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Repairable k-out-n system work model analysis from time response

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Abstract. A novel reliability-based work model of *k*/*n* (G) system has been developed. Unit failure probability is given based on the load and strength distributions and according to the stress-strength interference theory. Then a dynamic reliability prediction model of repairable *k*/*n* (G) system is established using probabilistic differential equations. The resulting differential equations are solved and the value of k can be determined precisely. The number of work unit k in repairable *k*/*n* (G) system is obtained precisely. The reliability of whole life cycle of repairable *k*/*n* (G) system can be predicted and guaranteed in the design period. Finally, it is illustrated that the proposed model is feasible and gives reasonable prediction.

Keywords: repairable system, dynamic reliability, prediction model, *k*/*n* (G) system

1. Introduction

A consecutive k - out - of- n (G) system is composed of n identical units such that the system works if and only if at least k units work. In other words, the system fails if less than k units work. Obviously, if $k=1$, the system is a parallel system whereas if $k = n$, the system is a series system. A consecutive k-out-of-n (G) system is a common type of system. It can be found in a variety of engineering systems, such as aircraft engine, power plant generator, etc.

Model and life cycle analysis of *k*/*n* (G) system has become essential for improving the system performance. Classical reliability and probabilistic risk assessment methods often consider that the state of *k*/*n* (G) system does not change with time (Atwood 1986, Ronold and Larsen 2000, Lewis 2001, Champiri *et al*. 2012). The reliability analysis of 1/2 (G) system under external impact was discussed in the article (Liudong *et al*. 2012, Mohammad and Saeed 2010). Besides that, the reliability of *k*/*n* (G) system under load which was equally burdened by its units was studied in the article (Scheuer 1988). The failure rate of the system was found to be increased with load and the result was presented in the article (Cojazzi 1996). However, the system failure rate was considered constant over time in the above articles, and thus the analysis of system reliability was unchanged with time. In fact, the state of the system and the units or components of the system are in a

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dynamic process.

On the other hand, reliability of a single degree of freedom nonlinear vibration system was considered to change with time (Zhang *et al*. 2003, Kim *et al*. 2013). The reliability and availability of simple system which was consisted of a single unit or two parallel units were studied when the failure rate was changed with time (Tao *et al*. 2009). The dynamic reliability of large complex system and dynamic probabilistic risk assessment was presented in the article (Yongfeng *et al*. 2012). The dynamic reliability model of failure relativity *k*/*n* (G) system was established by using the stress-strength interference theory and theory of order statistics in the articles (Yongfeng *et al*. 2012, Severo 1969). It was observed that the reliability of the systems was changed with time, but the application of the system reliability analysis was limited. The prediction model of epidemic infection was established using differential equations in the articles (Siskind 1965, Ida *et al*. 2007, Reza and Ted 2011). The model can be used to predict exactly the scale of epidemic infection event, outbreak time, duration and number of deaths, so the government's policy decision can be provided based on the theoretical basis. It was tested and verified using an epidemic infection event and the model was accurate.

Based on the above articles, a reliability-based work model of repairable *k*/*n* (G) system has been established by using the stress-strength interference theory, probability theory and stochastic process. The proposed model is analyzed and the derived probabilistic differential equations are solved. The number of work unit (k) can be determined theoretically from the proposed model. Finally, it is shown that the proposed model is feasible, practicable and effective.

2. The proposed work model of repairable *k*/*n* (G) system

In this paper, the assumptions of repairable *k*/*n* (G) system are given as follows

1) The total number of unit in the system remains unchanged, each unit can only be in one of the two possible states ‘work’ or ‘fail’, and each unit is independent of each other.

2) The number of unit at time t in ‘work’ state and ‘fail’ state are $r(t)$and $s(t)$, respectively, and $r(t) + s(t) = n$, $k \le r(t) \le n$.

The external random shock is usually considered to obey Poisson distribution with mean parameter λ_1, and the resulting stress δ is considered to obey normal distribution $\delta \approx N(\mu_\delta, \sigma_\delta)$. It is assumed that the repair order is first-fail, first-repaired and the repaired unit is as good as new. Repair time is considered to obey exponential distribution with mean parameters λ_2. The strength s of each unit in repairable *k/n* (G) system is equal and is considered to obey normal distribution $s \approx N(\mu_s, \sigma_s)$.

The reliability index *ß* and reliability *R* of each unit are calculated as follows

$$\beta = (\mu_s - \mu_\delta)\,(\sigma_s^2 + \sigma_\delta^2)^{-1/2}\ ,\ \ R = \Phi(\beta) \tag{1}$$

where $\Phi(\cdot)$ is the standard normal distribution function.

If *m*-series of external random shocks are applied on the system, the reliability of each unit can be determined by the maximum random shock. The reliability of the system or unit under repeated random shocks is equivalent to its reliability under the maximum random shock. Therefore, the reliability index and reliability can be calculated as follows

$$R_m = \Phi(\beta_m)\ ,\ \ \beta_m = \frac{\mu_s - \mu_{\delta_m}}{\sqrt{\sigma_s^2 + \sigma_{\delta_m}^2}} \tag{2}$$

Then, the reliability of the unit with respect to time is computed using fully probabilistic method

$$R(t)=\sum_{m=0}^{+\infty}P[N(t)-N(0)=m]R_m=\sum_{i=0}^{+\infty}\frac{(\lambda_1 t)^m}{m!}e^{-\lambda_1 t}R_m \tag{3}$$

Finally, the failure probability of the unit with respect to time can be determined as follows

$$\alpha(t)=1-R(t) \tag{4}$$

At time $(t,t+\Delta t)$, the following probabilities are obtained

(1) The unit failure probability of the *k*/*n* (G) system is $\alpha r(t)\Delta t+o(\Delta t)$

(2) The unit repaired probability of the *k*/*n* (G) system is $\lambda_1 s(t)\Delta t+o(\Delta t)$

(3) The failure or repaired probability for several units of the *k*/*n* (G) system is $o(\Delta t)$

(4) The probability that remains constant for all units of the *k*/*n* (G) system is $1-\alpha r(t)\Delta t-\lambda_1 s(t)\Delta t+o(\Delta t)$.

There are two transform styles for both the system states 'work' and 'fail'

$$(r(t)\to r(t)-1, s(t)\to s(t)+1) \quad \text{or} \quad (r(t)\to r(t)+1, s(t)\to s(t)-1) \tag{5}$$

The probability of the number of unit reliability $r(t)$ and failure $s(t)$ at time *t* is obtained as follows

$$\begin{aligned}P_{r(t)s(t)}(t)=\alpha(r(t)+1)P_{r(t)+1,s(t)-1}(t)\Delta t+(1-\alpha r(t)\Delta t-\\ \lambda_1 s(t)\Delta t+o(\Delta t))P_{r(t),s(t)}+(\lambda_1 s(t)\Delta t+o(\Delta t))P_{r(t)-1,s(t)+1}\end{aligned} \tag{6}$$

Eq. (6) can be collated as follows

$$\frac{dP_{r(t)s(t)}(t)}{dt}=\alpha(r(t)+1)P_{r(t)+1,s(t)-1}(t)-(\alpha r(t)+\lambda_1 s(t))P_{r(t),s(t)}+\lambda_1 s(t)P_{r(t)-1,s(t)+1} \tag{7}$$

Suppose the number of unit failure at time *t* is denoted as $i(t)$, then

$$\frac{dP_{r(t)i(t)}(t)}{dt}=-i(t)(n\alpha+\lambda_2)P_{r(t),i(t)} \tag{8}$$

The differentiation of dynamic reliability of the system with respect to time can then be obtained using Eqs. (7) and (8) and is shown as follows

$$\frac{dP_{r(t)s(t)}(t)}{dt}=\alpha(r(t)+1)P_{r(t)+1,s(t)-1}(t)-(\alpha r(t)+\lambda_2 s(t))P_{r(t),s(t)}+\lambda_2 s(t)P_{r(t)-1,s(t)+1} \tag{9}$$

where $r(t)+s(t)=n$, $0\le s(t)\le n-k$, $0\le r(t)\le k$, $0\le i(t)\le n-k$.

The initial condition of Eq. (9) is shown as follows

$$P_{n,0}(0)=1 \quad \text{and the others are 0} \tag{10}$$

3. Solving the work model of repairable *k*/*n* (G) system

Suppose $j(t)=(r(t),s(t),n)=\frac{1}{2}(n+1)(n+2)-(n+1)r(t)-s(t)+\frac{1}{2}(r(t)-1)r(t)$ (11)

Then Eq. (9) can be rewritten as follows

$$Q_{j(t)}^{'}(t)=(r(t)+1)Q_{j(t)-n+r(t)}(t)-(r(t)+\gamma s(t))Q_{j(t)}(t)+\gamma s(t)Q_{j(t)-1}(t) \tag{12}$$

where $\gamma=\lambda_2/\alpha$ which is the relative probability of failure.
The matrix form of Eq. (12) is shown as follows

$$Q^{'}(t)=AQ(t) \tag{13}$$

where $Q(t)=(Q_{j(t)}(t)),\ \ j(t)=1,2,\cdots,\frac{1}{2}n(n+1)$ and $A=(a_{u,v}),\ u,v=1,2,\cdots,\frac{1}{2}n(n+1)$
The initial condition of Eq. (13) is $Q(0)=(0,1,\cdots,0)$.
The matrix A in Eq. (13) can be partitioned such that

$$A=(A_{f,l}),\quad f,l=1,2,\cdots,n+1. \tag{14}$$

where $A_{f,l}$ is a $f\times l$ matrix, $A_{ll}=(a_{u,v}),\ u,v=1,\cdots l.$

$$a_{u,u}=-(l-u)(n-l+1+\gamma),\quad u=1,\cdots,l.$$

$$a_{u,u-1}=(l-u+1)\gamma,\ \ u=1,\cdots,l.$$

$$A_{l,l-1}=(a_{u,v}),\ \ u=1,\cdots,l;\ \ v=1,\cdots,l-1.$$

$$a_{u,u}=(l-u-1)(n-l-2),\ \ u=1,\cdots,l-1.$$

The others partitioned matrices in $A_{f,l}$ are zero matrices.
Eq. (13) can be derived as follow

$$Q(t)=Ce^{At} \tag{15}$$

$$C=c_{f,l}(e,w)=\frac{-\gamma^{e}(n-1)!(f-e-1)!}{(n-f+1)!(l-w-1)!}B_{l,w}D(f,l,e,w) \tag{16}$$

$$\begin{aligned} D(f,l,e,w)=&\sum_{g_{e-w}=e+1}^{f}\sum_{g_{e-w-1}=e}^{g_{e-w}}\cdots\sum_{g_1=w+2}^{g_2}\frac{(n-g_1-f+w+\gamma)!}{(g_1-f)!(n-2f+w+\gamma)!}\\ &\times\prod_{h=0}^{e-w-1}\frac{(g_{e-w-h}-e+1+h)(g_{e-w-t}-e+h)}{G_{e-w-h}(g_{e-w-h})} \end{aligned} \tag{17}$$

$$G_x(g_x)=\prod_{y=0}^{g_{x+1}-g_x}[(g_{x+1}-g-l-y)(n-g_{x+1}-l+w+1+y+\gamma)-(l-w)x] \tag{18}$$

$$\begin{aligned}B_{l,w}=(-1)^l\sum_{r(t)=w+1}^{l}\frac{(-1)^{r(t)-1}(n-2l+w+1+\gamma)!}{(l-r(t))!(n-r(t)-l+w+1+\gamma)!}\\ \times\sum_{s(t)=1}^{w-1}\sum_{\xi=s(t)+1}^{r(t)}\frac{(l-w-1)!}{(\xi-s(t)-1)!}B_{\xi,s(t)}D(r(t),\xi,w,s(t))\end{aligned} \tag{19}$$

$f,\ l=1,2,\cdots,n+1, l\le f;\ e=1,2,\cdots,l.\ l\le e.$

If $e<w$ or $f<l$, $c_{f,l}(e,w)=0$. This can be explained as follows

If $f<l$, $c_{f,l}(e,w)=0$ can be obtained by using the theorem 1 in the article

If $e<w$, Eq. (20) can be obtained by using the mathematical induction method as follows

$$c_{l+1,l}(1,2)=\frac{(l-1)(n-l+1)c_{l,l}(1,2)}{(f-e)(n-f+1+\gamma)-(l-w)(n-l+1+\gamma)} \tag{20}$$

The fact that $c_{l,l}(1,2)=0$, so $c_{l+1,l}(1,2)=0$. Eq. (20) can be established when the iteration $f_0=l+1,l+2,\cdots,f-1;\ e_0=1,2,\cdots,w-2.$ and $f_0=f;\ e_0=1,2,\cdots,e-1.$ is executed.

The following will prove that $f_0=f;\ e_0=e.$ is also established.

When $c_{f-1,l}(e,w)=0$ and $c_{f,l}(e-1,w)=0$ are substituted to Eq. (21)

$$c_{f,l}(e,w)=\frac{(m-e-1)(n-m+1)+(m-e+1)\cdot\gamma\cdot c_{f,l}(e-1,w)}{(f-e)(n-f+1+\gamma)-(l-w)(n-l+1+\gamma)} \tag{21}$$

Thus if $e<w$, then $c_{f,l}(e,w)=0$ can be obtained.

The general solution of Eq. (15) is solved as follows

If $(m,w)=(1,2)$, Eq. (22) is obtained by using the article

$$c_{2,2}(1,1)=c(2,2)=1 \tag{22}$$

Similarly, Eq. (23) can be obtained from iteration as follows

$$\begin{aligned}c_{f,2}(e,f)=\frac{-\gamma^{e-1}(n-1)!(f-e-1)!B_{2,1}}{(n-f+1)!}\\ \cdot\frac{D(f-1,2,e,1)+(f-e+1)(f-e)D(f,2,e-1,1)}{(f-e)(n-f+1+\gamma)-(l-w)(n-l+1+\gamma)}\end{aligned} \tag{23}$$

Eq. (23) is rewritten as follow

$$c_{f,2}(e,f)=\frac{-\gamma^{e-1}(n-1)!(f-e-1)!B_{2,1}}{(n-f+1)!}\cdot D(f,2,e,1) \tag{24}$$

So for $f_0=f;\ e_0=e.$, Eqs. (14)-(19) are all established.

Because

$$c_{l,l}(w,w)=c(\frac{1}{2}e(e-1)+w,\ \frac{1}{2}e(e-1)+w) \tag{25}$$

Its form of four coordinates is shown as follows

$$c_{l,l}(w,w)=-\sum_{s(t)=1}^{w-1}\sum_{\xi=s(t)+1}^{l}c_{l,\xi}(w,s(t))-\sum_{\xi=w+1}^{l-1}c_{l,\xi}(w,w) \tag{26}$$

Then

$$\begin{aligned}B_{l,w}=(-1)^{l}&\sum_{s(t)=1}^{w-1}\sum_{\xi=s(t)+1}^{l}\frac{(l-w-1)!}{(\xi-s(t)-1)!}B_{l,s(t)}D(l,\xi,w,s(t))\\&-\sum_{\xi=w+1}^{l-1}\frac{(l-w-1)!}{(\xi-w-1)!}B_{\xi,w}D(l,\xi,w,w)\end{aligned} \tag{27}$$

When $B_{\xi,w},\xi=w+1,\cdots,l-1.$ is substituted into Eq. (27), Eq. (28) is obtained as follows

$$\begin{aligned}\frac{-B_{l,w}}{(l-w-1)!}=-\sum_{s(t)=1}^{w-1}&((-1)^{w+1}D(l,\xi,w,w)\sum_{r(t)=w+1}^{w+1}\frac{(-1)^{r(t)-1}(n-2(w+1)+w+1+\gamma)!}{(w+1-r(t))!(\xi-s(t)-1)!}\\&\cdot\sum_{\xi=s(t)+1}^{r(t)}\frac{B_{\xi,w}}{(\xi-s(t)-1)!}D(l,\xi,w,w)+\cdots\\&+(-1)^{l-1}D(l,l-1,w,w)\sum_{r(t)=w+1}^{l-1}\frac{(-1)^{r(t)-1}(n-2(l-1)+w+1+\gamma)!}{(m-1-r(t))!(n-r(t)-(l-1)+w+1+\gamma)!}\\&\cdot\sum_{\xi=s(t)+1}^{r(t)}\frac{B_{\xi,s(t)}}{(\xi-s(t)-1)!}D(r(t),\xi,w,s(t))+D(l,l,w,w)\\&\cdot\sum_{\xi=s(t)+1}^{r(t)}\frac{B_{\xi,s(t)}}{(\xi-s(t)-1)!}D(l,\xi,w,s(t))\end{aligned} \tag{28}$$

Eq. (19) can be obtained by simplifying Eq. (28).

The method used to solve Eq. (16) is similar to Eq. (20). Finally the general solution of Eq. (15) can be obtained.

4. Examples

A repairable k/n (G) system where k = 2 and n = 4 is used as an example to verify the proposed dynamic reliability-based work model. The strength of each unit obeys the normal distribution and is given as $s\approx N(600,40)MPa$ whereas the external random shock S obeys Poisson distribution with mean parameter $\lambda_1=0.5/h$. Based on m-series of external random shocks which are applied on the system, the maximum random shock $\delta_{\max}$ can be obtained. The reliability index β_m and reliability R_m of each unit can be computed using Eq. (2). The results are shown in Table 1.

Then, the reliability of the unit at time *t* is obtained as follows

$$R(t)=\sum_{m=0}^{+\infty}P[N(t)-N(0)=m]R_m=\sum_{m=0}^{+\infty}\frac{(0.5t)^m}{m!}e^{-0.5t}R_m$$

It is assumed that the repair order is first-fail, first-repaired and the repaired unit is as good as new. Repair time is considered to obey exponential distribution. Two mean parameters of the exponential repair time distribution are studied, $\lambda_2=0.05$ and

4.1 Mean parameter of repair time, $\lambda_2=0.05$

The 'work' state of the system with $\lambda_2=0.05$ is shown as follows

$$P_{4,0}(t)=0.9223e^{-4t}$$

$$P_{3,1}(t)=0.3398e^{-4t}+14.9454e^{-1.554t}-14.85492e^{-1.667t}-0.2878e^{-3.108t}-0.0825e^{-5.554t}$$

$$P_{2,2}(t)=0.1324e^{-0.0014t}+2.9542e^{-0.00554t}-4.2556e^{-0.00667t}+0.5261e^{-0.00154t}+0.6559e^{-0.00105t}$$

$$P_{1,3}(t)=0.4221e^{-0.0004t}-13.3730e^{-0.00052428t}+12.9669e^{-0.0004530t}+0.1113e^{-0.006665t}-0.1189e^{-0.000399t}$$

$$P_{0,4}(t)=0.0905e^{-0.0004t}-6.0347e^{-0.0000625t}+6.1513e^{-0.0000435t}-0.2496e^{-0.0003108t}+0.0425e^{-0.00399t}$$

The 'work' state of the repairable 2/4 (G) system with $\lambda_2=0.05$ is shown in Table 2 up to *t* = 5000 hours. It can be concluded from Table 1 and Table 2 that when $\lambda_2=0.05$, the relative failure is more than 1 ($\gamma>1$) and thus the failure rate of the system is higher than the repair rate. The system loses it efficiency over time and does not perform as intended. However, if the *k* is changed from 2 to 1, the reliability of the system will be improved.

4.2 Mean parameter of repair time

The work state of the 2/4 (G) system with $\lambda_2=1.5$ is computed as follows

$$P_{4,0}(t)=0.9223e^{-0.004t}$$

$$P_{3,1}(t)=2.0632e^{-0.001275t}+2.212e^{-0.004877t}-3.2752e^{-0.001333t}-e^{-0.001417t}$$

$$P_{2,2}(t)=1.1330e^{-0.00004t}-4.0347e^{-0.00006246t}+5.1513e^{-0.00004131t}-2.2496e^{-0.00007815t}$$

$$P_{1,3}(t)=0.1248e^{-0.0000518t}+2.9437e^{-0.00001103t}-$$
$$10.0458e^{-0.00002015t}+11.9975e^{-0.00002997t}-5.0202e^{-0.00004t}$$

$$P_{0,4}(t)=0.8030e^{-0.00000518t}-3.2839e^{-0.00001179t}+$$
$$5.1565e^{-0.00002160t}-3.9992e^{-0.00003263t}+1.3263e^{-0.00004t}$$

The 'work' state of the repairable 2/4 (G) system with $\lambda_2=1.5$ is shown in Table 3 up to *t* = 10000 hours. For the case of $\lambda_2=1.5$, the relative failure is less than 1 ($\gamma<1$) and thus the failure rate of the system is slower than the repair rate. Therefore, it is shown that *k* = 2 is determined correctly and the system performs reasonably well.

5. Conclusions

A reliability-based work model of repairable *k*/*n* (G) system has been developed. The number of work unit, k in repairable *k*/*n* (G) system is normally set according to one's experience, but it can be determined theoretically from the proposed model. Firstly, failure probability of repairable *k*/*n* (G) unit is obtained using the stress-strength interference theory and probability distribution of random shocks which are applied on the unit. Secondly, the reliability prediction model of the repairable *k*/*n* (G) system is established by using the probability differential equations and according to the unit reliability and the failure state transition situation. Finally, the solution of the proposed model is given and the number of work unit *k* in repairable *k*/*n* (G) system is obtained. The examples have shown that the proposed model is accurate and feasible. The reliability of whole life cycle of repairable *k*/*n* (G) system can be predicted and guaranteed in the design period, so reasonable and economic design can be performed.

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