Computers and Concrete, *Vol. 12, No. 4 (2013) 499-518*
DOI: http://dx.doi.org/10.12989/cac.2013.12.4.499

An efficient method for the compressive behavior of FRP-confined concrete cylinders

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(*Received Jun 11, 2012, Revised February 14, 2013, Accepted May 24, 2013*)

Abstract. Fiber reinforced polymer (FRP) jackets have been widely used as an effective tool for the strengthening and rehabilitation of concrete structures, especially damaged concrete columns. Therefore, a clear understanding of the compressive behavior of FRP-confined concrete is essential. The objective of this paper is to develop a simple efficient method for predicting the compressive strength, the axial strain at the peak stress, and the stress-strain relationship of FRP-confined concrete. In this method, a compressive strength model is established based on Jefferson's failure surface. With the proposed strength model, the strength of FRP-confined concrete can be estimated more precisely. The axial strain at the peak stress is then evaluated using a damage-based formula. Finally, a modified stress-strain relationship is derived based on Lam and Teng's model. The validity of the proposed compressive strength and strain models and the modified stress-strain relationship is verified with a wide range of experimental results collected from the research literature and obtained from the self-conducted test. It can be concluded that, as a competitive alternative, the proposed method can be used to predict the compressive behavior of FRP-confined concrete with reasonable accuracy.

Keywords: FRP-confined concrete; strength model; strain model; stress-strain relationship

1. Introduction

It is well known that a large number of existing structures in practical engineering need rehabilitation or strengthening because of inappropriate design or construction, damage induced by natural disasters, and performance degradation. The urgent need for new and reliable construction systems promotes the development of advanced composite materials. In recent years, external confinement of concrete with fiber-reinforced polymer (FRP) composites has served as a popular method of column retrofit, which can significantly enhance the load bearing capacity and ductility. These materials have many advantages, such as light weight, high strength-to-weight ratio, and good corrosion resistance.

Due to the importance of FRP composites to structural rehabilitation and repair, much attention

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http://www.techno-press.org/?journal=cac&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

has been paid and many numerical methods have been developed in the past decades. The existing models can be divided on the whole into two categories (Lam and Teng 2003): “design-oriented” and “analysis-oriented”. In this paper, only the design-oriented model is discussed. The most important parameters for this type of models include the compressive strength and the axial strain at the peak stress of FRP-confined concrete. Fardis and Khalili (1982) used the formula proposed by Richart *et al*. (1928) to predict the compressive strength of FRP-confined concrete, but the result was overestimated, as indicated by Xiao and Wu (2000). Saadatmanesh *et al*. (1994) extended the confinement model for steel-confined concrete (Mander *et al*. 1988) to FRP-confined concrete. It was shown that the extension is inappropriate (Mirmiran and Shahawy 1997, Samaan *et al*. 1998). To remedy the drawbacks of the above-mentioned models, a number of new models were developed. Xiao and Wu (2000, 2003) considered the effect of the lateral stiffness provided by the FRP jacket. Lam and Teng (2003) and Matthys *et al*. (2005) used the actual failure strain instead of the nominal one. The model proposed by Spoelstra and Monti (1999) can predict the compressive strength with strain hardening and softening curves. After a systematic assessment of the existing models in the literature, Vintzileou and Panagiotidou (2008) proposed an empirical model for both circular and prismatic elements. Recently, Wu and Zhou (2010) developed a unified model for circular and square columns based on the Hoek-Brown failure criterion. With this model, the compressive strength of normal and damaged columns can be evaluated. Mousavi *et al*. (2010) proposed a data mining approach for estimating the strength of CFRP-confined concrete. However, most of these models are based on large-scale regression analysis of experimental data gathered from various sources. Their reliability depends highly on the selected experimental data and therefore large errors could be caused if they are directly applied to other experimental data. Some models for the compressive strength of circular columns are summarized in Table 1.

Saadatmanesh *et al*. (1994), Miyauchi (1997), and Kono *et al*. (1998) expressed the axial strain at the peak stress of FRP-confined concrete as a function of the lateral pressure provided by FRP. Further studies (De Lorenzis and Tepfers 2003, Samaan *et al*. 1998) indicated that the axial strain at the peak stress is related not only to the strength but also to the stiffness of the FRP jacket. Therefore, the effect of the FRP jacket stiffness needs to be considered (De Lorenzis and Tepfers 2003; Fahmy and Wu 2010; Fardis and Khalili 1982, Lam and Teng 2003, Saafi *et al*. 1999, Samaan *et al*. 1998, Spoelstra and Monti 1999, Teng *et al*. 2009, Toutanji 1999, Wu *et al*. 2006, Xiao and Wu 2000). Some models for the axial strain at the peak stress of FRP-confined concrete are summarized in Table 2.

Experimental results have indicated that, when FRP-confined concrete columns are subjected to sufficient confinement, the stress-strain curve features a monotonically ascending bilinear shape. Existing design-oriented stress-strain models for FRP-confined concrete adopt different approximations to the bilinear curve. Karbhari and Gao (1997) and Xiao and Wu (2000) expressed the bilinear curve by two straight lines, whereas Wu *et al*. (2006) used a trilinear formula. These models are simple but not realistic (Lam and Teng 2003). Samaan *et al*. (1998) employed the four-parameter stress-strain relationship proposed by Richard and Abbott (1975) to describe the response of FRP-confined concrete. The main feature of this model is that the stress-strain curve is represented by a single equation and the transition zone is controlled by a curve-shaped parameter. Toutanji (1999) and Saafi *et al*. (1999) proposed a similar model to characterize the stress-strain curve for FRP-wrapped and -encased cylinders. The model is composed of two portions. The first one is similar to that of plain concrete and the second one is controlled by the stiffness of FRP composites. These models are too complicated for practical application (Lam and Teng 2003).

Table 1 IAEs of various strength models

Model	Equation of f'_{cc}/f'_{co}	IAE(f)(%)
Xiao and Wu (2003)	$1.1+[4.1-0.45(f'^2_{co}/C_j)^{1.4}](f_l/f'_{co})$	38.9
Xiao and Wu (2000)	$1.1+[4.1-0.75(f'^2_{co}/C_j)](f_l/f'_{co})$	37.5
Fardis and Khalili (1982)	$1+4.1(f_l/f'_{co})$	35.5
Razvi and Saatcioglu(1999)	$1+6.7(f_l^{0.83}/f'_{co})$	35.5
Kumutha *et al*. (2007)	$1+0.93(f_l/f'_{co})$	32.6
Toutanji (1999)	$1+3.5(f_l/f'_{co})^{0.85}$	30.3
Shehata *et al*. (2002)	$1+1.25(f_l/f'_{co})$	26.7
Mirmiran and Shahawy (1997)	$1+4.269(f_l^{0.587}/f'_{co})$	25.9
Kono *et al*. (1998)	$1+0.0572f_l$	23.5
Miyauchi *et al*. (1999)	$1+2.98(f_l/f'_{co})$	18.4
Lam and Teng (2003)	$1+3.3(f_l/f'_{co})$	16.6
Lam and Teng (2001)	$1+2.0(f_l/f'_{co})$	16.5
Campione and Miraglia (2003)	$1+2.0(f_l/f'_{co})$	16.5
Miyauchi *et al*. (1997)	$1+3.485(f_l/f'_{co})$	15.9
Matthys *et al*. (2005)	$1+2.3(f_l/f'_{co})^{0.85}$	15.4
Saafi *et al*. (1999)	$1+2.2(f_l/f'_{co})^{0.84}$	15.3
Samaan *et al*. (1998)	$1+6.0(f_l^{0.7}/f'_{co})$	15.3
Karbhari and Gao (1997)	$1+2.1(f_l/f'_{co})^{0.87}$	15.2
Wu and Wang (2009)	$1+2.2(f_l/f'_{co})^{0.94}$	15.1
Spoelstra and Monti (1999)	$0.2+3.0(f_l/f'_{co})^{0.5}$	14.9
Rousakis and Karabinis (2008)	$(\rho_f E_f/f'_{co})(-0.4142E_f\times10^{-6}/E_{f\mu}+0.0248)+1$	14.8
Wu and Zhou (2010)	$f_l/f'_{co}+\sqrt{(16.7/f'^{0.42}_{co}-f'^{0.42}_{co}/16.7)f_l/f'_{co}+1}$	14.8
This paper	$1+(1+2\alpha+\gamma)/(1-\alpha)(f_l/f'_{co})$	14.8

Youssef *et al*. (2007) presented a model based on a comprehensive experimental program including large-scale circular, square, and rectangular short columns confined by CFRP and GFRP jackets with a wide range of confinement ratios. This model is of the general form for circular and rectangular sections but is still complicated for application. Lam and Teng (2003) developed a simple stress-strain model, which consists of a parabolic portion and a straight-line portion with a smooth transition. This model is appealing and simple and can capture the main feature of FRP-confined concrete (Fahmy and Wu 2010).

As compared with the previous studies, the main purpose of the present paper is to develop a simple and efficient method with fewer parameters for predicting the compressive behavior of FRP-confined concrete so that it can be conveniently used in practical engineering.

Table 2 IAEs of various strain models

Model	Equation of ε_{cc}	IAE(ε) (%)
Xiao and Wu (2000)	$(\varepsilon_{ju}-0.0005)/[7(f'_{co}/E_l)^{0.8}]$	73.2
Samaan *et al*. (1998)	$(f'_{cc}-f_o)/E_2$ $f_o=0.872f'_{co}+0.371f_l+6.258$ $E_2=245.61f'^{0.2}_{co}+0.6728E_l$	69.1
Toutanji (1999)	$\varepsilon_{co}[1+(310.57\varepsilon_{ju}+1.90)(f'_{cc}/f'_{co}-1)]$	64.1
Spoelstra and Monti (1999)	$\varepsilon_{co}(2+1.25E_0/f'_{co}\varepsilon_{ju}\sqrt{f_l/f'_{co}})$	59.2
Kono *et al*. (1998)	$\varepsilon_{co}(1+0.28f_l)$	47.7
Fardis and Khalili (1982)	$\varepsilon_{co}+0.0005(E_l/f'_{co})$	44.3
Saafi *et al*. (1999)	$\varepsilon_{co}[1+(537\varepsilon_{ju}+2.6)(f'_{cc}/f'_{co}-1)]$	42.4
De Lorenzis and Tepfers (2003)	$\varepsilon_{co}[1+26.2(f_l/f'_{co})^{0.8}E_l^{-0.148}]$ (FRP wraps) $\varepsilon_{co}[1+26.2(f_l/f'_{co})^{0.68}E_l^{-0.127}]$ (FRP tubes)	39.2
Miyauchi *et al*. (1997)	$\varepsilon_{co}[1+10.6(f_l/f'_{co})^{0.373}]$ (f'_{co} = 30 MPa) $\varepsilon_{co}[1+10.5(f_l/f'_{co})^{0.525}]$ (f'_{co} = 50 MPa)	38.9
Saadatmanesh *et al*. (1994)	$\varepsilon_{co}[1+5(f'_{cc}/f'_{co}-1)]$	34.0
Lam and Teng (2003)	$\varepsilon_{co}(1.75+12\rho_K\rho_\varepsilon^{1.45})$ $\rho_K=2E_{frp}t/[(f'_{co}/\varepsilon_{co})D]$ $\rho_\varepsilon=\varepsilon_j/\varepsilon_{co}$	30.1
Wu *et al*. (2006)	ε_{ju}/ν_u $\nu_u=0.56c_1(f_l/f'_{co})^{-0.66}$ $c_1=1$ ($E_{frp}\le 250$ GPa) $c_1=1\sqrt{250/E_{frp}}$ ($E_{frp}>250$ GPa)	29.7
Teng *et al*. (2009)	$\varepsilon_{cc}=\varepsilon_{co}(1.75+6.5\rho_K^{0.8}\rho_\varepsilon^{1.45})$ $\rho_K=2E_{frp}t/[(f'_{co}/\varepsilon_{co})D]$ $\rho_\varepsilon=\varepsilon_j/\varepsilon_{co}$	29.1
This paper	$\varepsilon_{cc}=f'_{cc}/E_{\sec}$ $E_{\sec}=E_0/[1+2\varepsilon_l/\beta]$ $E_0=3320\sqrt{f'_{co}}+6900$ $\beta=nf'_{co}/E_0$ $n=0.0588f'_{co}+0.8$	28.9

2. Effective failure strain of FRP jacket

When a FRP-confined concrete cylinder is subjected to axial compression, the lateral strain increases and a circumferential tensile stress is developed in the FRP jacket, which restricts the transverse dilation of the cylinder. Once the circumferential tensile stress in the FRP jacket reaches the tensile strength, failure occurs. Under the assumption of deformation compatibility between the FRP jacket and concrete surface, the lateral confining pressure f_l acting on the concrete cylinder is given by the equilibrium condition of forces as follows

$$f_l = \frac{2tf_{frp}}{D} = \frac{2tE_{frp}\varepsilon_f}{D} \tag{1}$$

Where D is the diameter of the confined concrete cylinder and t, f_{frp}, E_{frp}, and ε_f are the total thickness, tensile strength, elastic modulus, and tensile failure strain of the FRP jacket, respectively.

The compressive strength of FRP-confined concrete is closely related to the tensile failure strain of the FRP jacket wrapping on confined elements. In most existing models, it is assumed that the FRP jacket ruptures once the circumferential tensile stress reaches the tensile strength given by the flat coupon test. With this assumption, the maximum confining pressure can be obtained from Eq. (1). However, some experimental results (Lam and Teng 2004, Realfonzo and Napoli 2011, Smith *et al*. 2010) indicated that the tensile failure strain of the FRP jacket wrapping on confined elements, ε_f, is much smaller than the tensile failure strain, ε_{fu}, given by the flat coupon test. Therefore, it is obvious that the confinement model must rely on the actual tensile failure strain achieved in FRP-confined concrete rather than that given by the flat coupon test. For this reason, a strain efficiency factor $k_\varepsilon = \varepsilon_f / \varepsilon_{fu}$ is introduced. According to Realfonzo and Napoli (2011), k_ε can be taken as 0.6 for CFRP and GFRP jackets regardless of the strength level of concrete. Thus, f_l can be re-expressed as

$$f_l = \frac{2tE_{frp}k_\varepsilon\varepsilon_{fu}}{D} = \frac{2tk_\varepsilon f_{frp}}{D} \tag{2}$$

3. Development of new model

3.1 Strength model derived from Jefferson's failure criterion

To determine the compressive strength of FRP-confined concrete, f'_{cc}, the failure surface under multiaxial stresses proposed by Jefferson (2003) is adopted. The failure surface takes the same meridians as those used in the compressive case of Lubliner *et al*.'s (1989) model. To ensure the continuity of the curve on the π-plane, an elliptical curve is assumed (William and Warnke 1974). With the ellipse, all the conditions of smoothness, symmetry, and convexity are satisfied. If the eccentricity parameter is taken as $1/\sqrt{2}$, the curve is considerably simplified without losing accuracy. Thus, the failure surface is expressed as

$$F(I_1, J_2, \theta) = \sqrt{J_2}A_r(\theta) + \left(\alpha + \frac{\gamma}{3}\right)I_1 - f'_{co}(1-\alpha) = 0 \tag{3}$$

where I_1 is the first stress invariant, J_2 is the second deviatoric stress invariant, θ is the Lode angle (within 0-60 degree), f'_{co} is the uniaxial compressive strength of concrete, α and γ are material parameters determined from f'_{co} and the strength ratio of biaxial to uniaxial compression, b_r, and $A_r(\theta)$ is defined as

$$A_r(\theta)=\rho_c\left(\frac{2\cos(\theta)^2+b^2}{\cos(\theta)+\sqrt{2\cos(\theta)^2+c}}\right) \tag{4}$$

with $b=\sqrt{2}-1$, $c=(5-2\sqrt{2})/2$, and $\rho_c=(\gamma+3)/\sqrt{3}$.

The parameters α and γ are given by Lubliner *et al*. (1989)

$$\alpha=\frac{b_r-1}{2b_r-1},\quad \gamma=\frac{3(1-\rho)}{2\rho-1} \tag{5}$$

It is expected that b_r is generally in the range of 1.05 to 1.3 (Kupfer *et al*. 1969). However, recent studies on high-strength concrete indicated that b_r decreases by increasing f'_{co}. Based on various experimental results, b_r is expressed as a power function as follows (Papanikolaou and Kappos 2007)

$$b_r=1.5\left(f'_{co}\right)^{-0.075} \tag{6}$$

Substituting the principal stresses $\sigma_1=\sigma_2=-f_l$ and $\sigma_3=-f'_{cc}$ into Eq. (3), the relationship between the confined concrete strength f'_{cc} and the lateral confinement pressure f_l is given by

$$\frac{f'_{cc}}{f'_{co}}=1.0+\frac{1+2\alpha+\gamma}{1-\alpha}\frac{f_l}{f'_{co}} \tag{7}$$

Eq. (7) has the same form as the model proposed by Richart *et al*. (1928). From Eq. (7), it can be seen that the confinement effectiveness coefficient k is equal to $(1+2\alpha+\gamma)/(1-\alpha)$. Obviously, k is a function of the unconfined strength f'_{co} and can be determined from Eqs. (5) and (6). Combining with Eq. (2), the confined strength f'_{cc} can be obtained from Eq. (7).

3.2 Axial strain at peak stress

The behavior of FRP-confined concrete is different from that of steel-confined concrete. The FRP jacket is a linear elastic material and provides a continuously increasing confining pressure on concrete specimens up to failure, whereas the steel tube applies a constant confining pressure on concrete specimens after yielding. Therefore, the lateral confinement pressure provided by the FRP jacket is of the passive type.

Usually, the axial stress of concrete, σ_c, at any axial strain level, ε_c, can be expressed as

$$\sigma_c=E_{\sec}\varepsilon_c \tag{8}$$

where $E_{\sec}$ is the secant modulus of concrete and directly reflects the stiffness of concrete. When the concrete cylinder is subjected to axial compression, the core area expands laterally, resulting in microcracks, damage and loss of stiffness. Therefore, the secant modulus of concrete is closely related to the lateral expansion of the core area. In this paper, $E_{\sec}$ is expressed in terms of the area strain ε_a (for cylinders, $\varepsilon_a=2\varepsilon_l$) as follows (Pantazopoulou and Mills 1995, Lee and Hegemier 2009)

$$E_{\sec} = \frac{E_0}{1 + 2\varepsilon_l / \beta} \tag{9}$$

where β is the secant modulus softening rate and E_0 is the initial elastic modulus of concrete, which are determined as follows (Carrasquilio *et al*. 1981, Lee and Hegemier 2009)

$$E_0 = 3320\sqrt{f'_{co}} + 6900 \ \text{(in MPa)} \tag{10}$$

$$\beta = n \frac{f'_{co}}{E_0} \tag{11}$$

with $n = 0.0588 f'_{co} + 0.8$. From Eq. (8), the axial strain at the axial peak stress, ε_{cc}, is equal to

$$\varepsilon_{cc} = \frac{f'_{cc}}{E_{\sec}} \tag{12}$$

3.3 Stress-strain model

The design-oriented stress-strain model proposed by Lam and Teng (2003) is adopted in this paper. The model is based on the following assumptions (Lam and Teng 2003): (1) the stress-strain curve is composed of two portions: the first is a parabola and the second is a straight line, (2) the initial slope is equal to the initial elastic modulus E_0, (3) the parabola is affected by the presence of the FRP jacket, (4) the stress-strain curve is first-order continuous, and (5) the curve ends at a point where both the compressive strength f'_{cc} and corresponding axial strain ε_{cc} are reached. Thus, the stress-strain curve is expressed as

$$\sigma_c = \begin{cases} E_0 \varepsilon_c - \dfrac{(E_0 - E_2)^2}{4 f'_{co}} \varepsilon_c^2, & 0 \le \varepsilon_c < \varepsilon_t \\ f'_{co} + E_2 \varepsilon_c, & \varepsilon_t \le \varepsilon_c < \varepsilon_{cc} \end{cases} \tag{13}$$

where σ_c is the axial stress, ε_c is the axial strain, E_2 is the slope of the second linear portion, and ε_t is the axial strain at the transition point where the first and second portions meet. From the condition of continuity of the stress-strain curve, the transition strain ε_t is easily derived as follows

$$\varepsilon_t = \frac{2 f'_{co}}{E_0 - E_2} \tag{14}$$

The slope of the linear portion, E_2, is simply equal to

$$E_2 = \frac{f'_{cc} - f'_{co}}{\varepsilon_{cc}} \tag{15}$$

4. Model validation

To verify the developed models, a comprehensive database (Ahmad *et al*. 1991, Almusallam 2007, Berthet *et al*. 2005, Campione and Miraglia 2003, Carey and Harries 2005, Cui and Sheikh 2010, Fam and Rizkalla 2001, Fam *et al*. 2003, Fardis and Khalili 1982, Harries and Kharel 2002, Harries and Kharel 2003, Ilki *et al*. 2008, Karbhari and Gao 1997, La Tegola and Manni 1999, Lam and Teng 2003, Lam and Teng 2004, Lau and Zhou 2001, Li 2006, Liang *et al*. 2012, Mandal *et al*. 2005, Mastrapa 1997, Matthys *et al*. 2005, Matthys *et al*. 2006, Mirmiran *et al*. 1998, Miyauchi *et al*. 1999, Nanni and Bradford 1995, Pessiki *et al*. 2001, Purba and Mufti 1999, Rochette and Labossiere 2000, Shahawy *et al*. 2000, Shehata *et al*. 2002, Silva and Rodrigues 2006, Teng *et al*. 2007, Teng *et al*. 2009, Thériault *et al*. 2004, Toutanji 1999, Wang and Wu 2008, Watanabe *et al*. 1997, Wu *et al*. 2006, Xiao and Wu 2000, Xiao and Wu 2003) with 520 FRP-wrapped concrete circular columns is established. Among the 520 specimens, 12 CFRP-wrapped circular specimens were conducted by the authors and their dimensions, number of CFRP plies, and mechanical parameters are listed in Table 3. The database includes 313 CFRP-wrapped specimens and 207 GFRP-wrapped specimens with diameters D from 51 to 406 mm and with unconfined concrete strengths from 15.2 to 85.6 MPa. The elastic moduli and tensile strengths of FRP are in the range of 5 to 436 GPa and of 76.5 to 4493 MPa, respectively. The specific FRP properties were achieved from the flat coupon tensile test or provided by the manufacturer.

The performance of a model is evaluated in terms of the integral absolute error (IAE). This index can effectively demonstrate the deviation between the theoretical predictions and the experimental results. For the compressive strength f'_{cc}, the IAE is defined as

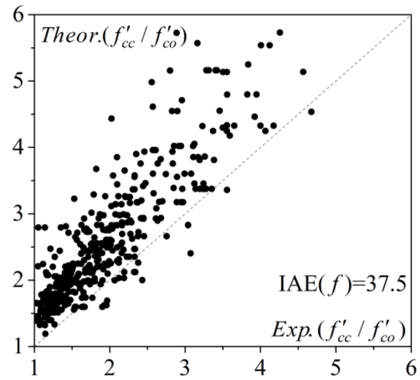
$$\mathrm{IAE}(f)=\sum\frac{\sqrt{[\mathrm{exp.}(f'_{cc}/f'_{co})-\mathrm{theor.}(f'_{cc}/f'_{co})]^2}}{\sum\mathrm{exp.}(f'_{cc}/f'_{co})} \tag{16}$$

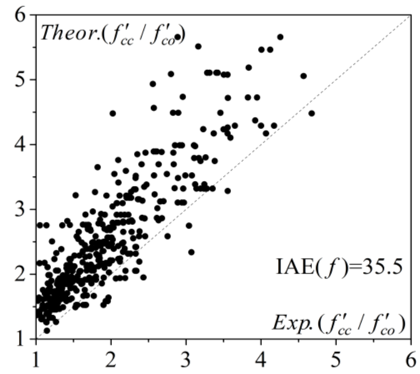
where $\mathrm{exp.}(f'_{cc}/f'_{co})$ and $\mathrm{theor.}(f'_{cc}/f'_{co})$ are the experimental and theoretical ratios of strength between confined and unconfined concrete, respectively. In a similar way, the IAE for the strain ε_{cc} is defined as

$$\mathrm{IAE}(\varepsilon)=\sum\frac{\sqrt{[\mathrm{exp.}(\varepsilon_{cc})-\mathrm{theor.}(\varepsilon_{cc})]^2}}{\sum\mathrm{exp.}(\varepsilon_{cc})} \tag{17}$$

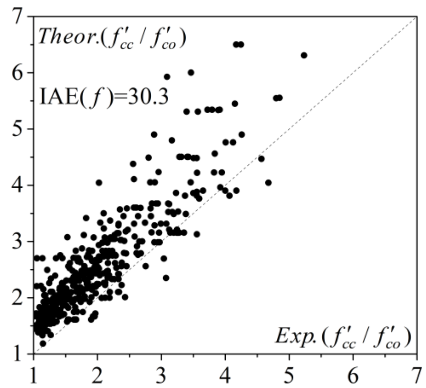
Table 3 Details of specimens and test results

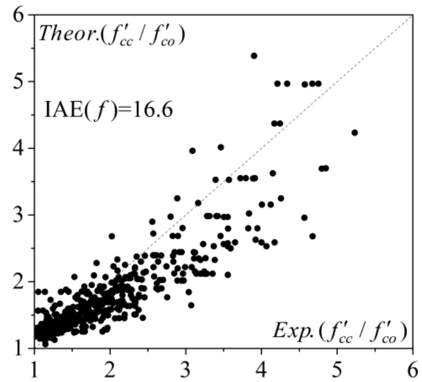
Specimen	Dimensions (mm)	Number of CFRP plies	Specimen	f'_{co} (MPa)	f'_{cc} (MPa)	$\bar{f}'_{cc}$ (MPa)	$\frac{f'_{cc}}{f'_{co}}$	$\frac{f_{l,j}}{f'_{co}}$	ε_{cca} (%)
S1	Φ100×200	1	25.9	64.3	64.6	2.48	0.39	2.31	9.63
S2	Φ200×400	2	22.7	64.3	64.9	2.83	0.45	2.29	10.41
S3	Φ300×600	3	24.5	58.8	60.5	2.40	0.39	1.84	8.36



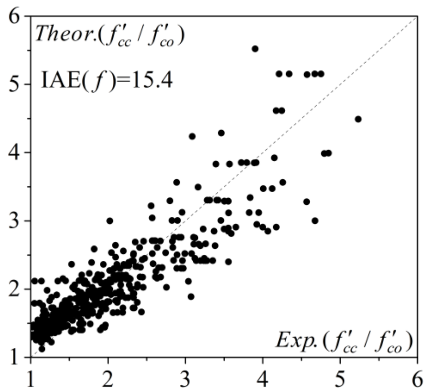


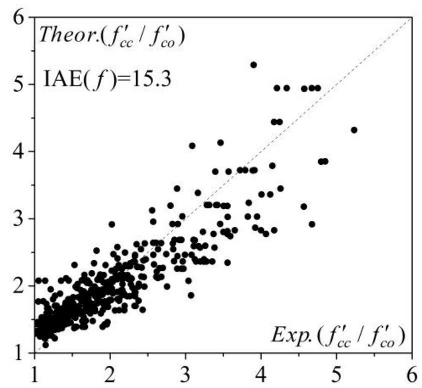
(a) Xiao and Wu (2000) (b) Fardis and Khalili (1982)





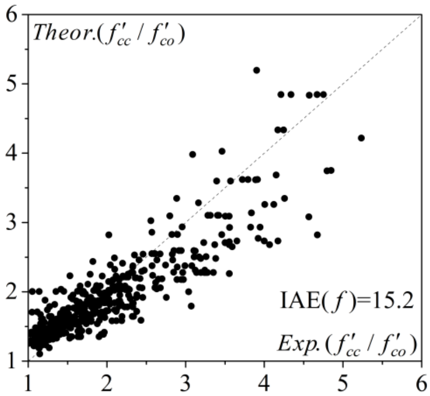
(c) Toutanji (1999) (d) Lam and Teng (2003)



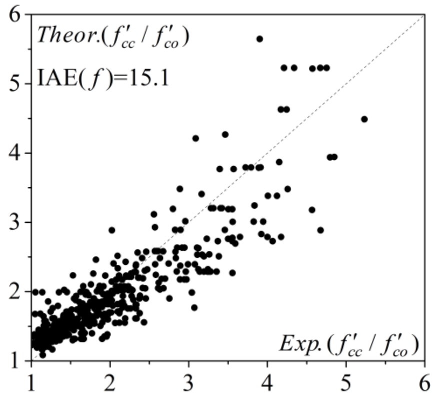


(e) Matthys *et al*. (2005) (f) Saafi *et al*. (1999)

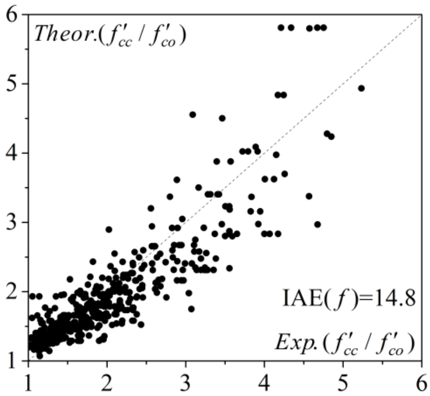
Fig. 1 Performance of various confined strength models



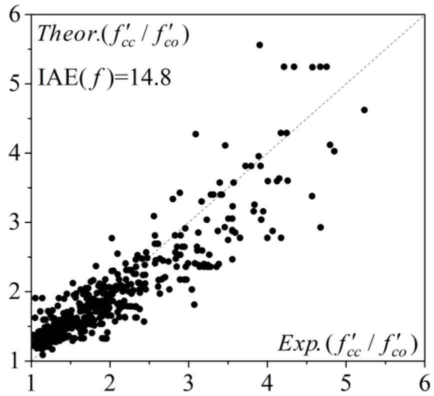
(g) Karbhari and Gao (1997)



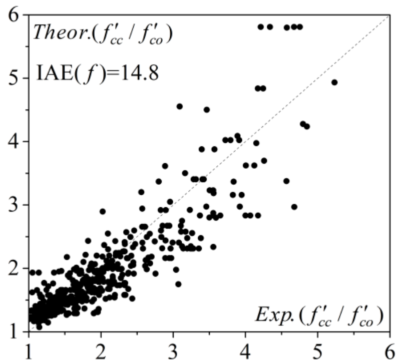
(h) Wu and Wang (2009)



(i) Rousakis and Karabinis (2008)

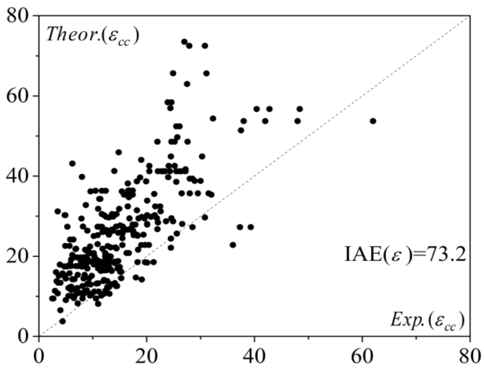


(j) Wu and Zhou (2010)

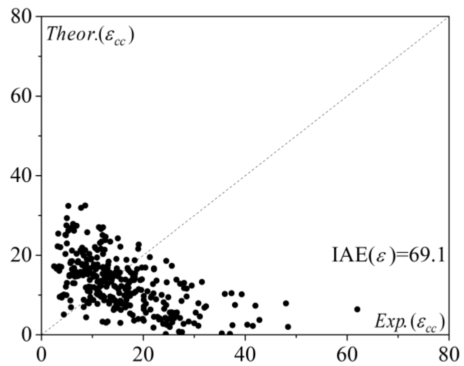


(k) This paper

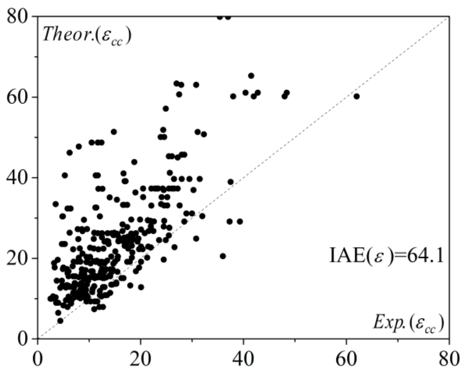
Fig. 1 Continued



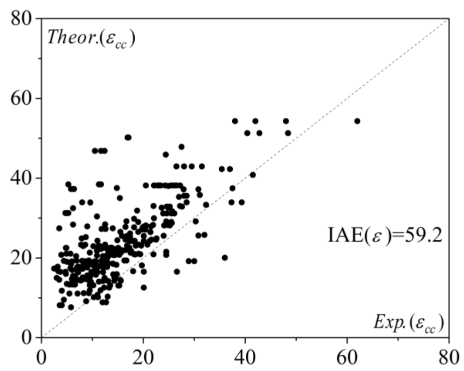
(a) Xiao and Wu (2000)



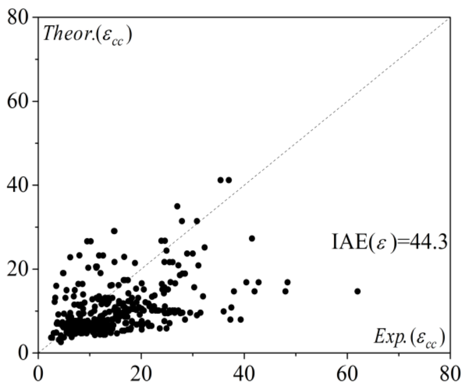
(b) Samaan *et al*. (1998)



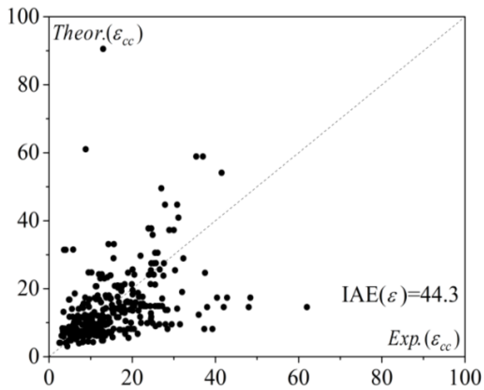
(c) Toutanji (1999)



(d) Spoelstra and Monti (1999)

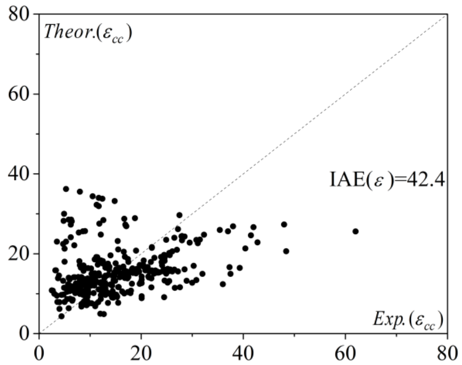


(e) Kono *et al*. (1998)

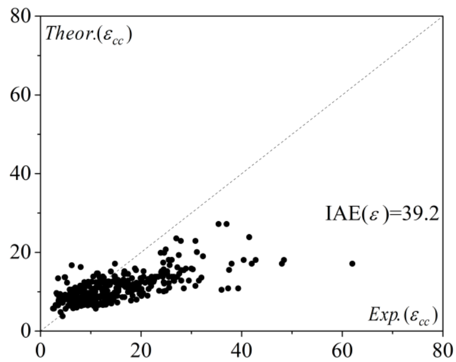


(f) Fardis and Khalili (1982)

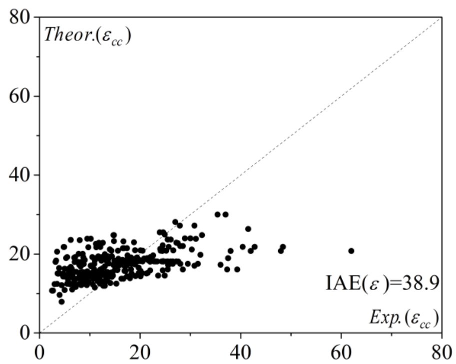
Fig. 2 Performance of various strain models



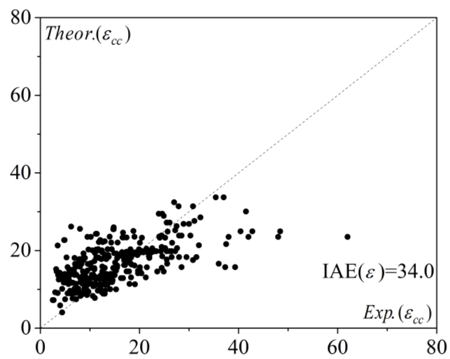
(g) Saafi *et al*. (1999)



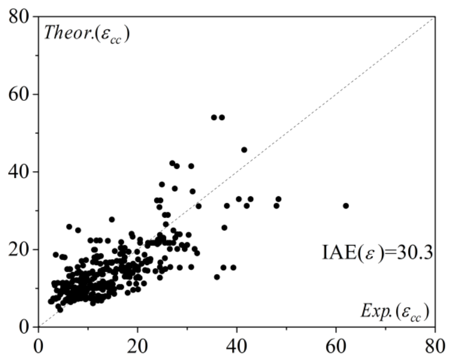
(h) De Lorenzis and Tepfers (2003)



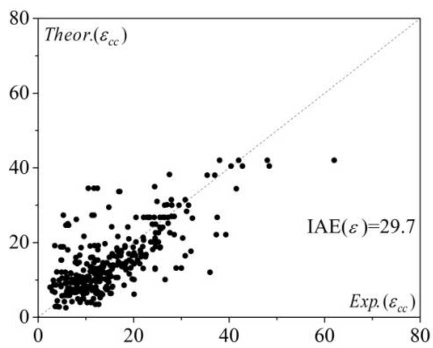
(i) Miyauchi *et al*. (1997)



(j) Saadatmanesh *et al*. (1994)

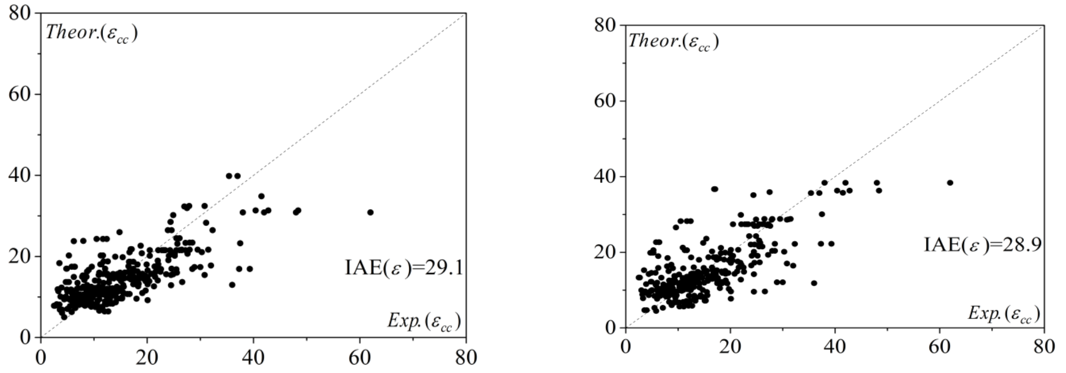


(k) Lam and Teng (2003)



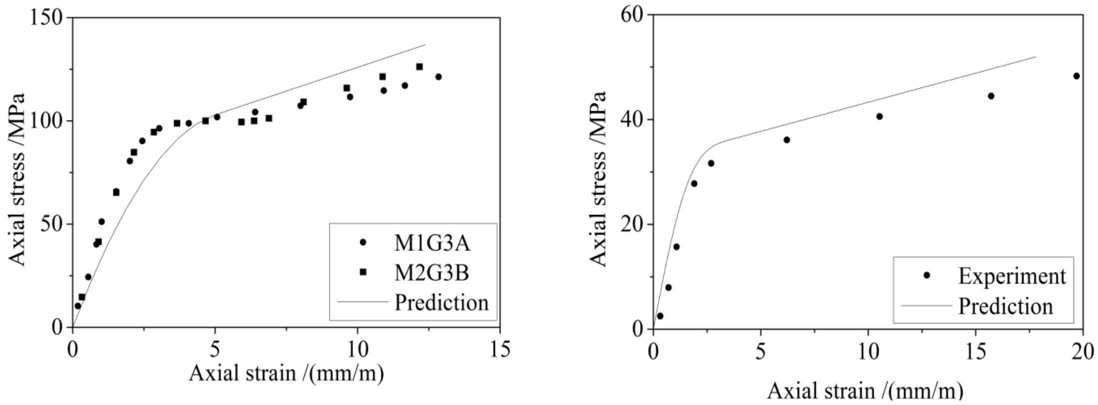
(l) Wu *et al*. (2006)

Fig. 2 Continued

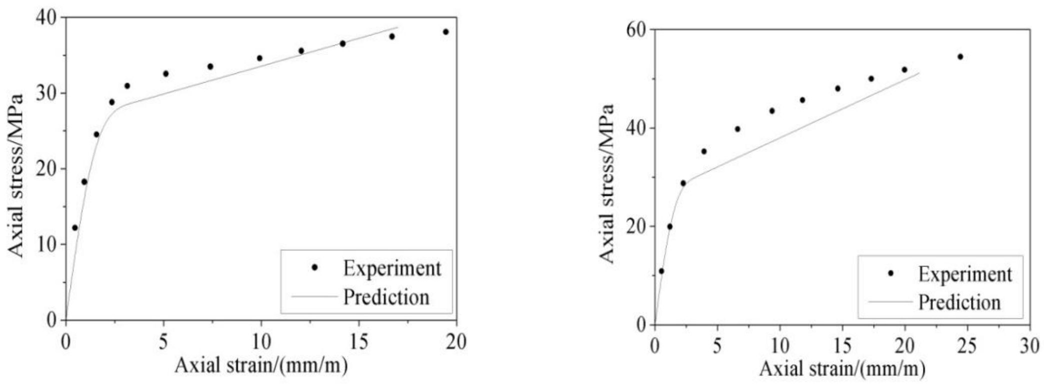


(m) Teng *et al*. (2009) (n) This paper

Fig. 2 Continued

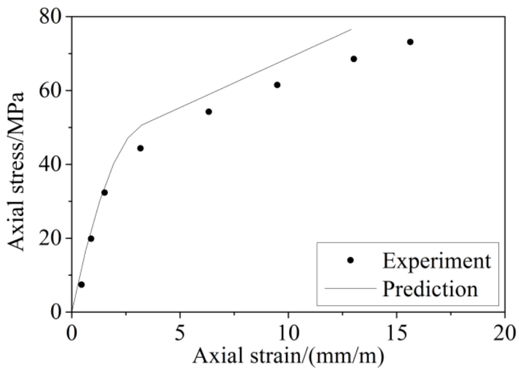


(g) Cui and Sheikh (2010) (case 2) (h) Demers and Neale (1999)

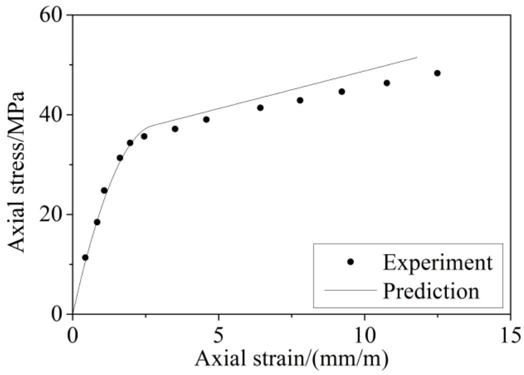


(i) Pessiki *et al*. (2001) (case 1) (j) Pessiki *et al*. (2001) (case 2)

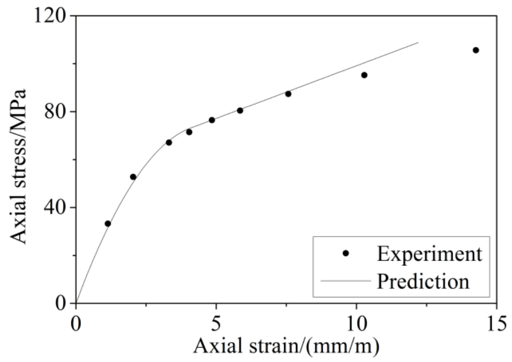
Fig. 3 Comparison of stress-strain curve between the theoretical predictions and experimental results obtained from the literature



(k) Rochette and Labossiere (2000)



(l) Xiao and Wu (2000) (case 1)



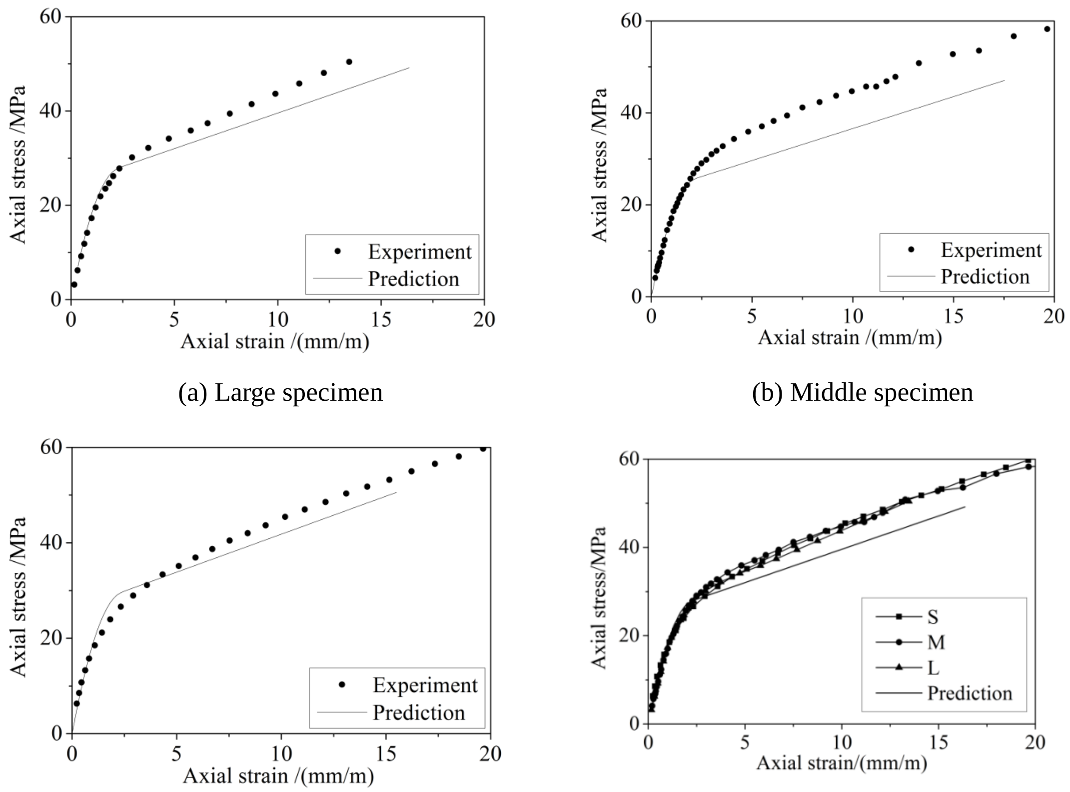
(m) Xiao and Wu (2000) (case 2)

Fig. 3 Continued

where $\exp.(\varepsilon_{cc})$ and $\mathrm{theor.}(\varepsilon_{cc})$ denote the experimental and theoretical failure strains FRP-confined concrete, respectively. To assess the stress-strain model, the IAE for the area under the stress-strain curve (i.e. the energy absorption at failure) is defined as

$$\mathrm{IAE}(A) = \sum \frac{\sqrt{[\exp.(A) - \mathrm{theor.}(A)]^2}}{\sum \exp.(A)} \tag{18}$$

The IAE for different strength models is listed in Table 1. The relationships between the theoretical predictions and the experimental results for some strength models are shown in Fig. 1, where the dash line represents perfect agreement and any point that falls below or above the dash line indicates that the predicted value is smaller or larger than the experimental one. As shown in Table 1 and Fig. 1, Rosakis and Karabinis's model, Wu and Zhou's model, and the model proposed in this paper have the lowest IAE value (14.8%), indicating that the three models yield the best results for the compressive strength of FRP-confined concrete. An important advantage of the proposed model is that the confined strength f'_{cc} can be predicted accurately and therefore only



(a) Large specimen (b) Middle specimen

(c) Small specimen (d) Large, middle, and small specimens

Fig. 4 Comparison of stress-strain curve between the theoretical predictions and experimental results obtained from self-conducted test

the uniaxial compressive strength f'_{co} of concrete needs to be measured.

It should be noted that most of the models listed in Table 2 include the strain ε_{co} at $\sigma = f'_{co}$ for unconfined concrete, which is evaluated by the regression formula (De Nicolo *et al*. 1994)

$$\varepsilon_{co} = 0.00076 + [(0.626 f'_{co} - 4.33) \times 10^{-7}]^{0.5} \tag{19}$$

The initial elastic modulus of concrete is used in Spoelstra and Monti's model, which is determined from Eq. (10).

The IAE for strain models and the relationships between the theoretical predictions and the experimental results are given in Table 2 and Fig. 2, respectively. It can be seen from Table 2 that the IAE for the axial strain at the peak stress, ε_{cc} is larger than that for the compressive strength f'_{cc} due to the larger scatter of measured strains. It is also seen that, Wu *et al*.'s model (2006), Teng *et al*.'s (2009) and the model proposed in this paper have the lowest IAE values, i.e., 29.7%, 29.1%, and 28.9%, respectively, which indicates that the three models yield the best results for the axial strain at the peak stress, ε_{cc}, of FRP-confined concrete.

To verify the modified stress-strain model, a wide range of experimental results on CFRP-confined cylinders (Berthet *et al*. 2005, Campione and Miraglia 2003, Rochette and Labossiere 2000, Xiao and Wu 2000) and on GFRP-confined cylinders (Cui and Sheikh 2010, Demers and Neale 1999, Pessiki *et al*. 2001) are collected from the research literature. Using the same parameters given in these papers, the stress-strain curves can be calculated as shown in Fig. 3, together with the experimental results. The IAE for the area of stress-strain curve is 12.8%. It can therefore be concluded that the modified design-oriented model proposed in this paper is fully capable of capturing the compressive behavior of FRP-confined concrete and correlated well with the experimental results.

To further verify the proposed stress-strain model, three CFRP-confined cylinders with the same slenderness ($L/D = 2$) and lateral confining pressure were tested in this study. The used fiber-reinforced polymer sheet was a type of unidirectional carbon fabric. The measured tensile strength and elastic modulus were 3248 MPa and 245 GPa, respectively. The details of the three specimens and the measured basic parameters are listed in Table 3. The results are shown in Fig. 4. It can be seen from Fig. 4 that the theoretical predictions are in good agreement with the experimental results. Therefore, the validity of the proposed stress-strain model is again verified. It is also interesting to note from Fig. 4 that no obvious size effect on the stress-strain relationship of FRP-confined concrete is observed.

It should be pointed out that the stress-lateral strain behavior of FRP-confined concrete is very crucial since 0.4% concrete lateral strain limit may be decisive. Therefore, a further study is needed to establish a more general tri-axial nonlinear constitutive model to estimate the lateral strain of FRP-confined concrete.

6. Conclusions

A simple and efficient method has been developed in this paper for predicting the compressive behavior of FRP-confined concrete. Based on Jefferson’s failure surface, the compressive strength has been expressed as a function of lateral confining pressure. The axial strain at the peak stress has been evaluated using the relationship between the secant modulus and lateral strain. By modifying Lam and Teng’s model, a stress-strain relationship of FRP-confined concrete has been established. The validity of the proposed compressive strength and strain models and the modified stress-strain relationship has been verified with a wide range of experimental results collected from the literature and obtained from self-conducted test. Based on the numerical results, it has been found that Rousakis and Karabinis’s model, Wu and Zhou’s model and the proposed model yield the best results for the compressive strength and that Wu *et al*.’s model, Teng *et al*.’s model, and proposed model yield the best results for the axial strain at the peak stress. It has also been found that the proposed stress-strain relationship is fully capable of capturing the compressive behavior of FRP-confined concrete.

Acknowledgements

The financial support from the National Natural Science Foundation with Grant No. 51121005, 50578025, and the Natural Science Foundation of Zhejiang Province with Grant No. LY12E08022, of the People’s Republic of China, are greatly acknowledged.

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