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Strength design criterion for asymmetrically reinforced RC circular cross-sections in bending

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Abstract. Asymmetrical reinforcement for circular sections in wall piles is an efficient construction component with reduced embodied energy. It has been proven that asymmetrical reinforced wall piles may save more than 50% of the reinforcement than the traditional symmetrically reinforced circular sections. The use of this new type of structural member increases the number of variables in the design problem, which makes its use by engineers more complicated. In order to facilitate the use of the asymmetrically reinforced piles, this paper presents a criterion for the design of this type of structural member. The chosen criterion has been analyzed with the help of flexural capacity-cost curves. The new criterion is similar to the design procedure traditionally used for RC beams.

Keywords: wall piles; asymmetric reinforcement; strength design criteria

1. Introduction

There are several optimal shapes for the cross-section of reinforced concrete structural members, the versatility is attributed to mouldability of concrete to any conceivable shape (Narayan and Venkataramana (2007)). Nevertheless circular sections are always attractive; often the circular shape is required for architectural or functional reasons. Circular cross-sections can be symmetrically reinforced (Belarby *et al.* (2009)) or asymmetrically reinforced (Hernández-Montes *et al.* (2010)). Asymmetrical reinforcement for wall piles (ARWP in what follows) is a system recently proposed by Gil-Martín *et al.* (2010a) for an energy efficient construction, see Fig. 1. Traditionally, longitudinal reinforcement for circular cross sections, used in earth retaining systems, consist of a number of bars of the same diameter spaced uniformly around the circumference of the section, inset from the face of the member by the required cover distance (see Fig. 2).

Initial practical experience with ARWP was obtained during the construction of a high-speed railway line between Madrid and Barcelona in 2009, for a segment of a cut-and-cover tunnel in Barcelona. Five piles were built using asymmetrically distributed reinforcement alongside many conventionally reinforced piles, over a 500 m stretch. The excavation was 10 m deep, and was braced using separated piles that were 18 m long, diameter of 1.20 m and spaced at 1.40 m on centre. ARWP saved nearly one tonne of steel per pile (51% of longitudinal steel with respect to

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Fig. 1 Asymmetrical reinforcement for wall pile at construction site

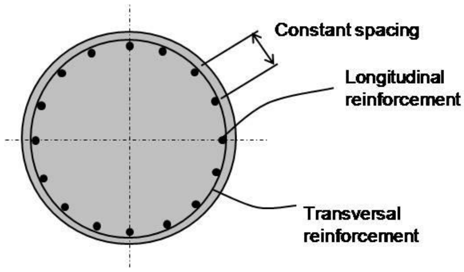


Fig. 2 Traditional reinforcement of a circular pile

the symmetrical case).

Strength design criteria of common sections are well established in engineering practice, such as rectangular sections or symmetrically reinforced circular sections. Asymmetrically reinforced circular sections (ARCS in what follows) are a novelty that was partially approached by Weber and Ernst (1989), and a design criterion for these sections has not yet been proposed. Serviceability limit states for symmetrically reinforced circular sections have been studied by Wiese *et al.* (2004).

In order to make the paper self contained, a brief review of the traditional criteria for strength design of rectangular sections and symmetrically reinforced circular sections has been made. Furthermore, a new design criterion for ARCS has been proposed.

2. Strength design assumptions in bending

In flexural analysis of reinforced concrete cross-sections for combined bending and axial load both the Bernoulli's hypothesis (i.e., that plane sections remain plane after deformation) and no

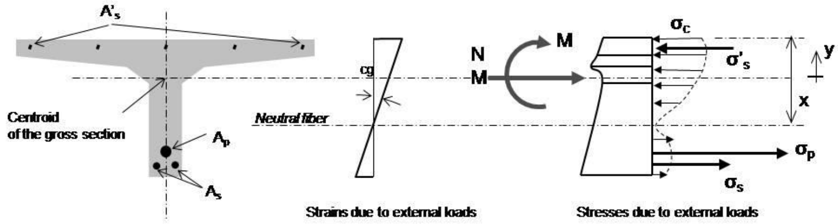


Fig. 3 Strains and stresses diagrams at cross section level

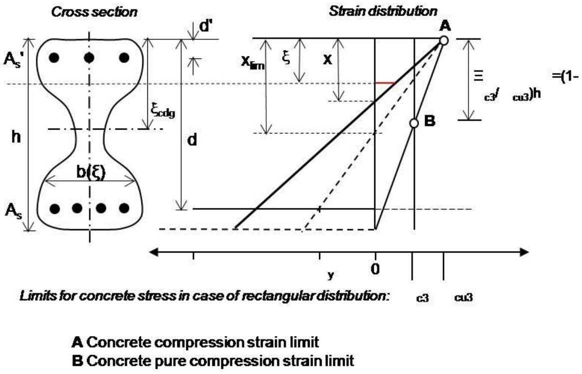


Fig. 4 Strain distributions in the ultimate limit state of bending

slip of reinforcement are accepted. The Bernoulli hypothesis allows the distribution of strain over the cross section to be defined by just two variables (such as the strain at the centroid (ε_{cg}) of the gross section and the curvature (ϕ) of the cross section as is indicated in Fig. 3).

For the ultimate strength design of a reinforced cross-section the strain limits are given in code provisions, which define the maximum usable strain at the extreme compression fiber and the tensile reinforcement strain. In this situation, if it is assumed that plane sections remain plane, the distribution of strain over the cross sections can be defined by just one variable, the unique variable considered in this paper is the neutral axis depth (x). So any fiber located at a distance ξ from the top fiber (see Fig. 4) presents a strain of

$$\varepsilon = \varepsilon(\xi, x) \tag{1}$$

The strain limits adopted in this paper are illustrated in Fig. 4, these limits correspond to Eurocode 2 (2002) in the the case of rectangular stress distribution for concrete. The maximum usable strain for concrete in bending is given $\varepsilon_{cu3,}$ the maximum strain for concrete in pure compression is ε_{c3} and there is no limit for the maximum usable strain for steel in tension. The assumption of no limitation for steel in tension is accepted by Eurocode 2 (2002) in case of bilinear model for the steel with no strain hardening. This approach is the basis of strain domains; wherein the strain diagram pivots about certain points located on the boundaries between adjacent domains, see points A and B in Fig. 4. With these limits, Eq. (1) is formulated as follows

$$\varepsilon(\xi,x)=\begin{cases}\varepsilon_{cu3}\dfrac{x-\xi}{x} & for \quad 0\le x<h\\ \varepsilon_{c3}\dfrac{x-\xi}{x-\Xi} & for \quad x\ge h\end{cases}$$
$$where\ \Xi=h\left(1-\frac{\varepsilon_{c3}}{\varepsilon_{cu3}}\right) \tag{2}$$

Steel and concrete behaviour models are expressed as *σ(ε)* functions. By means of mathematical composition of functions the constitutive relations (or stress-strain relations) of the concrete and the steel materials, for the case of strength design, can be expressed as a function of the variables x and ξ (neutral axis depth and position of any fiber, respectively).

$$\sigma=\sigma(\varepsilon)=\sigma(\varepsilon(\xi,x))=(\sigma\circ\varepsilon)(\xi,x) \tag{3}$$

$(\sigma\circ\varepsilon)$ represents the composition of the mathematical functions σ and ε. Obviously the *σ(ε)* model of concrete has to be valid for ultimate limit state of bending.

2.1 Cross sections with two layers of reinforcement

In the case of two levels of reinforcement, located at *d'* and *d* from the top fiber, as it is seen in Fig. 4, Eq. (3) are expressed as

$$\begin{aligned}\varepsilon_s'(x)&=\varepsilon(d',x)\\ \varepsilon_s(x)&=\varepsilon(d,x)\\ \sigma_s'(x)&=\sigma_s(\varepsilon_s'(x))=(\sigma_s\circ\varepsilon_s')(x)\\ \sigma_s(x)&=\sigma_s(\varepsilon_s(x))=(\sigma_s\circ\varepsilon_s)(x)\end{aligned} \tag{4}$$

Where ε_s' and σ_s' are the strain and stress of the upper steel, and ε_s and σ_s are the strain and stress of the bottom steel.

In the case of the ultimate state limit of bending, the equilibrium equations at the section level (e.g. Fig. 3) for a combination of bending moment, M_u, and axial force, N_u, acting simultaneously, can be expressed as function of the neutral axis depth, x, as

$$\begin{aligned}N_u&=\sum_{\text{int}}N=\int_0^h\sigma_c(\varepsilon(\xi,x))b(\xi)d\xi+\int_{A's}(\sigma_s\circ\varepsilon'_s)(x)\ dA'_s+\int_{As}(\sigma_s\circ\varepsilon_s)(x)\ dA_s\\ M_u&=\sum_{\text{int}}M=\int_0^h\sigma_c(\varepsilon(\xi,x))b(\xi)(\xi_{cdg}-\xi)d\xi+\int_{A's}(\sigma_s\circ\varepsilon'_s)(x)\big|_{x=d'}\ (\xi_{cdg}-d')\,dA'_s\\ &\quad+\int_{As}(\sigma_s\circ\varepsilon_s)(x)\big|_{x=d}\ (\xi_{cdg}-d)\,dA_s\end{aligned} \tag{5}$$

where the subscript "int" means internal resultants. Stresses and axial forces in (5) are positive in compression and negative in tension. Without loss of generality, the axial load, N_u, and moment, M_u, that equilibrate the internal stress resultants are presumed to act on the centroid of the gross section (see Fig. 3). The moment, M_u, is considered positive if it produces tensile strain on the bottom fiber. For consistency, in the case that the applied loads cause compression over the depth of the section, the moment is considered positive if the compressive strain at the bottom fiber is smaller than the compressive strain at the top fiber.

The first terms on the right side of Eq. (5) are termed $N_c(x)$ and $M_c(x)$ respectively. They represent the contribution of concrete to the internal resultant of axial force and flexural moment

$$\begin{aligned} N_c(x) &= \int_0^h \sigma_c(\varepsilon(\xi, x))\, b(\xi)\, d\xi \\ M_c(x) &= \int_0^h \sigma_c(\varepsilon(\xi, x))\, b(\xi)(\xi_{cdg} - \xi)\, d\xi \end{aligned} \tag{6}$$

Eurocode 2 (2002) (§3.1.7(3)) defines the rectangular stress distribution for concrete as a stress block having a constant compressive stress of $\eta \cdot f_{cd}$, a depth equal to the $\lambda \cdot x$, where x is the depth of the neutral axis and f_{cd}= the design strength of the concrete, for concrete in which resistance is between 25 and 55 MPa. The factor λ defines the effective height of the compression zone and the factor η defines the effective strength. The design strength of the concrete is given as a function of the specified characteristic strength, f_{ck}, where $f_{cd}=\alpha_{cc} f_{ck}/\gamma_c$. The term α_{cc} accounting for long term effects on strength and the rate at which the load is applied. The term γ_c is the partial safety factor for concrete, taken as 1.5. In this paper we have chosen $\lambda=0.8$ and $\eta=1.0$ and $\alpha_{cc}=0.85$ as these represent fairly typical values.

Eq. (5) constitute a nonlinear system of two equations with three unknowns: A_s, A'_s and x (Aschheim *et al.* (2008)). The infinite solutions of this system for reinforcement areas A_s and A'_s can be presented as functions of the neutral axis depth, x. The admissible, i.e., positive, solutions for A_s and A'_s, obtained from Eq. (5) can be plotted on a reinforcement sizing diagram (RSD) as function of the neutral axis depth, x, Hernández-Montes *et al.* (2005) and Lee *et al.* (2009). Gil-Martín *et al.* (2011) shown that the minimum total reinforcement solution generally differs from the symmetric reinforcement solution that is typically represented using conventional N-M interaction charts.

2.2 RC beams with two layers of reinforcement

In engineering practice, the ultimate strength design for rectangular RC beams subjected to bending with no axial force is made by imposing the following assumption:

- $A'_s = 0$ in case of $M_d < M_{lim}$, or
- balance conditions (i.e., $x = x_{lim}$, see Fig. 4) in case $M_d \geq M_{lim}$

Using the above assumption, a new condition is added to Eq. (5), so a unique solution can be calculated. The above assumptions constitute the traditional method for the strength design of RC beams.

M_{lim} is defined as the flexural moment given by the stresses in concrete in case $x=x_{lim}$ (i.e., $M_c(x_{lim})$ given by Eq. (6). x_{lim} is the depth of the neutral fiber for which the strain of the bottom reinforcement is the yield strain (ε_y), see Fig. 4.

Both cases given in the above assumption are particular solutions covered by the Theorem of Optimal Reinforcement for Reinforced Concrete Cross Sections (Hernández-Montes *et al.* (2008)).

Chakrabarty (1992) describes the optimal design of singly-reinforced concrete beams, which constitutes a particular case of the above. Furthermore, diagrams representing the interaction of axial load and moment on ultimate strength, known as *N-M* interaction diagrams, were presented originally by Whitney and Cohen (1956) and continue to be widely used today.

2.3 Symmetrically reinforced circular sections

The criterion developed in the previous section is only applicable to rectangular cross-sections subjected mainly to bending for which the steel reinforcement is usually placed on one or two layers. In these cross-sections the depth of the centroid of both, top and bottom reinforcement, from the top fiber of the cross-section are d' and d respectively, see Fig. 4.

In the case of circular sections, this criterion is not directly applicable, this is due to the fact that it is based on the use of x_{lim}, which depends on d. However, in circular sections there is not a clear layer of reinforcement and each bar has its own vertical position, i.e., x_{lim} defined in rectangular sections has no meaning for the case of circular sections. Traditionally, longitudinal reinforcement for circular cross section has consisted of multiple bars of the same diameter (∅) spaced uniformly around the circumference of the section, inset from the face of the member by the required cover distance (see Fig. 2). So, each bar has a different lever arm, which is substantially different from the others. In this case equations (5) result in

$$\begin{aligned} N_u &= \sum_{\text{int}} N = N_c(x) + \sum_{i=1}^{n} A_1 \sigma_s\left(\varepsilon\left(y_i, x\right)\right) \\ M_u &= \sum_{\text{int}} M = M_c(x) + \sum_{i=1}^{n} A_1 \sigma_s\left(\varepsilon\left(y_i, x\right)\right) \cdot y_i \end{aligned} \tag{7}$$

Where n is the number of bars and y_i is the position of bar i, see Fig. 5.

For this type of reinforcement (same bar, uniformly distributed) there is only one variable in the design: the bar diameter -for a fixed number of bars- or the number of bars –for a fixed bar diameter-. In the event that the number of bars is fixed, the size of the bar can be obtained by making the ultimate moment equal to or greater than the design moment. In the case that the diameter of the bar is fixed, the only one variable to be solved is n, the number of bars.

3. Asymmetrically reinforced circular cross-sections –ARCS-

Due to the fact that it is necessary to provide a flexural capacity for potential reversals of horizontal forces induced by wind or earthquake loading, unsymmetrical bar arrangements are not suitable for common practice in columns. In this case, central symmetrical reinforcement is required for circular sections. However, in some engineering constructions, such as retaining walls supported by circular piles, the asymmetrically-reinforced cross-section has an immediate application because these structural elements are unlikely to experience significant reversals of bending moments, Under these circumstances ARWP are very interesting.

Gil-Martín *et al.* (2010b) showed that when *N*-*M* demands lack symmetry i.e., *N*-*M* combinations for design represented in *N*-*M* graphs are not symmetric-, substantial savings are possible using ARCS. This optimization process was applied to two situations: using only one bar diameter or two different bar diameters for the longitudinal reinforcement of the circular pile (see Fig. 6).

ARCS are longitudinally reinforced sections with different bar diameters ($\varnothing_1$, $\varnothing_2$,…) at

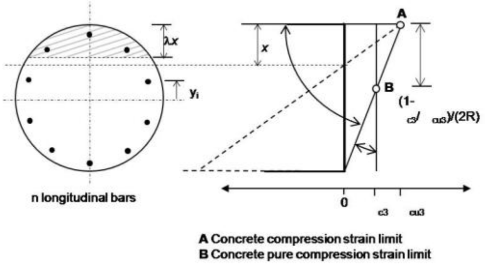
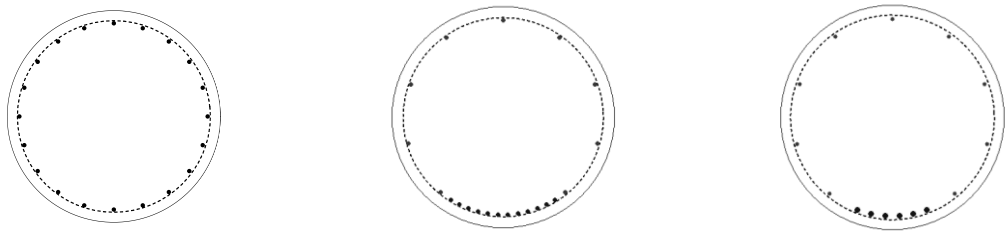


Fig. 5 Strain distributions considered at the ultimate limit state



(a) (b) (c)

Fig. 6 Piles. (a) Conventional reinforcement, (b) Optimized reinforcement using a single bar diameter and (c) Optimized reinforcement using two bar diameters

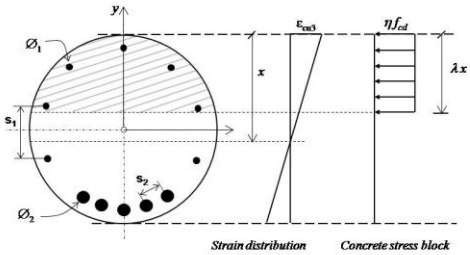


Fig. 7 Circular cross section reinforced with two different diameters of longitudinal bars

different spacing (s_1, s_2, …). In the event of two different bar diameters the number of longitudinal bars is partitioned into n_1 and n_2; n_1 is the number of $\varnothing_1$ bars separated by a spacing s_1, and n_2 is the number of $\varnothing_2$ bars separated by s_2. Areas A_1 and A_2 are the cross sectional areas of individual bars having diameters $\varnothing_1$ and $\varnothing_2$, respectively (see Fig. 7). For the case of ARCS the equilibrium equations, Eq. (5), can be expressed as

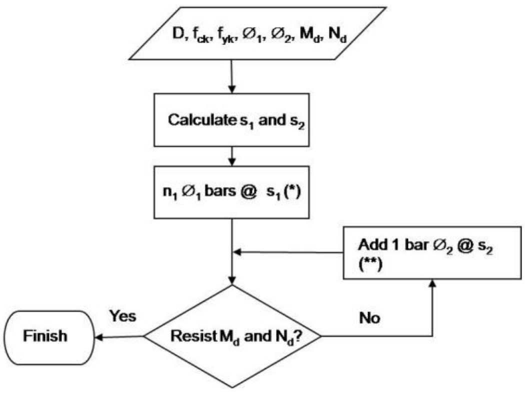
$$\begin{aligned} N_d &= N_c(x) + \sum_{i=1}^{n_1} A_1 \sigma_s(\varepsilon(y_i, x)) + \sum_{j=1}^{n_2} A_2 \sigma_s(\varepsilon(y_j, x)) \\ M_d &= M_c(x) + \sum_{i=1}^{n_1} A_1 \sigma_s(\varepsilon(y_i, x)) \cdot y_i + \sum_{j=1}^{n_2} A_2 \sigma_s(\varepsilon(y_j, x)) \cdot y_j \end{aligned} \tag{8}$$

Gil-Martín *et al.* (2010b) showed that the use of ARCS allows for substantial savings compared to conventionally reinforced sections of circular piles for retaining walls. These savings can represent more that 50% of the reinforcement and 10-15% of total cost (materials and labour) for the reinforced concrete portions of the work.

Section §2.3 showed that the design of central symmetrical reinforcement in circular sections is a problem with only one variable. At first sight, for the case of asymmetrically reinforced circular sections, the sizing problem is a more complicated problem due to the fact that more variables have to be considered. Nevertheless, two considerations are enough to reduce the design problem to a problem of one variable:

- In the compression zone, the bar diameter should be the minimum possible ($\varnothing_1$) and the distance between consecutive bars (s_1) the maximum possible.
- In the tension zone, the bar diameter should be the maximum possible ($\varnothing_2$) and the distance between consecutive bars (s_2) the minimum possible.

Taking these premises into consideration the design procedure turns into a problem of one variable and the algorithm to solve it is indicated in Fig. 8, (Gil-Martin *et al.* 2010b).



(*)The criterion to fix n_1 bars $\varnothing_1$ at @ s_1 is given by the Standard in use or by the constructional procedure.

(**) Bars $\varnothing_2$ added at spacing s_2 have to be included at the bottom, at the position that generates highest lever arm relative to the centroid of cross-section. When adding bars $\varnothing_2$ at s_2, there may be interference with bars located at a spacing s_1. Any bar ($\varnothing_1$) provided in the first step that is located within the region containing bars spaced at s_2 or within a distance less than s_2 from this region must be removed.

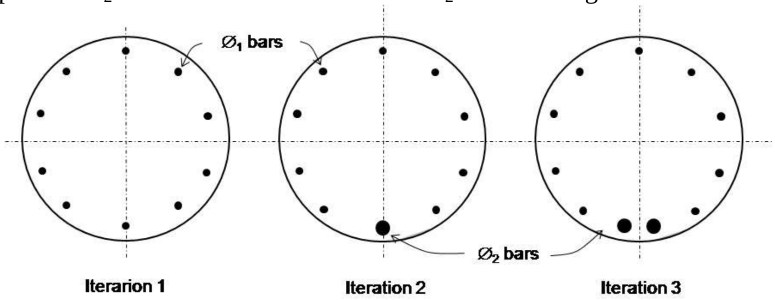


Fig. 8 Flow chart of the design procedure

In the particular case of circular RC cross-sections reinforced along their perimeter, as shown in Figs. 6(a) to 6(c), for both conventional and asymmetrical reinforcement, the design criterion given in §2.2 for rectangular cross-sections is not applicable. A new design criterion for ARCS is presented in this paper. The new criterion is as effective as the depth limit criterion in the case of rectangular cross-sections with two layers of reinforcement –i.e., $A'_s=0$ for $M_d < M_{lim}$ and $x=x_{lim}$ for $M_d \geq M_{lim}$-.

In this paper two possible strength design criteria for ARCS are analyzed, however only one of them is chosen as a valid criterion. The application of the proposed criterion allows the engineer to obtain the optimum pile diameter and the optimum amount of longitudinal reinforcement to resist any given applied bending moment, M_d.

4. Strength design criteria for ARCS in bending

Several interesting structural optimization processes are available in literature, at sectional level or at structural level (Topal and Uzman (2006)) or Guerra and Kiousis (2006) respectively. In order to understand the design process of ARCS, four circular cross-sections with nominal diameters, D, of 500, 600, 800 and 1000 mm subjected to pure bending, N_d=0, are considered. See Fig. 9(a). All the circular cross sections have been designed for a characteristic concrete strength (f_{ck}) equal to *25* MPa and characteristic steel strength (f_{yk}) of *500* MPa. The mechanical cover adopted is equal to 70 mm. Two different types of longitudinal reinforcement have been considered: one unique bar diameter or two bar diameters. In the event of only one bar diameter Ø16 is used for D=500 and 600 mm and Ø20 for D=800 and 1000 mm. For two different bar diameters Ø10 and Ø32 are used for all the diameters of the circular cross-section, D. The spacing –clear separation- between two bars is set as 300 mm in the compression zone (or light-reinforced zone) and 35 mm in the tension zone (or dense-reinforced zone) of the cross-section (see Figs. 9(b) and 9(c)).

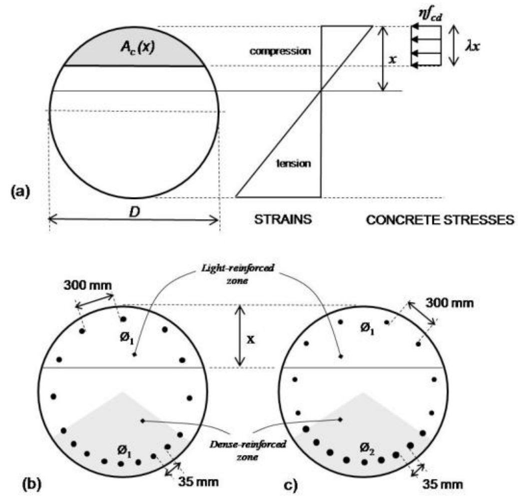


Fig. 9 ARCSs study

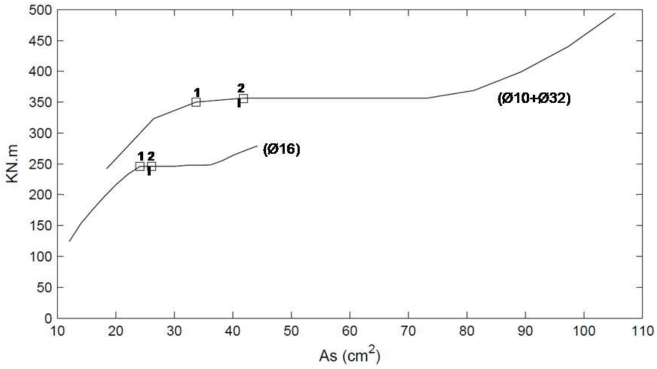


Fig. 10 D=500 mm

In order to identify the proposed design criterion, a study of the ultimate moment as function of the total area of reinforcement is carried out. The study is made for the four circular cross-section diameters mentioned before, see Figs. 10 to 13. In these figures the vertical axis represents the ultimate moment and the horizontal axis represents the total longitudinal reinforcement area (A_s) of the cross section. A_s is composed by the area of reinforcement in the light-reinforced zone plus the area of reinforcement in the dense-reinforced zone, see Fig. 9. A_s increases when the zone of dense-reinforcement increases, i.e., adding bars in the dense-reinforcement zone. Therefore we are merely operating as indicated in the flow chart shown in Fig. 8, computing the ultimate moment for each increment in A_s.

Two different design criteria are presented in this paper. The first criterion, identified by a little square and represented by the number 1 in Fig. 10 to 13, is determined as the last iteration (according to Fig. 8) for which the ultimate moment of the cross sections (M_u) causes all the bars in the dense-reinforced zone to be at yield stress (f_{yd}), i.e., all of them work in plasticity. For the next iteration, adding a bar in the dense-reinforced zone, the new M_u of this cross section will not cause yielding in all the bars of the dense-reinforced zone. In other words, the first criterion identifies the maximum ultimate moment that keeps all the bars of the dense-reinforced zone behaving in post-yield strains. This criterion is very similar to the notion of x_{lim} for rectangular cross section.

The second criterion, represented by the number 2 in Figs. 10 to 13, is determined as the first iteration in which the addition of a bar in the dense zone, as indicated in Fig. 8, causes a smaller ultimate moment than if the additional bar is located in the upper part of the light-reinforce zone –i.e., at the top of the cross section-.

Fig. 10 shows the flexural capacity of the circular cross-section, D= 500 mm, in pure bending as function of the total area of reinforcement, A_s. Two curves are represented in this figure. The upper curve corresponds to reinforcement using two different bar diameters (Ø10 and Ø32) and the bottom curve corresponds to reinforcement with only one bar diameter (Ø16). Fig. 10 shows that the use of two bar diameters is more efficient than the use of only one bar diameter. Furthermore, Fig. 10 illustrates that the range in the abscissa axis is greater when two bar diameters are used. Squares in Fig. 10 correspond to: the first proposed criterion (square 1) and the second proposed criterion (square 2).

Figs. 11-13 correspond to the same study for D=600, 800 and 1000 mm respectively.

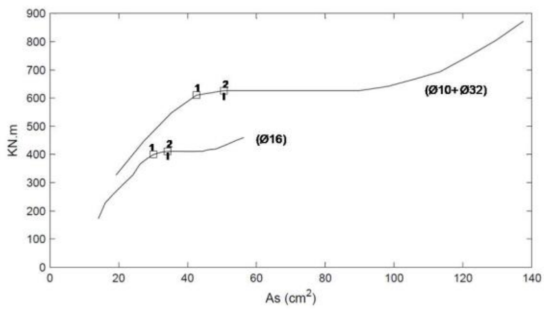


Fig. 11 *D*=600 mm

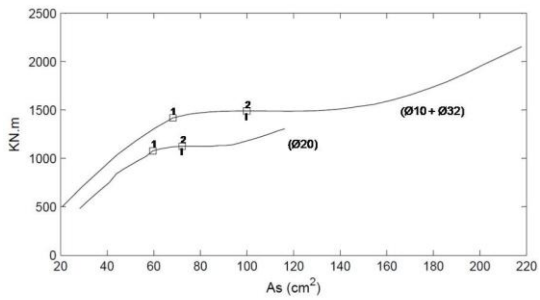


Fig. 12 *D*=800 mm

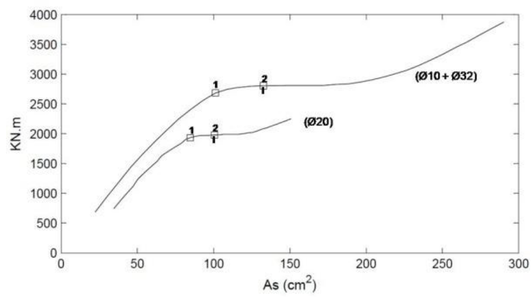


Fig. 13 *D*=1000 mm

All the curves in Figs. 10 to 13 show similar trends, these curves present three different domains. The initial domain corresponds to the first steep segment. In this domain little increment of the tension reinforcement steel placed in the dense-reinforced zone, away from the neutral axis, implies a considerable increase in the flexural strength of the cross-section. The boundary between the first domain and the second domain is the first design criterion. The second domain is identified in Figs. 10 to 13 as the horizontal part of the curve. In this second domain the increase of area of reinforcement does not lead to an increase in the flexural strength capacity of the cross-section. This blockage in the flexural strength capacity is due to the fact that the reinforcement bars are placed in the vicinity of the neutral axis –smaller stress-. In the third domain the slope increases again. In this final domain, the reinforcement is in compression and it is introduced far from the neutral axis, improving the flexural capacity of the cross section. It is interesting to note that this third domain corresponds to high amounts of reinforcement steel.

As shown in Figs. 10 to 13, the area of steel reinforcement obtained from the second strength design criterion (squares 2) is always bigger than the corresponding to the first one. Figs. 10 to 13 show that an increase in the area of steel does not always lead to an increase in the flexural strength of the cross-section (See Tables 1(a) and (b)), this is due to the fact that the reinforcement is added in the horizontal domain of the curves.

Tables 1(a) and (b) summarise the reinforcement and the bending moment capacity corresponding to both studied strength design criteria for all the cases studied. These values have also been represented in Figs. 10 to 13.

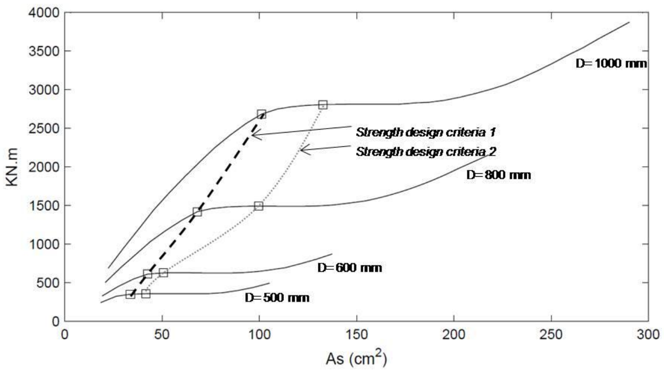
Table 1(a) Circular cross-sections asymmetrically reinforced using two bar diameters (Ø 10 @ 300 mm + Ø 32 @ 35 mm)

Diameter of the cross section	First criterion (squares 1 in Figs. 10 to 13)		Second criterion (squares 2 in Figs. 10 to 13)	
D (mm)	A_s (cm^2)	M_u (kN·m)	A_s (cm^2)	M_u (kN·m)
500	33.74 (2Ø 10+4Ø 32)	350.25	41.78 (2Ø 10+5Ø 32)	355.45
600	42.57 (3Ø 10+5Ø 32)	610.24	50.61 (3Ø 10+6Ø 32)	626.13
800	68.27 (5Ø 10+8Ø 32)	1420.32	99.65 (4Ø 10+12Ø 32)	1490.27
1000	101.22 (6Ø 10+12Ø 32)	2684.27	132.61 (5Ø 10+16Ø 32)	2807.19

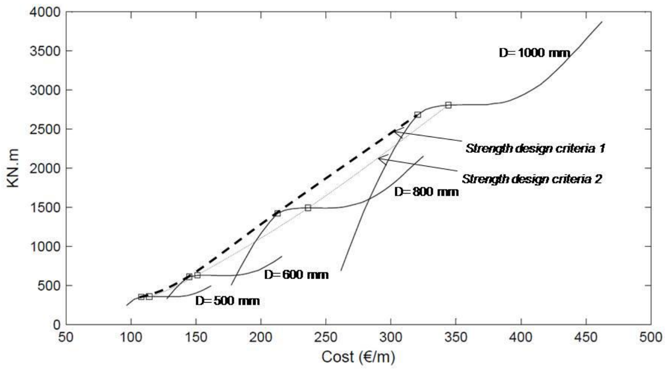
Table 1(b) Circular cross-sections asymmetrically reinforced using one bar diameter (At 300 mm in the compression zone and 35 mm in the tension zone)

Diameter of the cross section	First criterion (squares 1 in Figs. 10 to 13)		Second criterion (squares 2 in Figs. 10 to 13)	
D (mm)	A_s (cm^2)	M_u (kN·m)	A_s (cm^2)	M_u (kN·m)
500	24.13 (1Ø 16 @ 300 mm & 11Ø 16 @ 35 mm)	245.87	26.14 (1Ø 16 @ 300 mm & 12Ø 16 @ 35 mm)	246.20
600	30.16 (2Ø 16 @ 300 mm & 13Ø 16 @ 35 mm)	399.86	34.18 (2Ø 16 @ 300 mm & 15Ø 16 @ 35 mm)	410.59
800	59.69 (3Ø 20 @ 300 mm & 16Ø 20 @ 35 mm)	1079.82	72.25 (3Ø 20 @ 300 mm & 20Ø 20 @ 35 mm)	1126.34
1000	84.82 (4Ø 20 @ 300 mm & 23Ø 20 @ 35 mm)	1935.64	100.55 (3Ø 20 @ 300 mm & 29Ø 20 @ 35 mm)	1981.28

In order to obtain a complete vision of both criteria, all the curves that represent ARCS with two different bar diameters -upper curves in Figs. 10 to 13 are plotted together in Fig. 14(a). This figure shows that the first strength design criterion has a more linear behaviour than the second criterion. In fact, it can be concluded that only the first domain has practical consequences.



(a)



(b)

Fig. 14 Global results for circular piles reinforced using two bar diameters (Ø 10 @ 300 mm + Ø 32 @ 35 mm)

Table 2 Cost of a metre of pile

Unit	Price
Reinforcement steel	0.91 € / kg
Concrete (incluiding thixotropic mud, transportation, drilling and laying):	
D= 500 mm	78.85 €/ m
D= 600 mm	107.76 €/ m
D= 800 mm	153.81 €/ m
D= 1000 mm	232.95 €/ m
Aids	2%
Indirect costs	3%

Table 3 Summarize of cost of metre of no symmetrically reinforced circular cross section

Diameter of the cross section D (mm)	Reinforcement using two bar diameters		
	First criterion	Second criterion	Saving (%)
	Cost (€/m)	Cost (€/m)	
500	108.14	114.17	12.3
600	145.13	151.16	15.7
800	212.78	236.54	19.3
1000	320.63	344.17	18.1
	Reinforcement using one bar diameter		
500	100.93	105.45	7.1
600	135.85	138.85	8.5
800	206.35	215.77	10.9
1000	308.34	320.13	9.5

The proposed procedure could be summarized as: with the aid of Fig. 8 we try to reach a demand (M_d, N_d) by adding bars in the dense-reinforced zone. Fig. 14 helps us to choose the most convenient section diameter, in order to get as close as possible to the strength design criterion 1 by the left side (i.e., designing in first domain).

In order to estimate the cost per meter of pile, the prices of CYPE (2011) database have been consulted. Soft soil without cobbles or boulders and concrete poured without shoring has been considered. Prices per unit are summarized in Table 2.

Total costs of the reinforcement, per metre of pile, obtained from both criteria and for both types of reinforcement (using only one or two bar diameters) have been summarized in Table 3. Total cost includes excavation, concrete and steel. The last column in Table 3 shows the percentage of total savings for ARCS designed with the first criterion compared with the symmetrically reinforced cross-section, both presenting the same ultimate moment.

Flexural capacity of the cross sections has been represented with regards to cost per metre of pile in Fig. 14(b). Curves in Fig. 14(b) correspond to reinforcement using two different bar diameters for the entire studied cross-section diameters (D= 500, 600, 800 and 1000 mm)

The curves that link the first criterion points of each pile diameter, represented by a thick dash line in Fig. 14, can be used to choose the minimum pile diameter for a given design flexural moment, M_d. For example, for M_d=2500 kN the minimum diameter is 1000 mm.

5. Conclusions

Substantial reductions in the amount of steel reinforcement and concrete required in the design of pile walls can be achieved by coupling a novel solution approach with widely accepted assumptions for ultimate strength analysis. The present paper proposes a design criterion for the design of asymmetrical reinforcement for pile walls. The chosen solution of longitudinal reinforcement and pile diameter gives better flexural capacity to cost ratio. This criterion fixes an upper limit of the longitudinal reinforcement area for the design of asymmetrical reinforcement circular sections. The criterion is based on the existence of a local minimum in the graph bending capacity versus reinforcement area. Beyond this limit, the increment of the area of steel reinforcement will not produce an economical increase in flexural capacity. The most economical solution is obtained representing several graphs bending capacity versus reinforcement area for different pile diameter.

References

Aschheim, M., Hernández-Montes, E. and Gil-Martín, L.M. (2008), "Optimal domains for flexural and axial loading", *ACI Struct. J.*, **105**(6), 720-728.

Belarbi, A., Prakash, S. and You, Y.M. (2009), "Effect of spiral reinforcement on flexural-shear-torsional seismic behavior of reinforced concrete circular bridge columns", *Struct. Eng. Mech.*, **33**(2), 137-158.

Chakrabarty, B.K. (1992), "Model for optimal design of concrete beam", *J. Struct. Eng.*, **118**(11), 3238-3242.

Cype Ingenieros, S.A. (2011), *Software for architecture, engineering and construction*, www.cype.es.

Eurocode 2 (2002), *Design of concrete structures- Part 1: General rules and rules for buildings prEN*, European Committee for Standardization, Brussels.

Gil-Martín, L.M., Hernández-Montes, E. and Aschheim, M. (2010a), "Optimal reinforcement of RC columns for biaxial bending", *Mater. Struct.*, **43**(9), 1245-1256.

Gil-Martín, L.M., Hernández-Montes, E. and Aschheim, M. (2010b), "Optimization of piers for retaining walls", *Struct. Multidiscip. O.*, **41**(6), 979-987.

Gil-Martín, L.M., Aschheim, M., Hernández-Montes, E. and Pasadas-Fernández, M. (2011), "Recent developments in optimal reinforcement of RC beam and column sections", *Eng. Struct.*, **33**(4), 1170-1180.

Guerra, A. and Kiousis, P.D. (2006), "Design optimization of reinforced concrete structures", *Comput. Concrete*, **3**(5), 313-334.

Hernández-Montes, E., Gil-Martín, L.M. and Aschheim, M. (2005), "The design of concrete members subjected to uniaxial bending and compression using reinforcement sizing diagrams", *ACI Struct. J.*, **102**(1),150-158.

Hernández-Montes, E., Gil-Martín, L.M., Pasadas-Fernández, M. and Aschheim, M. (2008), "Theorem of optimal reinforcement for reinforced concrete cross sections", *Struct. Multidiscip. O.*, **36**(5), 509-521.

Hernández-Montes, E., Gil-Martín, L. and Aschheim, M. (2010), "Pilotes asimétricos para contención de tierras", *Revista de Obras Públicas*, **157**(3508), 31-38.

Lee, H.J., Aschheim, M., Hernández-Montes, E., and Gil-Martín, L.M. (2009), "Optimum RC column reinforcement considering multiple load combination", *Struct. Multidiscip. O.*, **32**(2), 153-170.

Narayan, K.S.B. and Venkataramana, K. (2007), "Shape optimization of steel reinforced concrete beams", *Comput. Concrete*, **4**(4), 317-330.

Topal, U. and Uzman, U. (2006), "Optimal design of laminated composite plates to maximise fundamental frequency using MFD method", *Struct. Eng. Mech.*, **24**(4), 479-491.

Weber, K. and Ernst, M. (1989), "Entwicklung von Interaktionsdiagrammen für asymmetrisch bewehrte Stahlbeton- reisquerschnitte", *Beton Stahlbetonbau*, **84**(7), 176-180.

Wiese, H., Curbach, M., Speck, K., Weiland, S., Eckfeldt, L. and Hampel, T. (2004), "Rissbreitennachweis für Kreisquerschnitte", *Beton Stahlbetonbau*, **99**(4), 253-261.

Whitney, C.S. and Cohen, E. (1956), "Guide for ultimate strength design of reinforced concrete", *ACI J.*, **28**(5), 445-490.

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