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Genetic-fuzzy approach to model concrete shrinkage

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Abstract. This work presents an approach to model concrete shrinkage. The goal is to permit the concrete industry's experts to develop independent prediction models based on a reduced number of experimental data. The proposed approach combines fuzzy logic and genetic algorithm to optimize the fuzzy decision-making, thereby reducing data collection time. Such an approach was implemented for an experimental data set related to self-compacting concrete. The obtained prediction model was compared against published experimental data (not used in model development) and well-known shrinkage prediction models. The predicted results were verified by statistical analysis, which confirmed the reliability of the developed model. Although the range of application of the developed model is limited, the genetic-fuzzy approach introduced in this work proved suitable for adjusting the prediction model once additional training data are provided. This can be highly inviting for the concrete industry's experts, since they would be able to fine-tune their models depending on the boundary conditions of their production processes.

Keywords: concrete; fuzzy logic; genetic algorithm; genetic-fuzzy; material modeling; shrinkage

1. Introduction

Concrete shrinkage is defined as decrease in concrete volume with time. This volume decrease does not depend on external stress and it is not completely reversible. Though shrinkage corresponds to a volumetric change, it is usually expressed in linear strain rather than volumetric units since real volumetric shrinkage is approximately three times the linear shrinkage (Brooks 2003). The occurrence of shrinkage in concrete is associated with a series of factors such as chemical reaction, gradient in temperature, and movement and loss of water. Each one of these leads to a different type of shrinkage, e.g., autogenous, plastic, drying and thermal shrinkage, among others. For a comprehensive review refer to Brooks (2003) and Mehta and Monteiro (2004). Although different types of shrinkage are known, there is no sharp boundary between them. This is because they occur simultaneously, especially those on fresh state. Therefore, dividing the total shrinkage, which corresponds to the sum of all types of shrinkage, in its specific parts is, in practice, a complex task, if not an impossible mission.

The occurrence of shrinkage in concrete leads to development of internal tension stresses,

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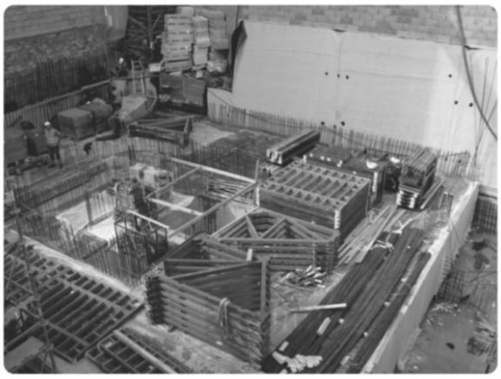
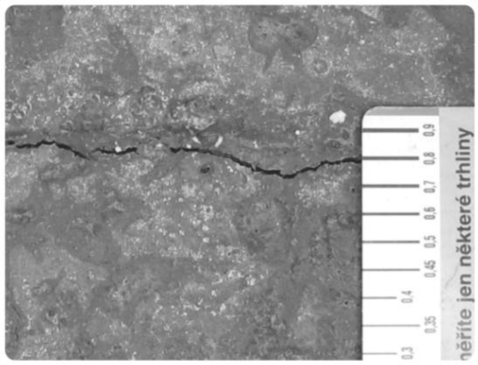


(a)



(b)

Fig. 1 Foundation block of a residential building in Prague, Czech Republic: (a) overall view of the construction side and (b) detail of ~0.8 mm crack width caused by concrete shrinkage

which can result in concrete cracking if the concrete mass has any restrains to deform and the developed stresses surpass the concrete tensile strength. In particular, restrains in concrete derive from the presence of reinforcements, connection members, formwork, and aggregates, to mention a few. Cracks in concrete create pathways for the easier access of aggressive agents, e.g., chloride ions, sulfates, and CO_2, that can cause corrosion in the steel reinforcement and chemical attack in the concrete, thereby contributing to the reduction in concrete's structure durability. An example of the effect of shrinkage on concrete is depicted in Fig. 1.

Furthermore, excessive shrinkage strain may reduce bounding tension, which results in loss of prestress in prestressed concrete, and increase deflection in asymmetrically reinforced concrete structures (Mehta and Monteiro 2004); therefore a reliable prediction of concrete shrinkage strain is an important factor in the entire designing process. Predicting shrinkage, especially during the period of construction, allows early counter-measures to be taken, e.g., premature loading or prestressing to compensate for the negative shrinkage effect. Consequently, accurate shrinkage prediction helps reduce maintenance costs and ensures that the designed structure will meet service life and durability requirements.

Apart from external factors such as ambient temperature and humidity, concrete shrinkage is caused by the hydration reaction in the cement paste. Thus, for a constant volume of concrete and environmental conditions, the higher the cement paste volume, the higher the shrinkage strain (Rozière *et al.* 2007). Because self-compacting concrete mixtures (SCC), i.e., concrete that can flow under its own weight and self-consolidate without any mechanical vibration (Skarendahl 2005), requires a high volume of cement paste in its composition to achieve excellent deformability and formwork filling capacity, this type of concrete is likely prone to higher shrinkage strain when compared to conventional concrete. Therefore measuring shrinkage strain is important when working with SCC.

Regardless the concrete type, the experimental measurement of shrinkage strain is not only a laborious and time consuming work, but also expensive. These characteristics, added to short deadlines, tight budgets, and the industry's trends of accelerating both designing and construction processes, sometimes make shrinkage experimental measurements unfeasible for all projects under development. For this reason, construction designers tend to use concrete shrinkage prediction models.

Shrinkage prediction models aim to determine concrete shrinkage strain in a faster and less expensive way when compared to experimental measurements. These models are based on several parameters, which depend on the approach that was considered when the model was developed. Commonly, prediction models are based either on analytical or empirical approach. Regardless the considered approach, developing a robust and reliable prediction model is not an easy task, because shrinkage is a complex phenomenon that is hard to be modeled due to the simultaneity of different types of shrinkage and interrelationship of factors affecting it. Furthermore, the fact that no common agreement has been found about the mechanism of shrinkage (Illston and Domone 2010) adds considerable difficulty and uncertainties to model shrinkage from a micro-mechanical perspective. According to Bažant and Baweja (2000), the abovementioned complexity of concrete shrinkage phenomenon explains why improvements in shrinkage prediction models are only gradual and slow with no sudden or significant breakthrough.

Although analytical and micro-mechanical based approaches have been presented in several researches, e.g., Benboudjema *et al.* (2005), Ling and Meyer (2008), and Pichler *et al.* (2007), experimental-based models remain as the most frequently used ones. The main reason for that is likely due to the simplicity of these models, which turns to be user-friendly. When using prediction models, it is important to keep in mind that complex models do not necessarily lead to better prediction than simple ones (McDonald and Roper 1993).

Among several existing shrinkage prediction models, highlights are given to ACI209, CEB FIP 1990, CEB MC 90-99, B3, GL2000, and EN 1992-1-1:2004. The details of these are described as follows:

ACI 209

The American Concrete Institute (ACI) model, namely ACI209, was developed by Branson and Christiason (1971). It consists in an empirical model that is based on experimental approach. The model has been updated through years by the ACI Committee 209, and its latest version dates back to 2008 (ACI 2008). The ACI209 model is limited to normal-weight, light-weight, and sand-light-weight concrete mixtures produced with cement content varying from 279 to 446 kg/m^3, relative humidity above 40.0%, and either steam or moist curing.

CEB FIP 1990

The Comité Euro-International Du Béton (CEB) recommends the CEB FIP 1990 prediction model for concrete shrinkage. CEB FIP 1990, namely CEB FIP, is an empirical model developed in 1990. Although an updated version of CEB FIP model was released in 1999 (CEB 1999), this model was considered in the present work because its equations are the basis for the shrinkage prediction model that is recommended by European Code (EN 2004). CEB FIP model is applicable for normal-weight concrete mixtures with average strength at 28 days ranging from 20.0 to 90.0 MPa, mean ambient temperature varying from 5.0 to 30.0 °C, and ambient relative humidity > 40.0%. Although curing type and time are not considered in the list of input parameters, the curing is limited to moist curing type for period of time no longer than 14 days (CEB 1993).

CEB MC90-99

The CEB MC90-99 model, namely CEB MC, is an updated version of previously described model (CEB FIP). Even with similar limitations regarding curing type and ambient relative humidity, CEB MC differs from CEB FIP because it covers a wider range of average compressive strength at 28 days (15.0 to 120.0 MPa), and a relatively narrower range of ambient temperature, (10.0 to 30.0°C) (CEB 1999).

Table 1 Physical properties and chemical compositions of raw materials

Concrete type	Input data	*N*	f_{model} [%]					
			ACI209	CEB FIP	CEB MC	EN 1992	B3	GL2000
CC	Imamoto and Arai (2008)	14	34.8	46.2	38.4	65.2	38.2	33.5
	Ballim (2000)	10	39.2	28.1	47.0	40.2	27.6	31.5
	White and Newtson (2006)	8	24.1	27.6	36.1	65.1	23.5	45.1
	$f_{all\text{-}CC}$ [%]		33.3	35.1	40.8	58.0	30.4	37.2
SCC	Guneyisi *et al.* (2010)	8	30.7	43.7	19.7	51.0	21.3	27.2
	Bouzoubaa and Lachemi (2001)	8	51.0	36.7	28.3	72.3	37.1	30.0
	Leemann *et al.* (2011)	3	32.1	49.1	27.3	63.1	22.3	44.8
	$f_{all\text{-}SCC}$ [%]		39.0	43.5	25.4	62.7	27.9	34.9
HSC	Ray *et al.* (2012)	6	38.9	51.3	33.1	64.9	37.9	28.9
	Bai *et al.* (2005)	3	35.5	26.9	40.9	29.9	60.4	53.3
	Guneyisi *et al.* (2008)	6	38.3	51.9	34.4	64.7	36.2	34.6
	$f_{all\text{-}HSC}$ [%]		37.6	44.9	36.3	55.7	46.2	40.3
	f_{all} [%]		36.7	41.4	34.8	58.9	35.7	37.5

EN1992-1-1:2004

The EN 1992-1-1:2004 model, here referred as EN1992, was designed based on CEB FIP and CEB MC prediction models. Although based on CEB models, the Eurocode indicates that the range of applicability for ambient temperature varies from -40.0 to 40.0°C (EN 2004). Apart from that, all limitations match those from CEB FIP and CEB MC models.

B3

This model was developed by Bažant (1995). The latest version of B3 model dates back to 2000 (Bažant and Baweja 2000). The use of the model is restricted to Portland cement concrete cured for at least one day. In particular, moist, sealed, and steam curing types are taken into account. The *w/c* and *a/c* (aggregate to cement) ratios are limited to a range that varies from 0.35 to 0.85, and 2.5 to 13.5, respectively. A limit to cement content and average strength at 28 days is also presented. In this case, these values range from 160 to 720 kg/m^3 and 17.0 to 70.0 MPa, correspondingly.

GL2000

The prediction model GL2000 is a modified version of the Atlanta 97 model that was first developed by Gardner and Zhao (1993). GL2000 is an experimental-based model that has been influenced by the CEB FIP model. The latest version of GL2000 was released in 2004. According to Gardner (2004), GL2000 model is applicable for concrete mixtures with average strength at 28 days lower than 82.0 MPa. Further, the model is suitable for concrete produced with any type of chemical or mineral admixtures and any kind of casting temperature.

The equations and detailed limitations of each of the discussed models are presented in the Eurocode (EN 2004) and the guide published by the ACI Committee 209 (ACI 2008).

Although constantly used, shrinkage prediction models may lead to different outputs even when the same input data is considered. Moreover, the total shrinkage strain obtained from the prediction models does not always match experimentally measured strains. To justify such statement, the experimental results from conventional concrete (CC) mixtures that were published

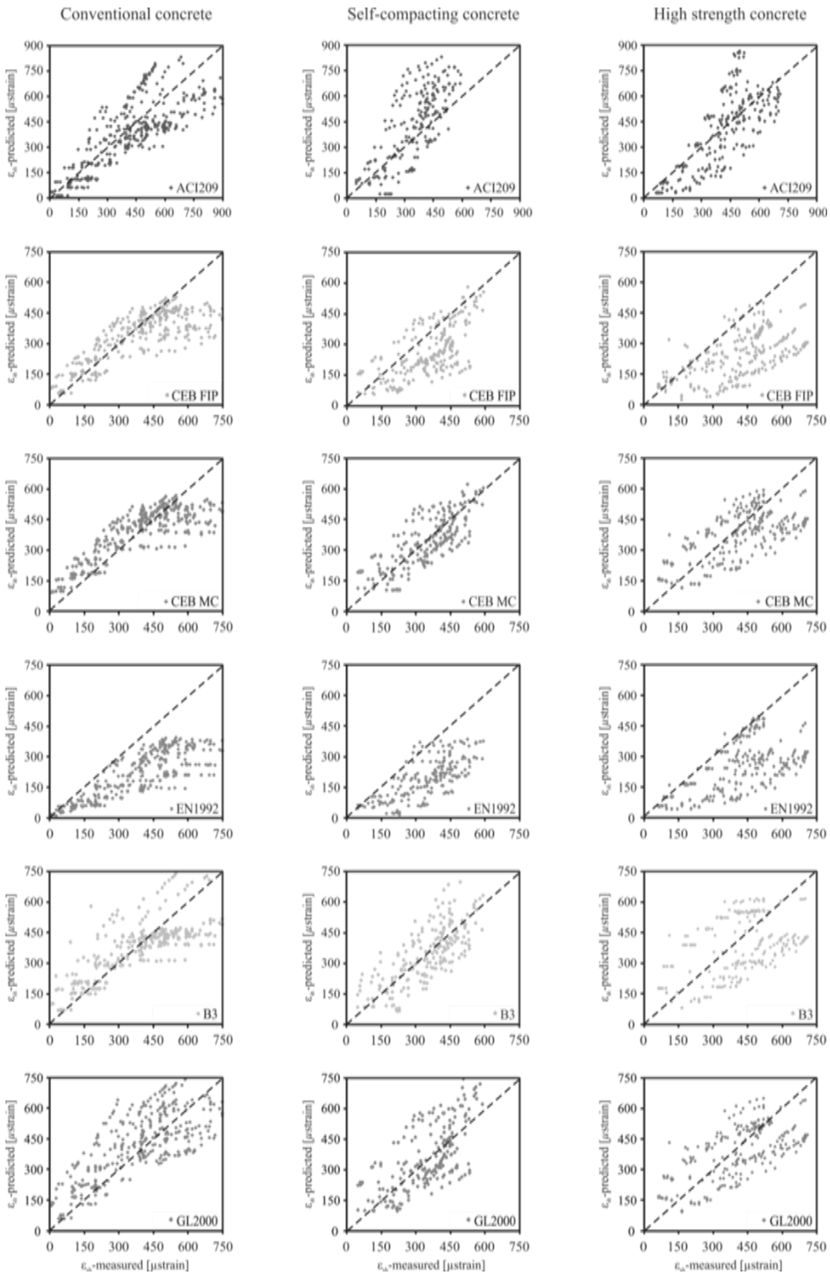


Fig. 2 Comparison of measured and predicted shrinkage strain for different concrete types

by different researchers were compared to the previously discussed prediction models. Besides CC, results from SCC and high-strength concrete (HSC) were also investigated. The interest in these types of concrete is because the application of theirs has dramatically increased in the concrete industry in the past few years, e.g., in the production of precast concrete. Three different papers were analyzed for each concrete type. In total, experimental results from sixty six different mixtures were assessed. The error from each model was computed through the mean squared error (MSE) equation that is described as follows

$$d_j = \frac{P_{ij} - M_{ij}}{M_{ij}} \cdot 100 \tag{1}$$

$$f_j = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} d_j^2} \tag{2}$$

$$f_{all} = \sqrt{\frac{1}{N} \cdot \sum_{j=1}^{N} f_j^2} \tag{3}$$

where P_{ij} is the predicted value of the shrinkage strain for the *i-th* data point in data set j, M_{ij} is the measured value of the shrinkage strain for the *i-th* data point in data set j, N and n are the number of data sets and data points, respectively, considered in the analysis, d_j is the percent difference between predicted and measured data point i, f_j is the MSE for data set j, and f_{all} is the overall MSE.

The results from the preliminary analysis are summarized in Table 1.

The results from Table 1 shows that CEB MC presented the lowest MSE, while EN1992 had the highest error. Nonetheless, an all-purpose classification can lead to misleading interpretation of the model since the same model, e.g., CEB MC ($f_{all\text{-}CC}$=40.8% and $f_{all\text{-}SCC}$=25.4%), sometimes exhibits different MSE when different types of concrete are investigated. For this reason, an independent analysis for each type of concrete is likely to improve the classification and identify the model that best fits each type of concrete.

The results for each type of concrete, see Table 1, indicates that B3, closely followed by ACI209 and CEB FIP, had the lower coefficient of variation for CC mixtures, while EN1992 presented the higher value. For SCC mixtures, CEB MC and B3 models had the best performance, whereas EN1992 had the higher coefficient of variation. Finally, for HSC mixtures, CEB MC, followed by ACI209 and GL2000, presented the best result, while EN1992 had the worst one. The measured and predicted strains (scatter plot) for different concrete types are presented in Fig. 2.

The graphs from Fig. 2 show that EN1992 tends to underestimate shrinkage strain values regardless the type of concrete that is analyzed. The other models, conversely, only present local trends for different types of concrete. The divergence of measured and predicted results, as well as of prediction models themselves, is likely connected to the unavoidable difference between properties of the materials used in each experimental study, thereby indicating that the reliability of the prediction models is open to discussion.

Instead of relying on the presented prediction models, which may not necessarily fit to certain applications, it would be interesting if the experts from concrete industry were able to develop their own prediction models based on a reduced number of experimental data. To improve the reliability of the predicted results, such experimental data would be closely related to the boundary conditions of an independent productions process, e.g., production of precast structural members.

In that light, the present work aims to develop an independent experimental-based modeling approach that focus on predicting concrete shrinkage. One of the tools that come in hand to develop experimental-based prediction models consists in using soft-computing techniques such as fuzzy logic and genetic algorithm (Idorn 2005). The main goal of the proposed methodology is to permit users to predict material's behavior based on a reduced number of experimental data, decreasing in this way costs associated with the performance of these tests. In order to provide an insight of the proposed approach, a simplified shrinkage prediction model for SCC was developed. The results from the developed model were compared against to published data that were not used in model development and to the prediction models that were discussed in the introduction of this work.

2. Soft computing

2.1 Fuzzy logic

The concept of fuzzy logic was first introduced by Zadeh (1965). It corresponds to a natural way of thinking where verbally expressed rules are applied to deal with imprecision and uncertainties commonly present in engineering applications. The use of linguistic expressions allows for dealing with partial truth-values, i.e., the truth-value ranges from completely true to completely false according to a degree of truth (or degree of membership). This characteristic makes fuzzy logic reasoning a very robust and flexible tool that can be used in control (e.g., Pourzeynali *et al.* 2007) and material modeling (e.g., Štemberk and Rainová 2011).

Within the applications of fuzzy logic, an emphasis is are put on Mamdani and Assilian (1975) publication, which was the first to demonstrate that fuzzy logic could be used in a profitable manner in controlling systems. After that, several applications followed, e.g., in commercial systems (Graham 1991) and improvement of car performance (Boverie *et al.* 1993). The application of fuzzy logic has also presented successful results in concrete industry and technology, e.g., control of cement kiln (Holmblad and Ostergaards 1982), diagnosis of concrete bridge deterioration (Zhao and Chen 2001), and prediction of bond strength of concrete (Tanyildizi 2009), and to control SCC and Ready-mixed concrete production (da Silva and Štemberk 2013 a,b), to mention a few.

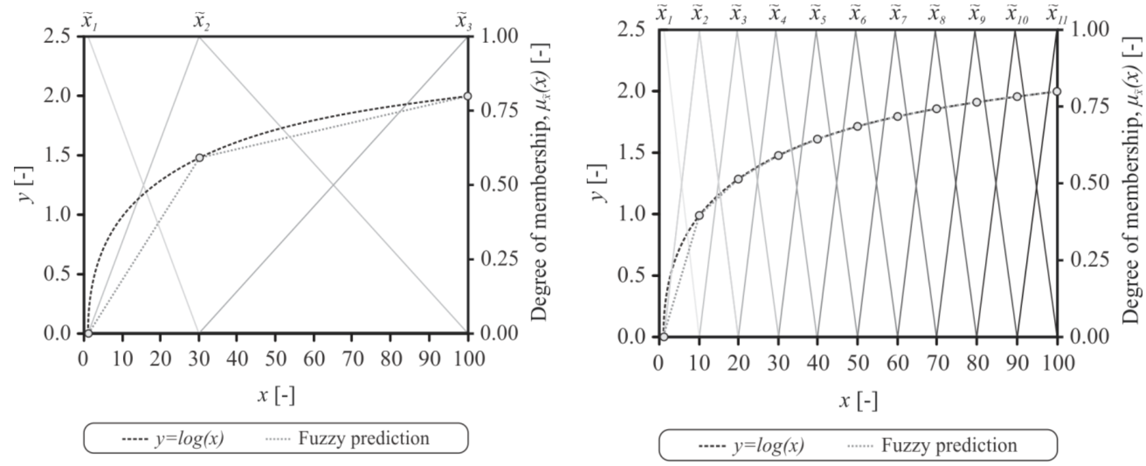
Fuzzy logic is represented by fuzzy expressions. These can be translated as mapping functions from $[0,1]^n \rightarrow [0,1]$, where n relates to the number of relations from the fuzzy sets. Fuzzy expressions are represented by *if-then* rules, e.g., "*if x is a, then y is b*". The rules are composed by an antecedent, i.e., the *if* part "*x is a*", and a consequent, i.e., the *then* part "*y is b*". In fuzzy logic, the antecedent and the consequent are usually linguistic variables.

The *if-then* rules can be declared by the relation R_n, which is generally expressed as

$$R_n : \tilde{A}(x) \rightarrow \tilde{B}(y) \tag{4}$$

where $\tilde{A}$ represents the input fuzzy set, defined on X, whereas $\tilde{B}$ is the output fuzzy set, defined on Y, and the index n corresponds to the number of rules of the system. The interpretation of each rule involves three basic steps: fuzzification, inference engine, and defuzzification.

First, the fuzzification process of maps the input data to compute the degree of membership $\mu(x)$ of an input x in each of the *n-th* rules of the system. Next, the inference engine deduces an output for a corresponding input by operating the n rules from the rule base of the system. The output from the inference engine depends on the type of operator that is used to deal with the input variables. Among several methods, emphases are put on Mamdani, Larsen, Tsukamoto, and Sugeno methods since they are usually applied in fuzzy systems (Chak *et al.* 1997, Lee 2004). Finally, the defuzzification converts a fuzzy value into a crisp value that will correspond to final



(a) (b)

Fig. 3 Comparison of the final shape of shrinkage strain curve predicted by the classical fuzzy logic approach with (a) three fuzzy sets ($f_{3\text{-}sets}$ = 25.0%) and (b) eleven fuzzy sets ($f_{11\text{-}sets}$ = 12.2%)

the output of the system. In general, the defuzzification methods can be classified in two types: composite moments and composite maximum. The former is connected to the first moment of inertia of the sets, whereas the latter is related to the maximum value from the output fuzzy sets (Nguyen *et al.* 2002).

Notice that selecting the most suitable inference engine and defuzzification technique is a context-dependent problem. Although strategies for selection are not properly defined, some criteria can be considered for guidance, e.g., continuity and computational simplicity. Additional criteria are discussed in Hellendoorn and Thomas (1993).

The combination of the described steps provides a decision-making framework of representing human expert rules to infer human decision. The key factors to achieve an acceptable performance in a fuzzy system are connected to the number of fuzzy sets and the size of the system's rule base. Commonly, there are m^k fuzzy rules, where m and k are the number of fuzzy sets and input variables, respectively.

In the classical fuzzy logic approach, the number of fuzzy rules can be reduced based on by the user's experience. Furthermore, the shape of the fuzzy sets is usually set as linear to simplify calculations. Examples of the classical approach in the designing of different material models can be found in Pokorná and Štemberk (2010) and Štemberk and Rainová (2011). Nevertheless, when this approach is implemented to model the behavior of non-linear materials, the final result is a rather roughly shaped piecewise curve. As a result, the predicted curve has discontinuous first derivative and cannot be readily used in finite element software.

Despite the drawbacks, the classical approach is feasible for material modeling; however, additional more linear fuzzy sets are required to obtain smoothed curves. The situation is illustrated in Fig. 3, where two models composed of three and eleven linear fuzzy sets, respectively, were developed to predict the results from the logarithmic function $y = log(\mathrm{x})$ for the interval $x \in [0,100]$. In both cases, the representative points of each fuzzy set were excluded from the calculations of the MSE.

The results shown in Fig. 3 indicate that using the classical approach demands the collection of

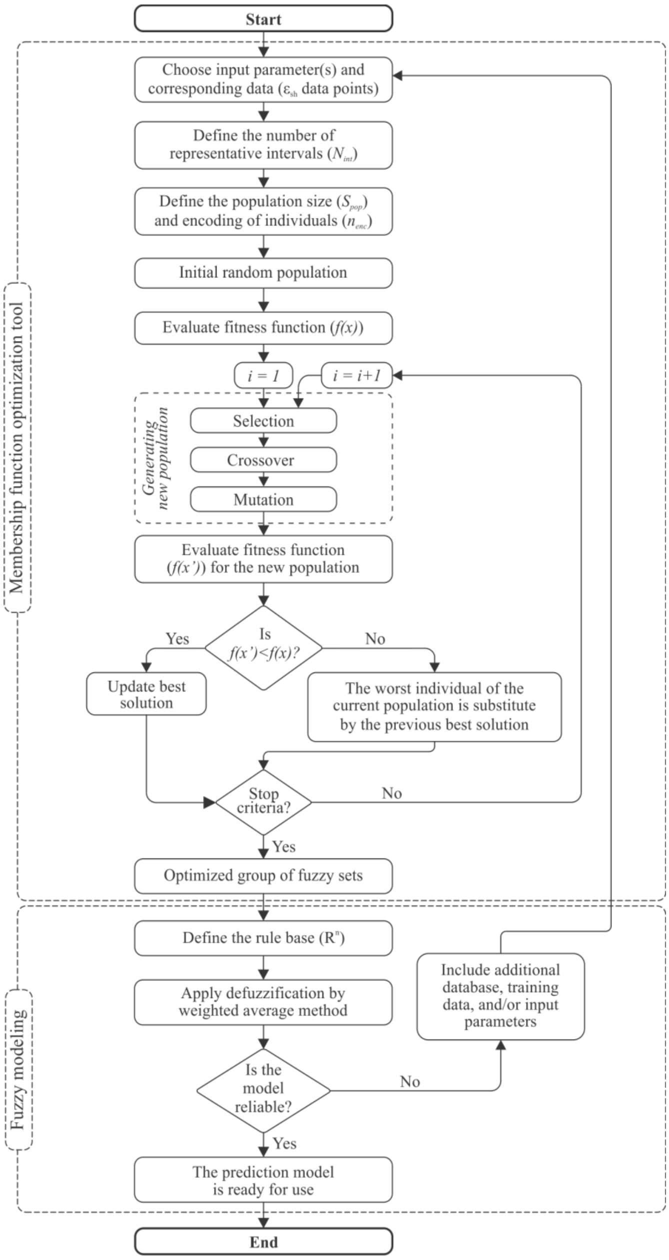


Fig. 4 Flowchart of the methodology for optimization of fuzzy sets by genetic algorithm

a large number of data points to obtain reliable results. In particular, the more complicated the shape of the curve to be modeled, the larger the amount of data to be collected. To improve the modeling process and thus reduce the user's workload, a modified approach is proposed in this work. The approach combines fuzzy logic and genetic algorithm for optimizing the shape of the membership functions.

2.2 Genetic algorithm

Genetic algorithms (GA) are stochastic adaptive methods that can be used for searching and optimization problems. They have been applied successfully for numerical optimization in engineering applications, e.g., Pezeshk *et al.* (2000) and Rokonuzzaman and Sakai (2010). Compared to traditional optimization methods, such as calculus-based and enumerative strategies, GAs are robust, global, and may be applied generally without recourse to domain specific heuristics.

Although performance is affected by these heuristics, GAs operate on a population of potential solutions, applying the principle of survival of the fittest to produce successively better approximations to a solution (Coley 1999, Han and Kim 2002). More specifically, individuals in a population compete for resources and mates. The most successful individuals in each competition will produce more offspring than those that performed poorly. Genes from fittest individuals propagate throughout the population so that two good parents will sometimes produce offspring that are better than either parent. Consequently, each successive generation will become more suited to their environment.

To sum up, it can be said that GAs aim to use selective breeding of the solutions to produce offspring better than the parents by combining information from the chromosomes (Mitchell 1998). The basic form of a GA involves the following three operators to achieve evolution: selection, or reproduction, crossover, and mutation (Coley 1999).

3. Genetic-fuzzy approach to model concrete shrinkage

The proposed approach is composed of the following parts: (i) membership function optimization tool and (ii) fuzzy modeling. The first part aims to optimize the fuzzy sets that will be used in the experimental-based model. The second part combines the optimized fuzzy sets to build the prediction model.

The framework of the proposed approach is illustrated in Fig. 4, and the details regarding each part of the approach are described in the following sections.

3.1 Membership function optimization tool

The membership function optimization tool combines the principles of fuzzy logic and Gas genetic algorithms to the optimize the fuzzy decision-making by minimizing the number of fuzzy sets and thereby fuzzy rules. This is achieved through optimization of the shape of the membership functions that compose the fuzzy subsets. Bearing in mind this idea, the following approach is proposed.

The adjustments are performed through the definition of the exponent value E_L and E_R of the general membership function illustrated in Fig. 5.

The general equation of the membership function to be optimized (Fig. 5) can be written as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < x_1 \text{ and } x > x_3 \\ \left((x - x_1)/(x_2 - x_1)\right)^{E_L}, & x_1 \le x \le x_2 \\ \left((x_3 - x)/(x_3 - x_2)\right)^{E_R}, & x_2 \le x \le x_3 \end{cases} \tag{5}$$

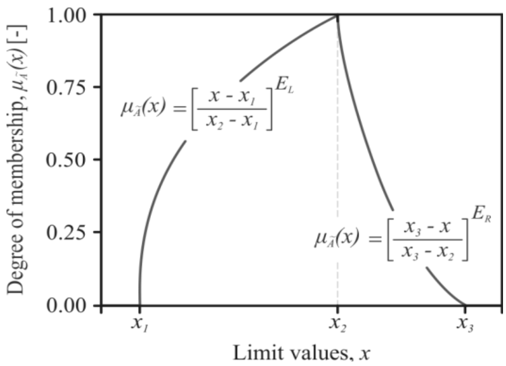


Fig. 5 General membership function to be optimized

Prior to starting the optimization process, the user has to determine the number of representative intervals (N_{int}) of the curve to be modeled. Each representative interval comprises an initial point (x_a , y_a) and a final point (x_b , y_b). The more complex the shape of the curve, the higher the number of intervals needed to achieve optimal results. After that, the user specifies the size of the population (S_{pop}) to be used in the genetic part of the algorithm.

From this point on the optimization process is automatic. Based on N_{int}, the encoding of each individual from the population is defined. It comprises a string of $n_{enc} = 2 \times N_{int}$ numbers, which corresponds to the exponent values (E_L and E_R) from the membership function to be optimized, see Eq. (5). Further, a random population is generated and the fitness function $f(x_i)$ is evaluated, where the index i represents the number of the population. The fitness function corresponds to the MSE between the predicted values and training data of the system, see Eqs. (1) and (2). Subsequently, three genetic operators (selection, crossover, and mutation) are applied to generate a new population. The selection operator (tournament selection) chooses the chromosomes for reproduction. To create a new individual to be included in the population, the selected chromosomes are crossed over by one-point crossover scheme with a probability C_{prob} = 90.0%. Further, mutation by disturbance is performed with a probability M_{prob} = 10.0%. After a new population is generated, the fitness function re-evaluates all individuals from the new population. The obtained results, i.e., $f(x_{i+1})$, are then compared with those from previous populations and the best overall solution is stored. In case none of base individuals from the new population shows better fitness than the stored solution, the individual with the worst solution from the new population is replaced with the best overall solution.

The automatic process of generating a new population and evaluating the best fit is repeated until convergence occurs. In particular, convergence is considered as achieved when more than 200 consecutive iterations do not lead to any improvements in the fitness function result. The final result consists of a group of optimized fuzzy sets that will compose the fuzzy decision-making.

3.2 Fuzzy modeling

The size of the rule base of the system depends on the number of input parameters selected for the prediction model and the number of optimized fuzzy sets. The general form of the rule base is presented as follows

$$R_n : if\ I_1\ is\ \tilde{I}_1, I_2\ is\ \tilde{I}_2, \cdots I_k\ is\ \tilde{I}_k, then\ O_n\ is\ \tilde{O}_n \quad (6)$$

where *n* is the number of curves in the database, *k* is the number of input parameters, $\tilde{I}_k$ and $\tilde{O}_n$ are the group of optimized fuzzy sets for the *k-th* input and *n-th* output of the system, respectively, and O_n is the crisp output of the *n-th* rule of the fuzzy system.

The decision-making of the system is based on the standard Sugeno inference engine. Nonetheless, the output from each fuzzy rule is composed of the group of optimized fuzzy sets, which represent the curves from the database of the system, rather than pre-defined equations that are considered in the Sugeno method (Lee 2004). After the degree of membership ($\mu_{R,n}$) of each rule is computed, the output of the system (O_{model}) is calculated using the following relation

$$O_{\mathrm{mod}el} = \sum_1^n \frac{\mu_{R,n} \cdot O_n}{\sum_1^n \mu_{R,n}} \quad (7)$$

4. Application of the proposed approach

The analysis shown in this section aims to present the practicality of the genetic-fuzzy approach, point out its advantages and drawbacks, and verify its reliability in designing experimental-based prediction models. Notice, however, that the development of an all-purpose shrinkage prediction model is not within the scope of this work; therefore only one input parameter was taken into account in the framework of the model.

The developed model focuses on predicting SCC shrinkage up to a period of 90 days. Such period corresponds to a construction time during which countermeasures can be taken. Because it is relevant for SCC mixtures, the volume of cement paste (V_{cp}) was chosen as input parameter of the prediction model. The developed model could, for example, be used in the production process of precast concrete during which curing is controlled and changes in the mixture composition are permitted but changes in material types are not.

The initial prediction model was based on the experimental data of shrinkage strain curves of SCC mixtures produced with different V_{cp}. Depending on the selected input parameters, training data, and Eqs. (6) and (7), the rule base and the output of the proposed shrinkage prediction model can be generally written as

$$R_n : if\ V_{cp}\ is\ \tilde{V}_{cp},\ then\ \varepsilon_{sh,n}\ is\ \tilde{\varepsilon}_{sh,n} \quad (8)$$

$$\varepsilon_{sh,output} = \sum_1^n \frac{\mu_{R,n} \cdot \varepsilon_{sh,n}}{\sum_1^n \mu_{R,n}} \quad (9)$$

where V_{cp} is the cement paste volume in l/m^3, $\tilde{V}_{cp,n}$ and $\tilde{\varepsilon}_{sh,n}$ are the optimized group of fuzzy sets for cement paste volume and shrinkage strain, respectively, $\mu_{R,n}$ is the firing strength assigned to each $\tilde{\varepsilon}_{sh,n}$ fuzzy set, and $\varepsilon_{sh,n}$ is the crisp output, namely shrinkage strain, from each rule R_n.

The obtained model was compared to other published data and to the prediction models that were discussed in Section 1. After an initial analysis, a third dataset was included in the database to investigate the ability of the proposed approach to adjust the prediction model when additional training data is used.

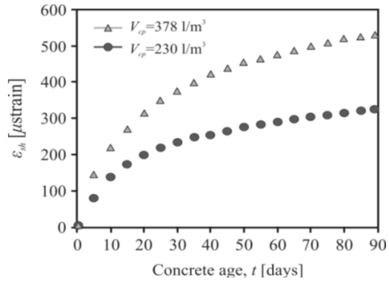


Fig. 6 Shrinkage strain of SCC produced with different V_{cp}

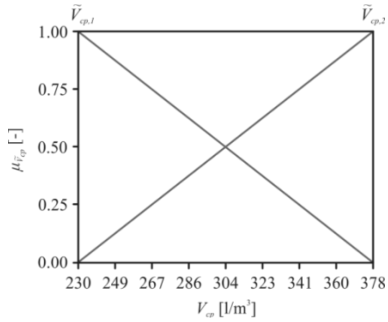


Fig. 7 $\tilde{v}_{cp}$ Fuzzy sets

Table 2 Representative intervals of the experimental data used to develop the model

V_{cp} [l/m^3]	Interval	t_a [days]	$\varepsilon_{sh,a}$ [μstrain]	t_b [days]	$\varepsilon_{sh,b}$ [μstrain]
230	I	0	0	15	175
	II	15	175	70	305
	III	70	305	90	325
328	I	0	0	20	315
	II	20	315	70	500
	III	70	500	90	530

4.1 Training data

The system illustrated in Fig. 4 was implemented for the experimental data presented in Leemann *et al.* (2011), where further details concerning the materials properties and curing conditions can be verified. The experimental results that are presented in Fig. 6 were considered as training data to develop the prediction model.

The number of representative intervals for the curves from Fig. 6 was defined as three (N_{int} = 3), each of these is composed of the values that are presented in Table 2. The population size was set at S_{pop} = 10.

4.2 Results and discussion

Once the input data has been defined, the proposed approach was implemented and convergence was achieved after approximately 500 iterations. The exponent values E_L and E_R of

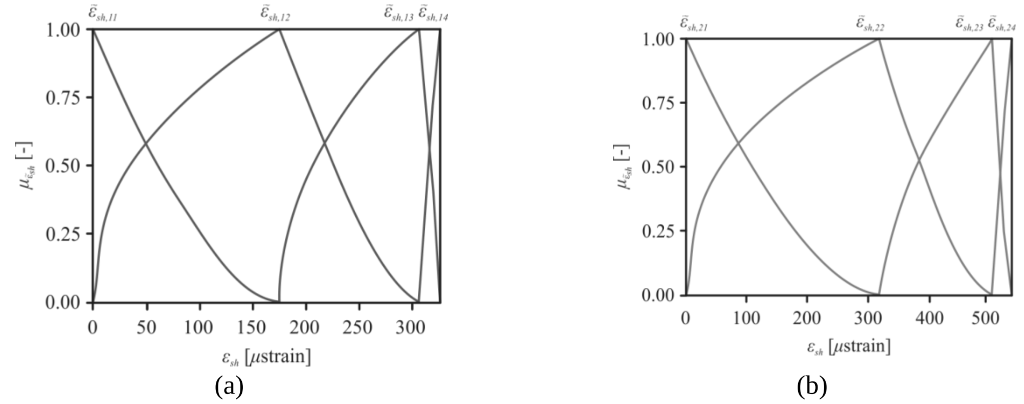


Fig. 8 Optimized fuzzy sets ($\tilde{\varepsilon}_{sh,n}$) for the concrete mixture with (a) V_{cp} = 230 l/m^3 and (b) V_{cp} = 378 l/m^3

Table 3 E_L and E_R values of the optimized fuzzy sets

Fuzzy set	$\mu(x)$	x_1 [μstrain]	x_2 [μstrain]	x_3 [μstrain]	E_L [-]	E_R [-]
$\tilde{V}_{cp}$	$\tilde{V}_{cp,1}$	-	230	378	-	1.000
	$\tilde{V}_{cp,2}$	230	378	-	1.000	-
$\tilde{\varepsilon}_{sh,1}$	$\tilde{\varepsilon}_{sh,11}$	-	0	175	-	1.573
	$\tilde{\varepsilon}_{sh,12}$	0	175	305	0.842	1.436
	$\tilde{\varepsilon}_{sh,13}$	175	305	325	0.498	0.726
	$\tilde{\varepsilon}_{sh,14}$	305	325	-	1.144	-
$\tilde{\varepsilon}_{sh,2}$	$\tilde{\varepsilon}_{sh,21}$	-	0	315	-	1.621
	$\tilde{\varepsilon}_{sh,22}$	0	315	500	0.393	1.519
	$\tilde{\varepsilon}_{sh,23}$	315	500	530	0.636	1.512
	$\tilde{\varepsilon}_{sh,24}$	500	530	-	0.762	-

the group of optimized fuzzy sets and their limits (x_1, x_2, and x_3, see Eq. (5)) are listed in Table 3.

Because only two sets of data were used in the optimization process, the fuzzy sets connected to V_{cp} had to be set as linear (Fig. 7). The shape of the optimized membership functions of the shrinkage strain curves ($\tilde{\varepsilon}_{sh,n}$) from the database is depicted in Fig. 8.

The obtained fuzzy logic prediction model is composed of the linear fuzzy sets that are shown in Fig. 7, the optimized fuzzy sets presented in Fig. 8, the rule base from Eq. (8), and the output equation (Eq. (9)). This model is suitable for predicting the total shrinkage strain up to 90 days of SCC mixtures with V_{cp} ranging from 230 to 378 l/m^3 and testing conditions defined in Leemann *et al.* (2011). The obtained prediction model will be regarded as FL model from this point on.

The decision-making from the FL model is composed of two rules, i.e., $n=2$ in Eq. (8), which comprises the group of optimized fuzzy sets that are illustrated in Figs. 6 and 7. The reduced number of fuzzy sets confirms the computational effectiveness of the FL model when compared to a model hypothetically obtained by the classical fuzzy logic approach that was discussed in Section 3. The final configuration of the obtained model is presented in Fig. 9.

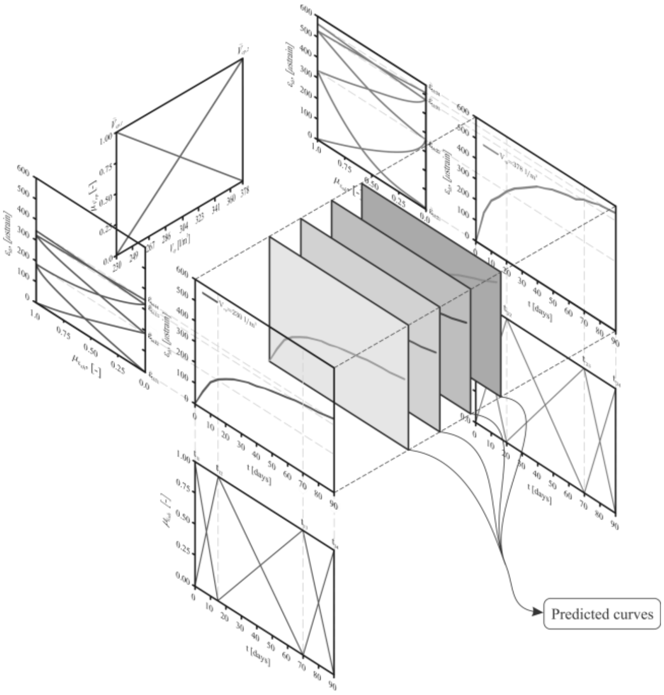
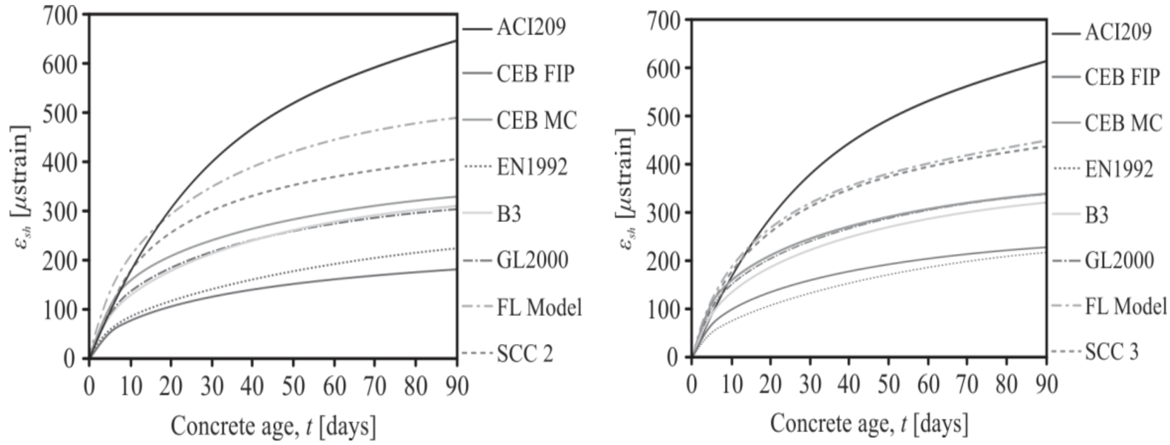


Fig. 9 Final configuration of the obtained prediction model



(a) (b)

Fig. 10 Predicted shrinkage strain curves for (a) SCC 2 and (b) SCC 3

In order to verify the quality of the FL model in predicting shrinkage strain, the experimental data published in Loser and Leemann (2009) was used for comparison. These data comprise shrinkage strain curves from five different SCC mixtures with composition and testing conditions similar to the experimental data that was used to develop the FL model. The volume of cement paste of each SCC, which is used as input in the FL model, is listed in Table 4. The experimental and predicted shrinkage strains were compared and the obtained MSE values are shown in Table 5.

As a complementary analysis, the shrinkage strain measurements of each SCC mixture from

Loser and Leemann (2009) were also compared to the values obtained from the prediction models discussed in Section 1. The input data that was used for these models are listed in Table 4. The obtained results are summarized in Table 5 together with those from the FL model.

The comparison between the shrinkage strain curves that presented extreme MSE values for the FL model, i.e., SCC 2 and SCC 3, is illustrated in Fig. 10.

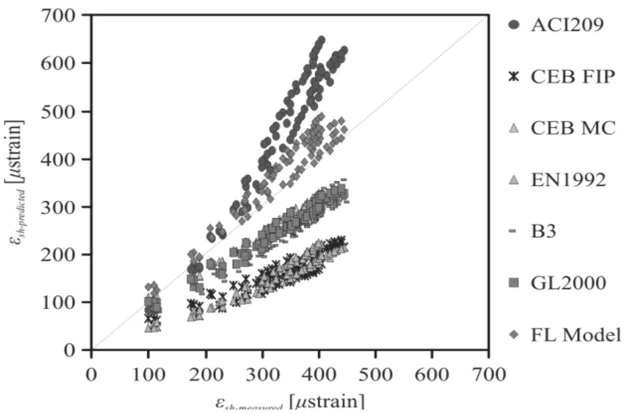


Fig. 11 Scatter plot of the prediction models

Table 4 Input data used to predict shrinkage strain (Loser and Leemann 2009)

Input Parameters	SCC 1	SCC 2	SCC 3	SCC 4	SCC 5
Cement paste volume, V_{cp} [l/m^3] (I)	329	349	316	342	332
Curing time, t_c [days]			1		
Curing type			Moist curing		
Relative Humidity, *RH* [%]			70.0%		
Cement type			CEM I 42.5		
Water content, w [kg/m^3]	178.0	179.0	181.0	199.0	166.0
Design compressive strength, f_c' [MPa]	53.3	63.1	51.0	49.4	66.0
Compressive strength at 28 days, f_{cm28} [MPa]	61.3	71.1	59.0	57.4	74.0
Elastic Modulus at 28 days, E_{cm28} [MPa]	35.7	38.2	35.4	35.5	40.6
Specimen size [mm]			120 × 120 × 360		
Member shape			Infinite prism		

(I) V_{cp} was used as input parameter in the FL model

Table 5 MSE values for different shrinkage prediction models

f_{model} [%]	SCC 1	SCC 2	SCC 3	SCC 4	SCC 5	f_{all} [%]
ACI209	34.2	45.9	30.3	35.6	46.3	39.0
CEB FIP	52.3	58.1	50.0	46.7	60.3	53.7
CEB MC	22.3	19.0	22.1	19.7	18.1	20.3
EN1992	56.8	51.1	57.1	55.9	49.7	54.2
B3	31.3	27.4	29.0	17.2	32.9	28.1
GL2000	25.3	26.8	23.2	19.5	27.6	24.7
FL Model	6.9	19.1	4.4	16.2	14.1	13.4

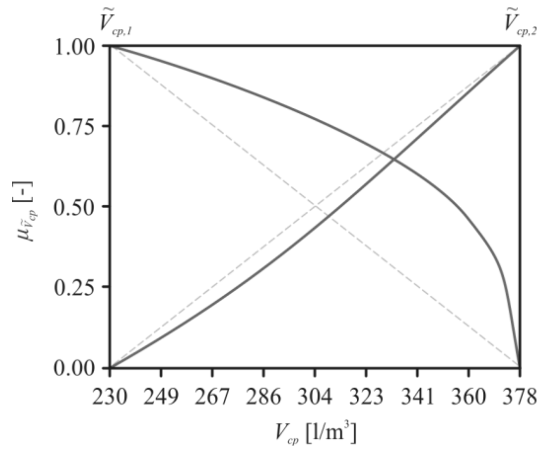


Fig. 12 Optimized fuzzy sets for $\tilde{v}_{cp,n}$

Table 6 EL and ER of the optimized group of fuzzy sets for FL2 model

Fuzzy set	$\mu(x)$	x_1 [μstrain]	x_2 [μstrain]	x_3 [μstrain]	E_L [-]	E_R [-]
$\tilde{V}_{cp}$	$\tilde{V}_{cp,1}$	-	230	378	-	0.251
	$\tilde{V}_{cp,2}$	230	378	-	2.099	-
$\tilde{\varepsilon}_{sh,1}$	$\tilde{\varepsilon}_{sh,11}$	-	0	175	-	1.573
	$\tilde{\varepsilon}_{sh,12}$	0	175	305	0.842	1.436
	$\tilde{\varepsilon}_{sh,13}$	175	305	325	0.498	0.726
	$\tilde{\varepsilon}_{sh,14}$	305	325	-	1.144	-
$\tilde{\varepsilon}_{sh,2}$	$\tilde{\varepsilon}_{sh,21}$	-	0	315	-	1.621
	$\tilde{\varepsilon}_{sh,22}$	0	315	500	0.393	1.519
	$\tilde{\varepsilon}_{sh,23}$	315	500	530	0.636	1.512
	$\tilde{\varepsilon}_{sh,24}$	500	530	-	0.762	-

Table 7 Individual and overall MSE for different fuzzy logic-based models

f_{model} [%]	SCC 1	SCC 2	SCC 3	SCC 4	SCC 5	f_{all} [%]
FL	6.9	19.1	4.4	16.2	14.1	13.4
FL2	8.9	3.8	11.2	- (II)	3.9	7.6

(II) SCC 4 was excluded from the analysis because it was used as training data to develop the FL2 Model.

The scatter plot of measured versus predicted values for all models that were considered in the analysis is presented in Fig. 11.

From Table 5, it can be noticed that, except for SCC 2 at which CEB MC had the lowest results, the FL model presented the lower MSE in all cases and thus the lower overall MSE error. The FL model was followed by CEB MC, GL2000, B3, ACI209, CEB FIP, and EN1992 models. Except for the fact that GL2000 presented lower MSE than B3, the classification of the models shown in Table 5 is similar to the one that was presented in the preliminary analysis (Table 1).

Although the MSE presented by FL model was lower than all the considered models, its value is still considered high, around 14.0%. This is likely because only two experimental data sets were used to develop the model, which led to linear membership functions for $\tilde{V}_{cp,n}$.

If an intermediary data set were included as training data, the linear shape of the fuzzy set depicted in Fig. 7 would be optimized. As a consequence, the MSE of FL model would be lower than the current value of 13.4%. To verify that, one of the experimental curves from Loser *et al.* (2009), particularly SCC 4, was included as training data and the optimization process was performed again. The optimized fuzzy sets related to V_{cp} are illustrated in Fig. 12. The corresponding exponent values of all fuzzy sets are listed in Table 6. The combination of these sets leads to a modified prediction model, namely FL2 model. Notice that, once the experimental data from SCC 4 was used as training data to optimize the shape of the fuzzy set connected to the input variable V_{cp} , the optimized fuzzy sets for shrinkage strain (i.e., $\tilde{\varepsilon}_{sh,n}$) that were shown in Fig. 8 remained unchanged.

Once again, experimental and predicted strains FL2 model were compared and the MSE values were computed. The obtained results are shown in Table 7 together with the MSE from FL model.

From Table 7, it can be noticed that the overall MSE for FL2 model was considerably reduced when compared with the first version of the model. This confirms the assumption that the inclusion of additional training data would lead to a prediction model with lower overall error. Nonetheless, the same is not verified for the individual MSE values (f_{model}). In particular, when compared with FL model, FL2 yielded to an increase of 2.0% and 6.8% in the MSE of SCC 1 and 3, respectively; whereas SCC 2 and 5 had a decrease of 15.3% and 10.2%, respectively. Such variation indicates that the genetic-fuzzy approach likely tends to even the individual MSE values so as to reduce the overall error of the prediction model. Notice, however, that although the individual MSEs from the FL2 model were sometimes higher than the FL model, they were still lower than those from the other predictions models (Table 6). Although the range of application of the developed prediction model is limited, the FL models were able to predict the shrinkage behavior of SCC with a reasonably low error. This allows for excluding the need of additional experimental analyses, thus contributing to reduce costs.

It is important to observe that, because few experimental data have been used for developing the model, the obtained model has restrictions in its range of applications. However, the further inclusion of a larger amount of experimental data, supported by the user's expertise, will lead to a more robust prediction model. This model could consider, for instance, include long-term shrinkage experimental measurements, e.g., up to 365 days, to build a model for different applications from the one presented in this work.

Furthermore, additional input parameters that are relevant on concrete shrinkage, such as relative humidity, curing time, compressive strength, among others, could be included in the prediction model depending on the user's needs. The possibility of enlarging or even changing the training data of the system arises as an advantage of the materials model obtained from the proposed approach against prediction models that are based on equations and its constants.

Finally, the lower MSE values from FL and FL2 models against the prediction models that were considered in the analysis confirm their quality in simulating the materials behavior. Also, the results indicate the success in combining fuzzy logic and GA to develop optimized material models.

5. Conclusions

The approach presented in this work is based on the combination of fuzzy logic and genetic algorithm. The aim is to optimize the decision-making of the fuzzy system by reducing the number of rules and fuzzy sets in the expert system. The performance of the system was verified through the definition of an experimental-based shrinkage prediction model. The obtained conclusions are presented as follows.

The genetic-fuzzy approach allowed for including relevant experimental data in the modeling process and for combining these data through optimized decision-making rules. The application of the proposed approach provided quantitative results (i.e., shrinkage prediction model) that can be readily applicable in existing finite element codes. The group of optimized fuzzy sets led to a proper prediction of shrinkage curves with a reduced number of rules, therefore making the entire modeling process more effective.

The comparison between experimental measurements and predicted values from the FL model yielded to an overall MSE of 13.4%. This indicates that the model properly represents the material's behavior, and it can be used to predict SCC shrinkage as long as the limits of the model are respected. As a result, the number of experimental tests can be considerably reduced when a new material model is required, leading thus to cost reduction.

A complementary analysis indicates that the obtained model will tend to reach a higher robustness and lower error once a larger number of experimental data points are included. In particular, the MSE of the FL model was reduced to ~8.0% after including an additional curve as training data. This proves the flexibility of the proposed genetic-fuzzy approach in adjusting the model according to the training data. Such flexibility is considered as a great advantage against other prediction models that were discussed in this work.

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References

American Concrete Institute (ACI) (2008), ACI 209.2R-08: Guide for modeling and calculating shrinkage and creep in hardened concrete, ACI Committee, Technical document.

Aydin, A.C., Tortum, A. and Yavuz, M. (2006), “Prediction of concrete elastic modulus using adaptive neuro fuzzy inference system”, *Civ. Eng. Environ. Syst.*, **23**(4), 295-309.

Bai, Y., Darcy, P. and Basheer, P.A.M. (2005), “Strength and drying shrinkage properties concrete containing furnace bottom ash fine aggregate”, *Constr. Build. Mater.*, **19**(9), 691-697.

Ballim, Y. (2000), “The effect of shale in quartzite aggregate on the creep and shrinkage of concrete – A comparison with RILEM model B3”, *Mater. Struct.*, **33**(4), 235-242.

Bažant, Z.P. (1995), “Creep and shrinkage prediction model for analysis and design of concrete structures – Model B3”, *Mater. Struct.*, **28**(6), 357-365.

Bažant, P.Z. and Baweja, S. (2000), “Creep and shrinkage prediction model for analysis and design of concrete structures: Model B3", *Adam Neville Symposium: Creep and Shrinkage – Structural Design Effects*, *ACI SP 194*, Farmington Hills, MI.

Benboudjema, F., Meftah, F. and Torrenti, J.M. (2005), "Interaction between drying, shrinkage, creep and cracking phenomena in concrete", *Eng. Struct.*, **27**(2), 239-250.

Bouzoubaa, N. and Lachemi, M. (2001), "Self-compacting concrete incorporating high volumes of class F fly ash – Preliminary results", *Cement Concrete. Res.*, **31**(3), 413-420.

Boverie, S., Demaya, B., Le Quellec, J.M. and Titli, A. (1993), "Contribution of fuzzy logic control to the improvement of modern car performances", *Control Eng. Pract.*, **1**(2), 291-297.

Branson, D.E. and Christiason, M.L. (1971), "Time dependent concrete properties related to design – Strength and elastic properties, creep and shrinkage", *Symposium on Creep*, Shrinakge and Temperature Effects, SP-27-13, American Concrete Institute, Detroid, Jan.

Brooks, J. (2003), *Elasticity, shrinkage, creep and thermal movement*, (Eds. Newman, J. and Choo, B.S.), Advanced Concrete Technology – Testing and Quality, Elsevier, London.

Chak, C.K., Feng, G. and Palaniswami, M. (1997), *Implementation of fuzzy systems*, (Ed. Leondes, C.T.), Fuzzy logic and expert system applications, Vol. 6, 1st Edition, Academic Press, London.

Coley, D.A. (1999), *An introduction to genetic algorithms for scientists and engineers*, World Scientific, Singapore.

Comité Euro-International Du Béton (CEB) (1993), CEB-FIP model code 1990, CEB Bulletin d'Information No. 213/214, Lausanne, Switzerland.

Comité Euro-International Du Béton (CEB) (1999), Structural concrete: Behaviour, design and performance – Updated knowledge of the CEB FIP model code 1990, fib Bulletin 2, V. 2, Fédération Internationale du Béton, Lausanne, Switzerland.

da Silva, W.R.L. and Štemberk, P. (2013)a, "Expert system applied for classifying self-compacting concrete surface finish", *Adv. Eng. Soft.*, **64**(0), 47-61, DOI: 10.1016/j.advengsoft.2013.04.005.

da Silva, W.R.L. and Štemberk, P. (2013)b, "Shooting-inspired fuzzy logic expert system for ready-mixed concrete plants", *J. Intel. Fuzzy Sys.*, **25**(2), 481-491, DOI: 10.3233/IFS-120655.

European Committee for Standardization (EN) (2004), Eurocode 2 (BS EN 1992), Design of concrete structures - Part 1-1: General rules and rules for buildings, European Committee for Standardization, Brussels.

Gao, F.L. (1997), "A new way of predicting cement strength – Fuzzy logic", *Cement Concrete. Res.*, **27**(6), 883-888.

Gardner, N.J. and Zhao, J.W. (1993), "Creep and shrinkage revisited", *ACI Mater. J.*, **90**(3), 236-246.

Gardner, N.J. (2004), "Comparison of prediction provisions for drying shrinkage and creep of normal strength concretes", *Can. J. Civil Eng.*, **31**(5), 767-775.

Graham, I. (1991), "Fuzzy logic in commercial expert systems – results and prospects", *Fuzzy Set. Syst.*, **40**(3), 451-472.

Guneyisi, E., Gesoglu, M. and Ozbay, E. (2010), "Strength and drying shrinkage properties of self-compacting concretes incorporating multi-system blended mineral admixtures", *Constr. Build. Mater.*, **24**(10), 1878-1887.

Guneyisi, E., Gesoglu, M. and Mermerdas, K. (2008), "Improving strength, drying shrinkage, and pore structure of concrete using metakaolin", *Mater. Struct.*, **41**(5), 937-949.

Han, K.H. and Kim, J.H. (2002), "Quantum-inspired evolutionary algorithm for a class of combinatorial optimization", *IEEE T. Evolut. Comput.*, **6**(6), 580-593.

Hellendoorn, H. and Thomas, C. (1993), "Defuzzification in fuzzy controllers", *J. Int. Fuzzy Syst.*, **1**(2), 109-123.

Holmblad, L.P. and Ostergaard, J.J. (1982), *Control of a cement kiln by fuzzy logic*, (Eds. Gupta, M.M. and Sanchez, E.), Fuzzy Information and Decision Processes, North-Holland, Amsterdam.

Idorn, G.M. (2005), "Innovation in concrete research – review and perspective", *Cement Concrete Res.*, **35**(1), 3-10.

Illston, J.M. and Domone, P.L.J. (2010), *Construction materials: Their nature and behaviour*, Spon Press, London.

Imamoto, K. and Arai, M. (2008) “Specific surface area of aggregate and its relation to concrete drying shrinkage”, *Mater. Struct.*, **41**(2) 323-333.

Lee, K.H. (2004), *First course on fuzzy theory and applications*, Advances in Soft Computing, Vol. 27, 1st Edition, Springer, Berlin.

Leemann, A., Lura, P. and Loser, R. (2011), “Shrinkage and creep of SCC – The influence of paste volume and binder composition”, *Constr. Build. Mater.*, **25**(5), 2283-2289.

Ling, F. and Meyer, C. (2008), “Modeling shrinkage of Portland cement paste”, *ACI Mater. J.*, **105**(3), 302-311.

Loser, R. and Leemann, A. (2009), “Shrinkage and restrained shrinkage cracking of self-compacting concrete compared to conventionally vibrated concrete”, *Mater. Struct.*, **42**(1), 71-82.

Mamdani, E.H. and Assilian, S. (1975), “An experiment in linguistic synthesis with a fuzzy logic controller”, *Int. J. Man Machine Studies*, **7**(1), 1-13.

Mcdonald, D.B. and Roper, H. (1993), “Accuracy of prediction models for shrinkage of concrete”, *ACI Mater. J.*, **90**(3), 265-271.

Mehta, P.K. and Monteiro, P.J.M. (2006*), Concrete: Microstructure, Properties and Materials*, McGrawHill, NY.

Mitchell, M. (1998), *An introduction to genetic algorithms (Complex adaptive systems)*, A Bradford Book, MIT Press MA.

Nguyen, H.T., Prasad, N.R., Walker, C.L. and Walker, E.A. (2002), *A First Course in Fuzzy and Neural Control*, Chapman and Hall/CRC, London.

Pezeshk, S., Camp, C.V. and Chen, D. (2000), “Design of nonlinear framed structures using genetic optimization”, *ASCE J. Struct. Eng.*, **126**(3), 382-388.

Pichler, C., Lackner, R. and Mang, H.A. (2007), “A multiscale micromechanics model for the autogenous-shrinkage deformation of early-age cement-based materials”, *Eng. Fract. Mech.*, **74**(1-2), 34-58.

Pokorná, N. and Štemberk, P. (2010), "Fuzzy Logic Model for Description of Fatigue Behavior of Concrete", *Proceedings of 15th International Conference - Mechanika 2010*, pages 351-355, Kaunas Univ. Technol., Kaunas, Lithuania.

Pourzeynali, S., Lavasani, H.H. and Modarayi, A.H. (2007), “Active control of high rise building structures using fuzzy logic and genetic algorithms”, *Eng. Struct.*, **29**(3), 346-347.

Ray, I., Gong, Z., Davalos, J.F. and Kar, A. (2012), “Shrinkage and cracking studies of high performance concrete for bridges decks”, *Constr.. Build. Mater.*, **28**, 244-254.

Rokonuzzaman, Md. and Sakai, T. (2010), “Calibration of the parameters for a hardening-softening constitutive model using genetic algorithms”, *Comput. Geotech.*, **37**(4), 573-579.

Rozière, E., Granger, S., Turcry, Ph. and Loukili, A. (2007), “Influence of paste volume on shrinkage cracking and fracture properties of self-compacting concrete”, *Cement Concrete. Compos.*, **29**, 626-636.

Skarendahl, A. (2005), “Changing concrete construction through use of self-compacting concrete”, *Proceedings of the 1st Int. Symposium on design, performance and use of self-consolidating concrete*, China, May.

Štemberk, P. and Rainová, A. (2011), “Simulation of hydration and cracking propagation with temperature effect based on fuzzy logic theory”, *Mechanika*, **17**(4), 358-362.

Tanyildizi, H. (2009), “Fuzzy logic model for the prediction of bond strength of high-strength lightweight concrete”, *Adv. Eng. Softw.*, **40**(3), 161-169.

White, A.J. and Newtson, C.M. (2006), “Effects of mixture proportions on concrete shrinkage”, *Proceedings of the 2nd International Congress*, Naples, June.

Zadeh, L.A. (1965), “Fuzzy sets”, *Inform. Control*, **8**(3), 338-353.

Zhao, Z. and Chen, C. (2001), “Concrete bridge deterioration diagnosis using fuzzy inference system”, *Adv. Eng. Softw.*, **32**(4), 317-325.

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