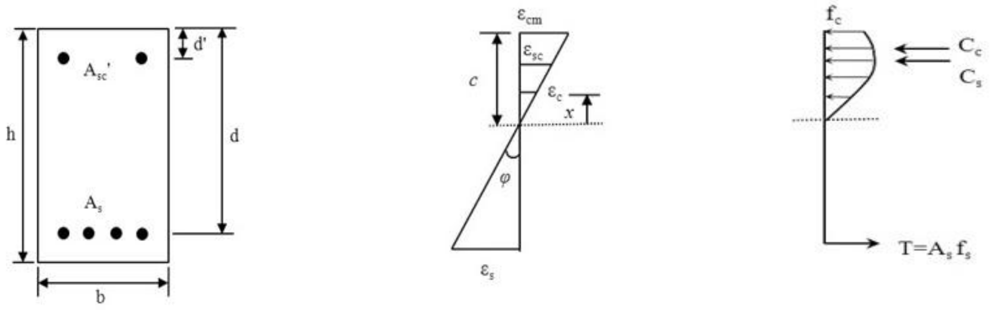
Theoretical moment-curvature relationships for reinforced concrete beam sections with flexure are derived on the basis of assumptions similar to those used for determination of the flexural strength (Park and Paulay 1975). It is assumed that plane sections before bending remain plane after bending and the stress-strain curves for concrete and steel are known. Additionally, it is assumed that the tensile stress in concrete may be neglected and concrete and steel is perfectly bonded. The curvature associated with a range of bending moment may be determined using these assumptions and from the requirements of strain compatibility and force equilibrium.

Fig. 1 shows a RC beam section with flexure. The calculation procedures of moment-curvature relationship are follows. For any concrete strain in the extreme compression fiber ε_{cm}, assume the neutral axis depth c, then the concrete strain ε_c at x and the steel strain ε_s and ε_{sc} are determined from similar triangles of the strain diagram. The corresponding stresses f_c, f_s and f_{sc} developed in the concrete, compression and tension steel, respectively, can then be evaluated from the respective stress-strain curves of materials. The compressive concrete force C_c, the compressive steel force C_s, and tensile steel force T are calculated and the axial equilibrium condition is checked. If the equilibrium condition is satisfied, the resisting moment M can be obtained from moment equilibrium condition about the neutral axis.

The corresponding curvature φ of the beam is defined by considering a small deformation as follows

$$\varphi = \frac{\varepsilon_{cm}}{c} \tag{5}$$

where ε_{cm} is the concrete strain in the extreme compression fiber and c is the neutral axis depth from the top. By carrying out repeated calculation of this procedure until the concrete strain in the extreme compression fiber ε_{cm} reaches the ultimate concrete strain ε_{cu}, then the full-range moment-curvature curve of a RC beam section is obtained.



(a) Cross section of RC beam (b) Strain (c) Stress and forces

Fig. 1 Theoretical moment-curvature determination

3.2 Curvature ductility factor

The ductility of a structural member may be defined as its ability to deform at or near the failure load without a significant loss in its load-carrying capacity. In the case of a flexural member, sectional ductility based on curvature and/or member ductility based on deflection is usually considered. Generally, the deflection ductility (member ductility) is more difficult to calculate accurately than the curvature ductility (sectional ductility). The curvature ductility factor is defined as the ratio of the curvature at ultimate state to the curvature at yielding of the tensile reinforcing steel (Park and Paulay 1975). The curvature ductility factor μ_φ of a RC beam section may be expressed as

$$\mu_\varphi = \frac{\varphi_u}{\varphi_y} \tag{6}$$

where φ_u is the curvature at ultimate state and φ_y is the curvature when the tension reinforcement first reaches the yielding strength. The ultimate state is defined when the concrete strain in the extreme compression fiber reaches a specified limiting value.

The value of curvature ductility factor is influenced by a definition of ultimate state because the value of ultimate curvature depends on a definition of ultimate state. Many researches for studying on the prediction of the curvature ductility factor of RC beam sections have been conducted (Pam *et al*. 2001a, Kwan *et al*. 2002, Arslan and Cihanli 2010, Bai and Au 2011). The ultimate states in each research are defined as different forms. In researches by Pam *et al*. (2001a) and Kwan *et al*. (2002), the ultimate curvature is taken as the curvature at which the resisting moment has, after reaching the peak, dropped to 0.80 of the peak moment and the yield curvature is defined as the curvature at the hypothetical yield point of an equivalent elasto-plastic system whose equivalent elastic stiffness is taken as the secant stiffness at 0.75 of the peak moment before the peak moment is reached. In research by Bai and Au (2011), the ultimate curvature is defined as the curvature of the section when its resisting moment has dropped to 85% of the peak moment after reaching the peak and the yield curvature is taken as that at the hypothetical yield point of an equivalent linearly elastic-perfectly plastic system with an elastic stiffness equal to the secant stiffness of the section at 75% of the peak moment. In research by Arslan and Cihanli (2010), the ultimate curvature φ_u is taken as the curvature when the extreme compression fiber of the unconfined concrete core reaches a strain capacity ε_{cu} expressed as Eq. (7)

$$\varepsilon_{cu} = 0.003 + 1.44\frac{1}{f_{ck}^{\,2}} + 0.00054\left(\frac{\rho'}{\rho}\right) \tag{7}$$

where f_{ck} is the compressive strength of concrete, ρ and ρ' are the tension and compression reinforcement ratio, respectively. In research by Bai and Au (2011), the ultimate curvature φ_u is defined as the curvature of the section when its resisting moment has dropped to 85% of the peak moment after reaching the peak and the yield curvature φ_y is taken as that at the hypothetical yield point of an equivalent linearly elastic-perfectly plastic system with an elastic stiffness equal to the secant stiffness of the section at 75% of the peak moment.

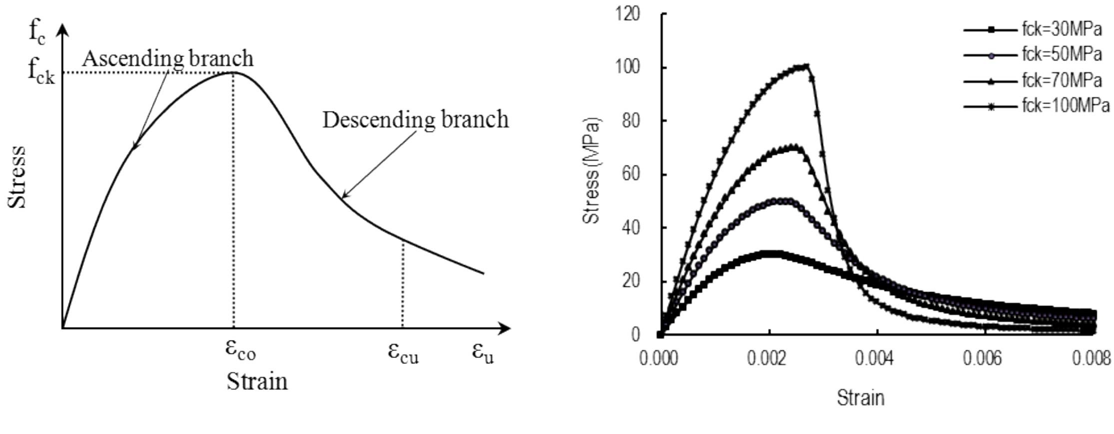
In each design code, when the resisting moment of RC beams in flexure design is evaluated, the ultimate state is defined as a specified limiting value of the concrete strain in the extreme compression fiber. In Eurocode (CEN 2004), the compressive strain in the concrete shall be limited to ε_{cu} depending on the compressive strength of concrete as Eq. (8)

$$\varepsilon_{cu} = 0.0035, \qquad for\ f_{ck} \le 50MPa \tag{8a}$$

$$\varepsilon_{cu} = 0.0026 + 0.035\left[\left(90 - f_{ck}\right)/100\right]^4, \quad for\ f_{ck} > 50MPa \tag{8b}$$

where f_{ck} is the compressive strength of concrete (MPa). In American Code (ACI 2008) and Korean Code (Korea Concrete Institute 2007), the ultimate state is defined as the concrete strain in the extreme compression fiber reaches a specified limiting value of 0.003 without regard to the compressive strength of concrete.

In this study, in practical design of RC beams made of high-strength materials according to ACI or KCI design code, in order to provide a relevant data about ductility, the ultimate curvature φ_u is taken as the curvature when the concrete strain in the extreme compression fiber reaches 0.003.



(a) Plot of stress-strain model (b) Typical stress-strain curve

Fig. 2 Stress-strain curves for concrete in compression

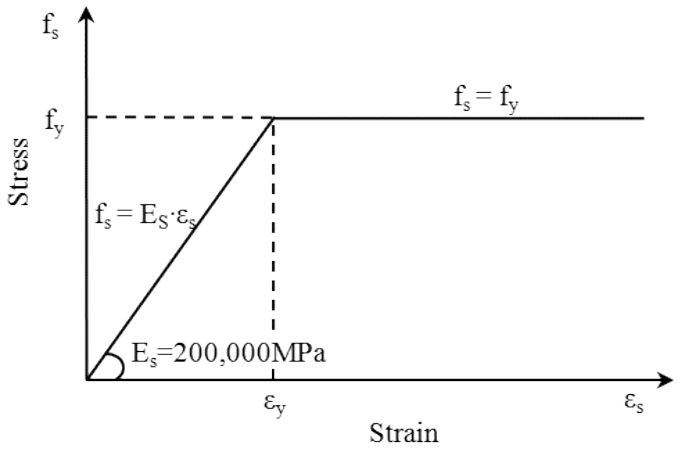


Fig. 3 Stress-strain curves of reinforcing steel

3.3 Stress-strain relationship of materials

3.3.1 Concrete

The stress-strain curve of concrete effects on the flexural behavior of RC beams. So, it is important to know the details of stress-strain relationship of high strength concrete in order to determine the full-range behavior of RC beams. Various stress-strain curves for normal- and high-strength concrete under uniaxial compression have been proposed in the literature. In unconfined concrete, the stress-strain relationships for normal and high strength concrete very different in the strain at the peak stress and ductility. In this study, for the ascending branch, the stress-strain curve of concrete proposed by Hognestad, which is widely used, is employed. And, for the descending branch and the strain at peak stress, the stress-strain curve developed by Attard and Setunge (1996), which is applicable to a broad range of concrete strength from 20 to 130 MPa, is employed in order to consider the post-peak behavior of high strength concrete. Fig. 2(a) shows the stress-strain model of concrete. The stress–strain relationships of concrete used in this study are as follows.

For the ascending branch where $\varepsilon_c \le \varepsilon_{co}$

$$f_c = f_{ck}\left[2\frac{\varepsilon_c}{\varepsilon_{co}} - \left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] \tag{9}$$

For the descending branch where $\varepsilon_c > \varepsilon_{co}$

$$f_c = f_{ck}\left[A\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right) + B\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] \Big/ \left[1 + (A-2)\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right) + (B+1)\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] \tag{10}$$

$$\varepsilon_{co} = 4.11\left(f_{ck}\right)^{0.75} / E_c \tag{11}$$

$$E_c = 4,370\left(f_{ck}\right)^{0.52} \tag{12}$$

$$A = \frac{f_{ci}(\varepsilon_{ci} - \varepsilon_{co})^2}{\varepsilon_{co}\varepsilon_{ci}(f_{ck} - f_{ci})}; \quad B = 0 \tag{13}$$

$$f_{ci} = f_{ck}\left\{1.41 - 0.17\ln(f_{ck})\right\} \tag{14}$$

$$\varepsilon_{ci} = \varepsilon_{co}\left\{2.50 - 0.30\ln(\varepsilon_{co})\right\} \tag{15}$$

where f_{ck} is the concrete compressive strength (MPa), ε_{co} is the strain of concrete at peak stress. Fig. 2(b) shows a plot of some typical stress-strain curves of concrete with the compressive strength from 30 MPa to 100 MPa, which is calculated by Eqs. (9) - (10).

3.3.2 Reinforcing steel

In this study, for the stress-strain curve of reinforcing steel with yield strength of 300-600MPa, a simple bilinear model as shown in Fig. 3 is employed. The strain hardening of the steel in the stress-strain curve is not taken into account and it is idealized an identical behavior of the reinforcing steel in tension and compression. When the steel strain ε_s increase, the stress f_s in the steel is calculated by

$$f_s = E_s\varepsilon_s, \qquad \textit{for} \;\; \varepsilon_s < \varepsilon_y = f_y / E_s \tag{16a}$$

$$f_s = f_y, \qquad \textit{for} \;\; \varepsilon_s \geq \varepsilon_y \tag{16b}$$

where E_s and f_y are the Young's modulus and yield strength of reinforcing steel, respectively.

4. Numerical analysis for ductility of RC beam sections

4.1 Sections analysed and variables

The beam sections analysed are rectangular type as shown in Fig. 1(a). A typical beam section has a width b = 300 mm and total depth h = 600 mm, and the compression and tension reinforcement is provided at depth d' = 50 mm and d = 550 mm from the top, respectively. For parametric study, the cylindrical compressive strength of concrete f_{ck} is varied from 30 MPa to 100 MPa to cover both normal- and high-strength concretes, the yield strength of reinforcing steel f_y ranges 300 MPa to 600 MPa to cover all type of reinforcement steel and Young's modulus of reinforcement steel E_s = 200 GPa. The tension reinforcement ratio ρ ($\rho = A_s/bd$) is varied 10.0% to 100.0% of the balanced reinforcement ratio (ρ_b) for all of singly and doubly reinforced concrete beam sections, and the compression reinforcement ratio ρ' ($\rho' = A_{sc}/bd$) for doubly reinforced concrete beam sections is varied 0.0% to 100.0% of the tension reinforcement ratio (ρ). It is assumed that steel yield strength of tension and compression reinforcement is the same value.

The balanced reinforcement ratio (ρ_b) for a singly reinforced beam section has been obtained from simplified equivalent rectangular concrete stress block as follows (ACI 2008, Korea Concrete Institute 2007) and the results calculated are listed in Table 1.

Table 1 Balanced reinforcement ratios for different strengths of materials

Type		Compressive strength of concrete f_{ck} (MPa)							
		30	40	50	60	70	80	90	100
Yield strength oftension reinforcement f_y(MPa)	300	0.047	0.058	0.066	0.074	0.086	0.098	0.111	0.123
	350	0.038	0.047	0.053	0.060	0.070	0.080	0.090	0.100
	400	0.032	0.039	0.044	0.050	0.058	0.066	0.075	0.083
	450	0.027	0.033	0.038	0.042	0.049	0.056	0.063	0.070
	500	0.023	0.028	0.032	0.036	0.042	0.048	0.054	0.060
	550	0.020	0.025	0.028	0.031	0.037	0.042	0.047	0.052
	600	0.018	0.022	0.025	0.028	0.032	0.037	0.041	0.046

4.2 Moment-curvature curves

The theoretical moment and curvature for RC beam sections are calculate by the method described previously. Fig. 4 shows some theoretical moment-curvature curves of the singly reinforced concrete beam sections with concrete strength f_{ck} = 70 MPa, steel yield strength f_y = 400 MPa and different tension reinforcement ratio (ρ). In the case of singly reinforced concrete beam sections, the moment-curvature curve is almost linear before the peak moment is reached, but the moment-curvature curves after peak moment have different shapes according to tension reinforcement ratio. In the case of beam sections with relatively low tension reinforcement ratio, the moment-curvature curve has lower stiffness at peak stage and a fairly long yield plateau at the post-peak stage before the resisting moment drops more rapidly until complete failure. However, in the case of beam sections with relatively high tension reinforcement ratio, the plateau of curve becomes gradually shorter and the curve becomes sharper around the peak moment indicating remarkable reduction in ductility. Comparing the moment-curvature curves for different steel ratio, it can be seen that the tension reinforcement ratio for singly RC beam sections basically affects both the shape of moment-curvature curve and the ductility of a beam section. Fig. 5 shows some selected moment-curvature curves of the singly RC beam sections with steel yield strength f_y = 400 MPa and tension reinforcement ratio ρ = 0.75ρ_b for different concrete strengths f_{ck}. In the case of beam sections with high strength concrete, the moment-curvature curve has higher stiffness at peak stage and a short plateau at the post-peak stage, and the curves become sharper around the peak moment indicating drastic reduction in ductility. In this case, the higher strength of concrete, the higher stiffness before post-peak and longer plateau after post-peak of the moment-curvature curves. Which means that the increase of concrete strength leads to the increase of ductility of beam sections provided the same tension reinforcement. Fig. 6 shows some selected moment-curvature curves of the singly RC beam sections with compressive concrete strength f_{ck} = 70 MPa and tension reinforcement ratio ρ = 0.75ρ_b for different yield strengths of reinforcement f_y. In the case of beam sections with high yield strength of steel, the moment-curvature curve has lower stiffness at peak stage and a long plateau at post-peak stage.

Fig. 7 shows some selected moment-curvature curves of the doubly reinforced concrete beam sections with concrete strength f_{ck} = 70 MPa, steel yield strength f_y= 400 MPa and tension reinforcement ratio ρ = 0.75ρ_b for the different compression reinforcement ratio (ρ'). In the case of doubly reinforced concrete beam sections, also, the moment-curvature curves are almost linear before the peak moment is reached, but the moment-curvature curves after peak moment have different shapes according to compression reinforcement ratio. The increase of compression

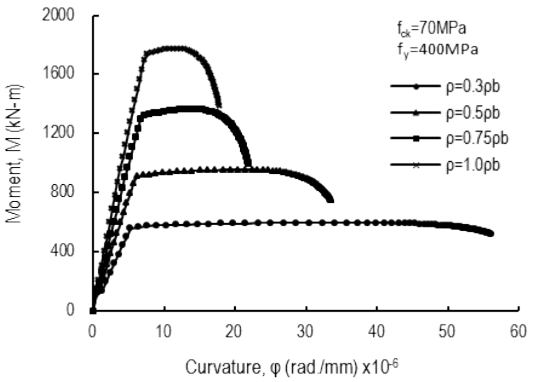


Fig. 4 Moment-curvature curves for singly RC beam sections with different tension steel ratio

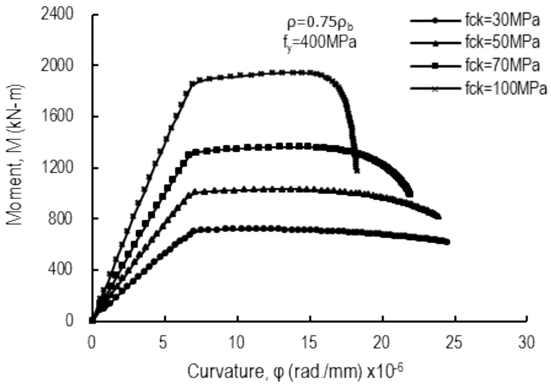


Fig. 5 Moment-curvature curves for singly RC beam sections with different strength of concrete

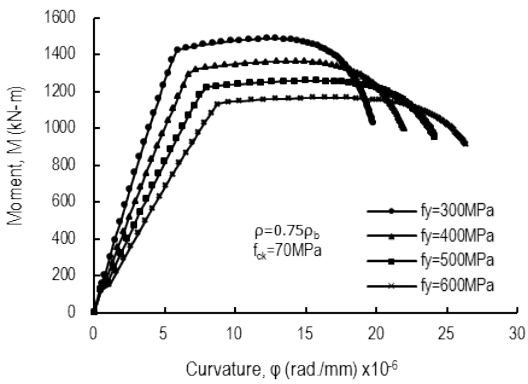


Fig. 6 Moment-curvature curves for singly RC beam sections with different yield strength of steel

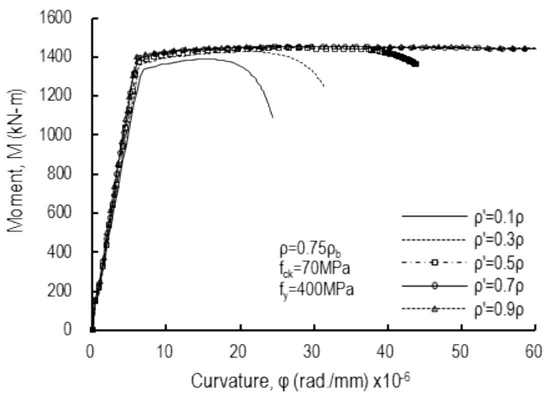


Fig. 7 Moment-curvature curves for doubly RC beam sections with different compressive reinforcement ratio

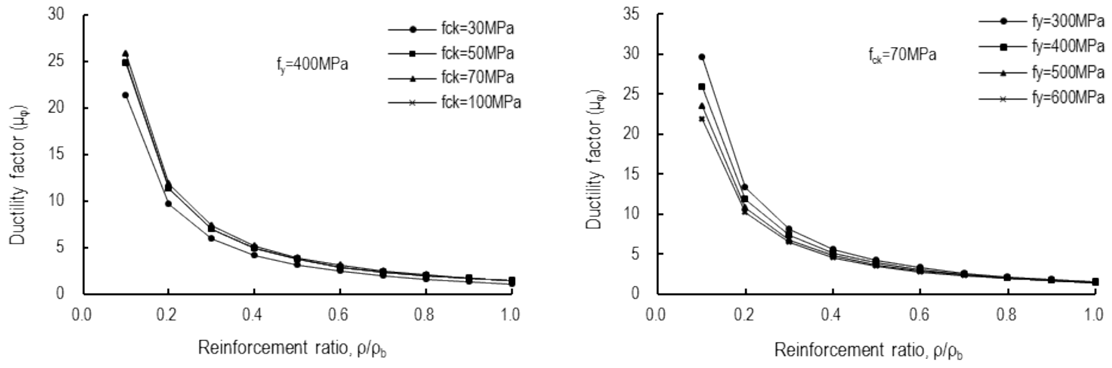
reinforcement ratio leads to a small increase of ultimate moment capacity, but a long plateau after post-peak stage indicating remarkable increase in ductility. In the case of doubly RC beam sections, also, the effects of the yield strength of reinforcement on the moment-curvature curve are similar to those in singly RC beam sections.

It is thus evident that the concrete strength, the reinforcement yield strength, and the ρ/ρ_b ratio in a singly RC beam section and, in addition to, the compression reinforcement ratio (ρ') in a doubly RC beams section have certain effects on the moment-curvature curve and the flexural ductility of reinforced concrete beam sections.

4.3 Curvature ductility factor of reinforced concrete beam sections

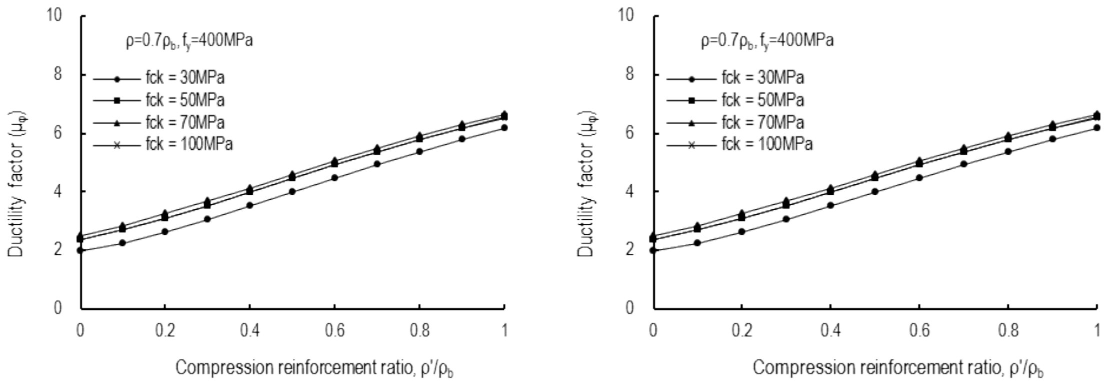
The curvature ductility factor μ_φ of RC beam sections is calculated by Eq. (6). The ultimate curvature μ_φ is taken the curvature when the concrete strain in the extreme compression fiber reaches 0.003 by American Code (ACI 2008) and Korean Code (Korea Concrete Institute 2007).

Fig. 8(a) and (b) show a plot of curvature ductility factor as a function of tension reinforcement

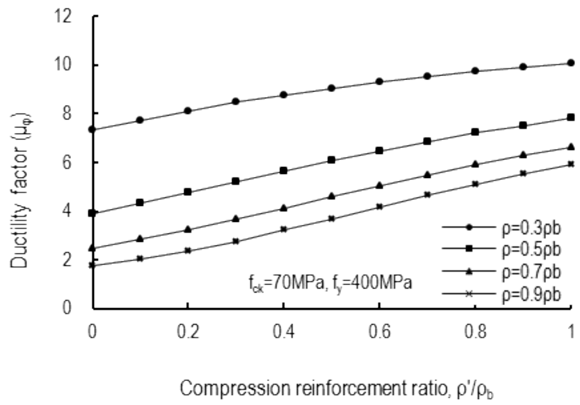


(a) f_y = 400 MPa (b) f_y = 400 MPa

Fig. 8 Ductility factor μ_φ of singly RC beam sections against tension reinforcement ratio



(a) $\rho = 0.7\rho_b$, f_y = 400MPa (b) $\rho = 0.7\rho_b$, f_{ck} = 70 MPa



(c) f_{ck} = 70MPa, f_y = 400MPa

Fig. 9 Ductility factor μ_φ of doubly RC beam sections against compression reinforcement ratio

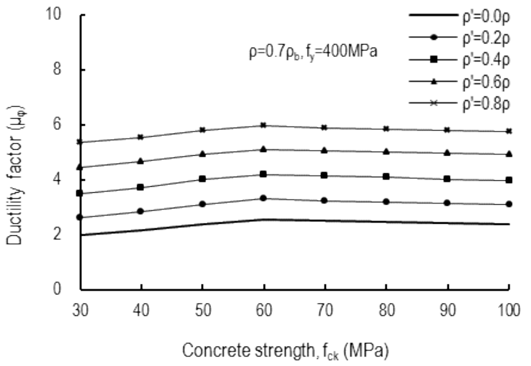


Fig. 10 Ductility factor μ_φ of doubly RC beamsections against concrete strength

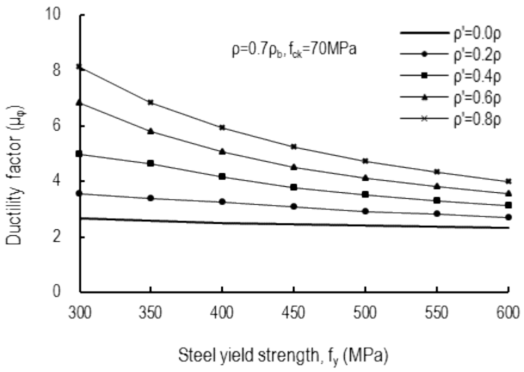


Fig. 11 Ductility factor μ_φ of doubly RC beam sections against steel yield strength

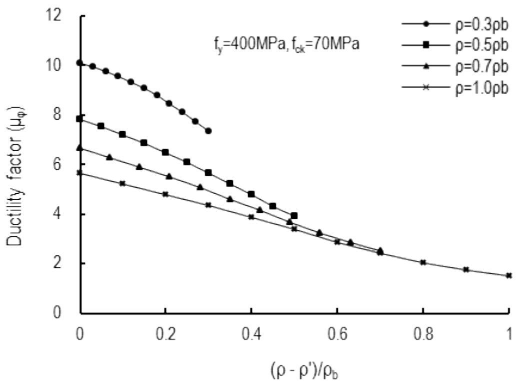


Fig. 12 Relationship of ductility factor μ_φ and $(\rho - \rho')/\rho_b$ for doubly RC beam sections

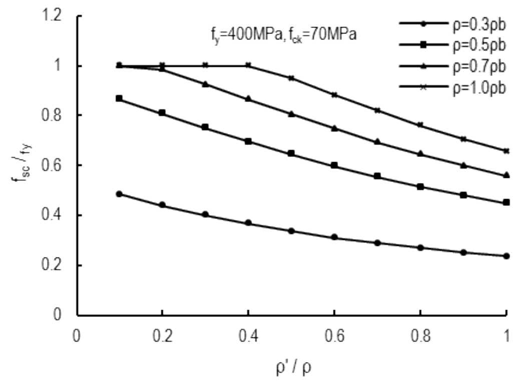


Fig. 13 Stress of compression reinforcement against ρ' / ρ in doubly RC beam sections

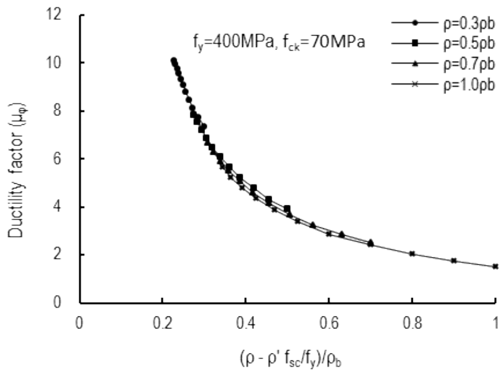


Fig. 14 Relationship of ductility factor μ_φ and $(\rho - \rho' f_{sc}/f_y)/\rho_b$ for doubly RC beam sections(f_{ck} = 70 MPa, f_y = 400 MPa)

ratio for the singly RC beam sections with steel yield strength f_y = 400 MPa and different concrete strengths and with concrete strength f_{ck} = 70 MPa and different steel yield strengths, respectively.

The curvature ductility factor decreases with increasing tension reinforcement ratio and a rate of decrease of ductility in relatively low steel ratio is larger than that in relatively high steel ratio. It may be seen that the tension reinforcement ratio is the most critical factor.

The curvature ductility factor in doubly RC beam sections may be affected by such as tension reinforcement ratio, compression reinforcement ratio, concrete strengthand steel yield strength.
Fig. 9 shows a plot of curvature ductility factor μ_φ as a function of compression reinforcement ratio ρ' for the doubly RC beam sections with different concrete strengths, steel yield strengths and tension reinforcement ratio. It indicates that the ductility factor of doubly RC beam sections increases with an increase in the compressive steel ratio. Under the same tension reinforcement ratio, the ductility factor almost linearly increases without regard to concrete strength and steel yield strength as the compressive reinforcement ratio is increased as shown Fig. 9 and a rate of increase of ductility factor in relatively low strength of steel is larger than that in relatively high strength of steel.

Fig. 10 shows a plot of ductility factor as a function of concrete strength for the doubly RC beam sections with tension reinforcement ratio $\rho = 0.70\rho_b$, steel yield strength $f_y = 400$MPa. At the same reinforcement conditions and steel yield strength, the ductility increases with an increase in concrete strength, but up to a certain level of concrete strength, about 60 MPa. Beyond this level, the ductility factor slightly decreases as the concrete strength is increased. Such results are similars with other observations by several researchers (Ashour, 2000, Rashid and Mansur, 2005). Fig. 11 shows a plot of ductility factorasafunctionoftensionsteel yield strength for the doubly RC beam sections with tension reinforcement ratio $\rho = 0.70\rho_b$, concrete strength $f_{ck} = 70$MPa. Under the same reinforcement conditions and concrete strength, the ductility factor of RC beam sections decreases with an increase in the steel yield strength. A rate of decrease of ductility is larger when a relatively low compression reinforcement ratio or high tension reinforcement ratio is provided.

Fig. 12 shows the effects of the difference tension reinforcement ratio between compression reinforcement ratio $(\rho - \rho')/\rho_b$ on the ductility factor of the doubly RC beam sections with concrete strength $f_{ck} = 70$MPa and steel yield strength $f_y = 400$MPa. The ductility factor decreases as the difference $(\rho - \rho')/\rho_b$ is increased.And the rate and trend of decrease of ductility with increasing of the difference $(\rho - \rho')/\rho_b$ are somewhat different and depend on the tension reinforcement ratio. In the case of low tension reinforcement, the rate of decrease of ductility is large with increasing of the difference $(\rho - \rho')/\rho_b$. The stress of compression reinforcement in doubly RC beams (f_{sc}) at ultimate state varies in dependence on the tension and compression reinforcement ratio. Fig. 13 shows a plot of the ratio of stress of compression reinforcement at ultimate state to yield strength (f_{sc}/f_y) in doubly RC beam sections against the value of reinforcement ratio (ρ'/ρ)when concrete strength is $f_{ck} = 70$MPa and steel yield strength is $f_y = 400$MPa. As the tension reniforcement ratio ρ is below $0.7\rho_b$, the compression reinforcement does not yield without regard to the amount of compression reinforcement. As the tension reinforcement ratio ρ is $1.0\rho_b$, the compression reinforcement yields when the compression reinforcement ρ' is below 0.4ρ and the compression reinforcement does not yield when the compression reinforcement ρ' is over 0.5ρ.

In the case of the doubly RC beam sections with an arbitrary combination of concrete strength and steel yield strength, the relations of curvature ductility factor and the value of $(\rho - \rho')/\rho_b$ are consisted of several curves according to the differenttension reinforcement ratio as shown in Fig. 12. However, if the stress of compression reinforcement at ultimate state is incorporated in the term of difference $(\rho - \rho')/\rho_b$ such as $\{\rho - \rho'(f_{sc}/f_y)\}/\rho_b$, the values of factor plotted against the corresponding values of reinforcement term $\{\rho - \rho'(f_{sc}/f_y)\}/\rho_b$ can be expressed almost a single curve as shown in Fig. 14. The shape and tendency of the curves plotted the relationship of {ρ -

$\rho'(f_{sc}/f_y)\}/\rho_b$ and μ_φ for doubly RC beam sections are almost similar to that of the relationship of ρ/ρ_b and μ_φ for singly RC beam sections. In other combinations of concrete strength and steel yield strength, the same results can be obtained.

Therefore, if the stress of compression reinforcement at ultimate state f_{sc} is incorporated in the term of difference $(\rho - \rho')/\rho_b$ such as $\{\rho - \rho'(f_{sc}/f_y)\}/\rho_b$ in doubly RC beam sections, the relation of ductility factor and reinforcement including compression reinforcement can be obtained with similar to that in singly RC beam sections. Also, when the stress of compression reinforcement is incorporated in way of $\{\rho - \rho'(f_{sc}/f_y)\}/\rho_b$, it indicates that the correlation of ductility factor and amount of reinforcement including compression reinforcement is fairly good.

5. Derivation of curvature ductility prediction equation for doubly RC beam sections

5.1 Parametric study

A parametric study is conducted to identify the influence of the parameters on the curvature ductility factor of doubly RC beam sections as previous mentioned. From the numerical analyses results, it is estimated that the concrete strength, the steel yield strength, the amount of reinforcing steel including compression reinforcement and the stress of compression reinforcement at ultimate stage have influence on the curvature ductility factor of doubly RC beam sections. Thus, the basic formation of the curvature ductility factor equation for doubly RC beam sections can be expressed as following.

$$\mu_\varphi = f\left(f_{ck}, f_y, (\rho - \rho'\frac{f_{sc}}{f_y})/\rho_b\right) \tag{17}$$

where f_{ck} is the concrete strength, f_y is the steel yield strength, ρ and ρ' are the tension and compression reinforcement ratio, respectively, ρ_b is the balanced steel ratio of the beam section without compression reinforcement, and f_{sc} is the stress of compression reinforcement which is calculated at ultimate stage.

According to the numerical analyses results as described in the previous, the curvature ductility factor μ_φ and the amount of reinforcing steel including the stress of compression reinforcement $\{\rho -\rho' (f_{sc}/f_y)\}/\rho$ are mutually related with the type of exponential function, the curvature ductility factor and the steel yield strength are also mutually related with the type of exponential function, and the curvature ductility factor and the concrete strength are mutually related with a polynomial function. Thus, the prediction equation of the curvature ductility factor for doubly RC beam sections can be expressed as following.

$$\mu_\varphi = k_1\left\{(\rho - \rho'\frac{f_{sc}}{f_y})/\rho_b\right\}^{k_2} \times \left(f_y\right)^{k_3} \times \left\{k_4\left(f_{ck}\right)^2 + k_5\left(f_{ck}\right) + k_6\right\} \tag{18}$$

where the coefficient k_i are determined from multiple regression analysis (Arslan and Gihanli, 2010). The multiple regression analysis is conducted using the numerical analysis results over 5,000 points.

5.2 Multiple regression analysis for the parameters

A regression analysis is carried out to verify the influence of $\{\rho - \rho' (f_{sc}/f_y)\}/\rho_b$ on curvature ductility factor μ_φ for doubly RC beam sections. The relationship of curvature ductility factor and parameter $\{\rho - \rho' (f_{sc}/f_y)\}/\rho_b$ is shown in Fig. 15. It indicates that the curvature ductility factor decreases with increasing of the value of $\{\rho - \rho' (f_{sc}/f_y)\}/\rho_b$ and the value of $\{\rho - \rho' (f_{sc}/f_y)\}/\rho_b$ has a remarkable effect on the curvature ductility of doubly RC beam sections. Based on the numerical analysis results, the relations of curvature ductility factor and parameter $\{\rho - \rho' (f_{sc}/f_y)\}/\rho_b$ can be obtained by regression analysis as follows

$$\mu_\varphi = 1.395 \left\{ (\rho - \rho' \frac{f_{sc}}{f_y}) / \rho_b \right\}^{-1.283} \quad (19)$$

The Eq. (19) clearly shows that the curvature ductility factor of doubly RC beam sections can be expressed as a function of $[\{\rho - \rho' (f_{sc}/f_y)\}/\rho_b]^{-1.283}$. The effect of yield strength of reinforcing steel on the ratio of the curvature ductility factor μ_φ to $[\{\rho - \rho' (f_{sc}/f_y)\}/\rho_b]^{-1.283}$ is shown in Fig. 16. The numerical analysis results indicate that the curvature ductility factor decreases with increasing of yield strength of reinforcement. By regression analysis, the relationship between the ratio of curvature ductility factor μ_φ to $[\{\rho - \rho' (f_{sc}/f_y)\}/\rho_b]^{-1.283}$ and the yield strength of reinforcement can be obtained as follows

$$\mu_\varphi / \left\{ (\rho - \rho' \frac{f_{sc}}{f_y}) / \rho_b \right\}^{-1.283} = 5.665 \left(f_y \right)^{-0.230} \quad (20)$$

The effect of concrete strength on the ratio of the curvature ductility factor to $[\{\rho - \rho'(f_{sc}/f_y)\}/\rho_b]^{-1.283} \times (f_y)^{-0.230}$ is shown in Fig. 17. The numerical analysis results indicate that the ductility factor increases with an increase in concrete strength, but up to certain level of concrete strength, that is, about 60 MPa. Beyond this level, the ductility factor slightly decreases as the concrete strength is increased. By regression analysis, the relationship between the ratio of the curvature ductility factor μ_φ to $[\{\rho - \rho' (f_{sc}/f_y)\}/\rho_b]^{-1.283} \times (f_y)^{-0.230}$ and the concrete strength can be obtained as follows

$$\mu_\varphi / \left[\left\{ (\rho - \rho' \frac{f_{sc}}{f_y}) / \rho_b \right\}^{-1.283} \times \left(f_y \right)^{-0.230} \right] = -0.0006 \left(f_{ck} \right)^2 + 0.0952 \left(f_{ck} \right) + 2.5062 \quad (21)$$

This equation clearly indicates that the curvature ductility factor of doubly RC beam sections can be expressed as a function of $\{-0.6(f_{ck})^2 + 95.2 f_{ck} + 2{,}506.2\} \times 10^{-3}$.

5.3 Proposed curvature ductility prediction equation for doubly RC beam sections

In doubly RC beam sections, considering the influence of parameters such as material strength, reinforcement ratio and stress of compression reinforcement, the prediction equation of curvature ductility factor can be offered based on the previous parametric study as follows

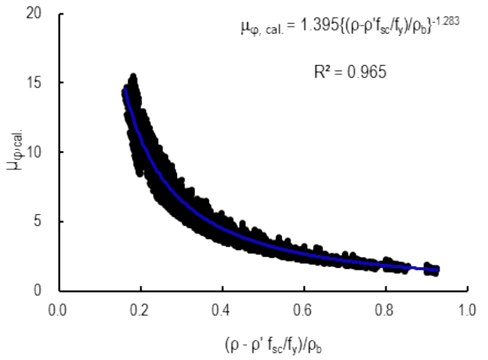


Fig. 15 Influence of parameter $\{\rho - \rho'(f_{sc}/f_y)\}/\rho_b$ on the curvature ductility factor μ_φ

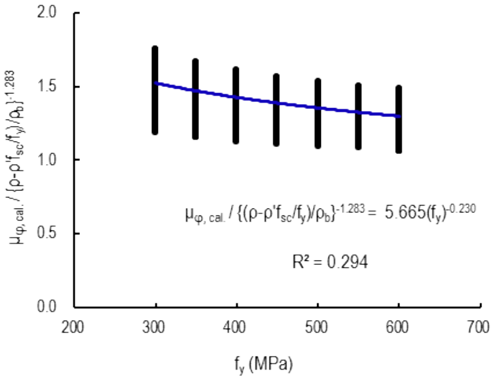


Fig. 16 Influence of f_y on the ratio of curvature ductility factor μ_φ to $\{(\rho - \rho' f_{sc}/f_y)/\rho_b\}^{-1.283}$

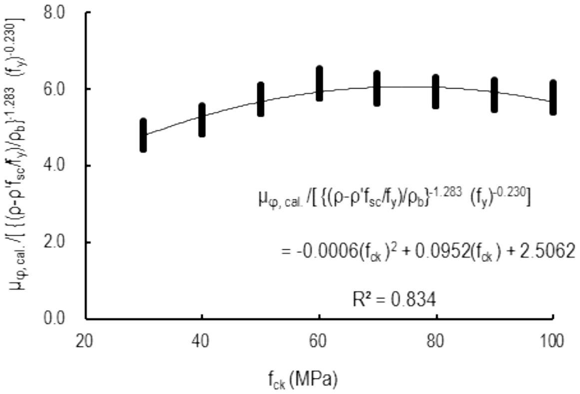


Fig. 17 Influence of f_{ck} on the ratio of curvature ductility factor μ_φ to $[\{(\rho - \rho' f_{sc}/f_y) / \rho_b\}^{-1.283} \times (f_y)^{-0.230}]$

$$\mu_\varphi = \left\{(\rho - \rho'\frac{f_{sc}}{f_y})/\rho_b\right\}^{-1.283} \times \left(f_y\right)^{-0.230} \times \left\{-0.6\left(f_{ck}\right)^2 + 95.2\left(f_{ck}\right) + 2{,}506.2\right\} \times 10^{-3} \qquad (22)$$

where f_{ck} is the concrete strength (MPa), f_y is the steel yield strength (MPa), ρ and ρ' are the tension and compression reinforcement ratio, respectively, ρ_b is the balanced steel ratio of the beam section when no compression reinforcement is provided, and f_{sc} is the stress of compression reinforcement which is calculated at ultimate stage.

5.4 Verification of proposed prediction equation for curvature ductility factor

Fig. 18 shows the comparison of the proposed curvature ductility factor obtained by Eq. (22) with the numerical results. The proposed predictions show excellent agreement as evident from coefficients correlation R^2 well above 0.99. Also, the mean value (MV) and the standard deviation (SD) for the ratio of the proposed curvature ductility factor obtained by Eq. (22) to the numerical result are 1.030 and 0.037, respectively. Within all the ranges of parameters in this study, the proposed equation is accurate to within 8.0% error for practical applications.

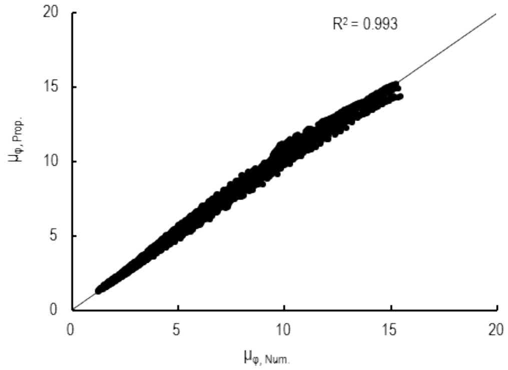


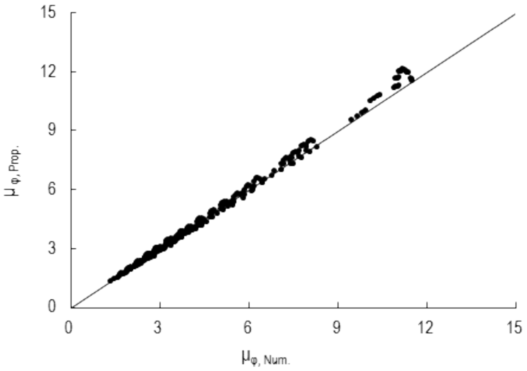
Fig. 18 Performance of proposed formula in predicting μ_φ

Table 2Verification of curvature ductility factor predictions

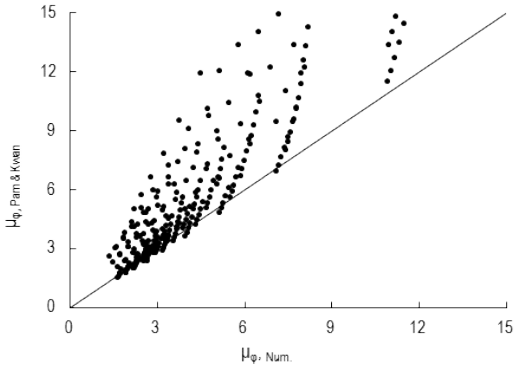
Parameters	f_{ck} = (30~100)*MPa*, f_y =450*MPa*, ρ = (0.2~1.0)ρ_b, ρ'= (0.1~0.4)ρ			f_{ck} = (60~100)*MPa*, f_y =450*MPa*, ρ = (0.2~1.0)ρ_b, ρ'= (0.1~0.4)ρ		
	MV*	SD**	COV***	MV	SD	COV
Proposed result /Num.****	1.018	0.031	0.029	1.028	0.030	0.030
Pam's *et al.*(2001b)result/Num.	1.391	0.416	0.405	1.186	0.368	0.446

*MV = mean value, **SD = standard deviation,*** COV=coefficient of variation, ****Num.=Numerical result

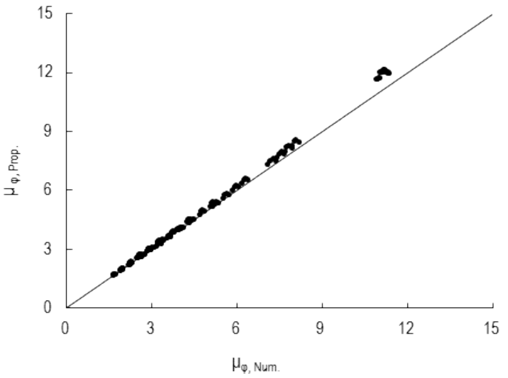
Pam's *et al*. (2001b) prediction Eq. (3) is derived based on the results for the sections with the concrete strength f_{ck} from 30 MPa to 100 MPa, the steel yield strength f_y = 460 MPa, the tensile reinforcement ratio ρ from 1.0% to 5.0% and the compression reinforcement ratio ρ' from 0.0% to 1.5%. Hence, in the case of the beam sections which the yield strength f_y is 450 MPa and the compression reinforcement ratio ρ' is provided less than 0.40ρ, the results obtained numerically are compared with the prediction results by the proposed equation and Pam's *et al*. (2001b) predictions by Eq. (3). Fig. 19 shows the comparison of the curvature ductility factors obtained numerically with the Pam's *et al.* predictions and the proposed predictions. It can be seen that the proposed predictions are well agree with the numerical results within all the ranges of parameters, but the Pam's *et al.* predictions are on the whole overestimation. It seems, due to that the stress-strain curve of concrete and the definition of ultimate curvature are different in both studies, that there are some discrepancies between the results obtained from the proposed equations and those from Pam *et al*. (2001b). Also, Table 2 shows the comparison of the ratio of proposed predictions to numerical results and the ratio of Pam's *et al.* predictions to numerical results, it can be seen that the proposed predictions produce the lower mean value, standard deviation and coefficient of variation (COV) than Pam's *et al.* predictions, and hence it provides better results than Pam's *et al.* predictions, based on the comparison with the numerical results. The COV obtained for the ratio of proposed predictions to numerical results is about 7% of thatobtained for the ratio of Pam's *et al.* predictions to numerical results. The discrepancy between Pam's *et al.* predictions and numerical results increases as the concrete strength is decreased or the compression steel ratio is increased, but the results predicted by the proposed formula considering of the stress of compression reinforcement at ultimate state are excellent agreement within all the ranges of the parameters.



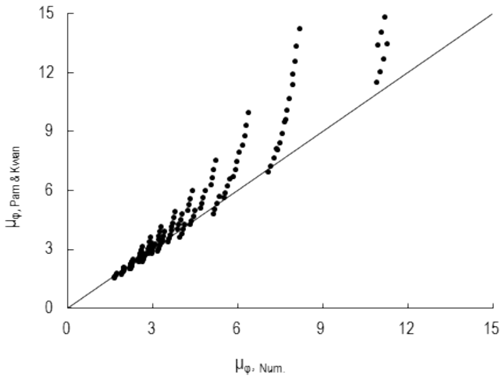
(a) Proposed formula (f_{ck} = (30~100) MPa)



(b) Pam's *et al*. (f_{ck} = (30~100) MPa)



(c) Proposed formula (f_{ck} = (60~100) MPa)



(d) Pam's *et al.* (f_{ck} = (60~100) MPa)

Fig. 19 Comparison of predicting μ_φ and numerical results (parameters:f_y = 450 MPa, ρ = (0.2~1.0)ρ_b, ρ' = (0.1~0.4)ρ)

6. Conclusions

The effects of concrete strength, steel yield strength, and amount of reinforcement including compression reinforcement on the complete moment-curvature behavior and the curvature ductility factor of doubly RC beam sections have been studied and a newly prediction formula for curvature ductility factor of doubly RC beam sections has been developed considering the stress of compression reinforcement at ultimate stage. The curvature ductility factor of doubly RC beam sections increases with an increase in concrete strength, but up to a certain level of concrete strength about 60 MPa and decreases with an increase of steel yield strength. Also, the curvature ductility factor increases as the tensile reinforcement ratio is decreased or the compresssion reinforcement ratio is increased. In the case of doubly RC beam sections, the compression reinforcement does not yield when the compression reinforcement is provided over a certain value, which depends on the concrete compressive strength, the steel yield strength and the tension reinforcement ratio. The relations of curvature ductility factor μ_φ and a value of $(\rho - \rho')/\rho_b$ are consisted of several curves according to the different tension reinforcement ratio ρ under the combination of concrete strength and steel yield strength, however the relations of curvature ductility factor and a value of reinforcement term $\{\rho - \rho'(f_{sc}/f_y)\}/\rho_b$ considering the stress of compression reinforcement at ultimate state can be expressed almost single curve such as that in singly RC beam sections.

Based on the numerical analysis results, the proposed predictions for the curvature ductility factor considering the stress of compression reinforcement is accurate to within 8.0% error for practical applications and the proposed formula was verified by comparisons of its predictions with the numerical results and other predictions. The proposed formula offers fairly accurate and consistent predictions of curvature ductility factor for doubly RC beam sections.

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