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Reinforcement design for the anchorage of externally prestressed bridges with "tensile stress region"

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Abstract. Two-dimensional tensile stresses are occurring at the back of the anchorage of the tendons of prestressed concrete bridges. A new method named "tensile stress region" for the design of the reinforcement is presented in this paper. The basic idea of this approach is the division of an anchor block into several slices, which are described by the tensile stress region. The orthogonal reinforcing wire mesh can be designed in each slice to resist the tensile stresses. Additionally the sum of the depth of every slice defined by the tensile stress region is used to control the required length of the longitudinal reinforcement bars. An example for the reinforcement design of an anchorage block of an external prestressed concrete bridge is analyzed by means of the new presented method and a finite element model is established to compare the results. Furthermore the influence of the transverse and vertical prestressing on the ordinary reinforcement design is taken into account. The results show that the amount of reinforcement bars at the anchorage block is influenced by the layout of the transverse and the vertical prestressing tendons. Using the "tensile stress region" method, the ordinary reinforcement bars can be designed more precisely compared to the design codes, and arranged according to the stress state in every slice.

Keywords: end anchorage beam; tensile stress region; slice; slice depth; externally prestressed concrete bridge; reinforcement design

1. Introduction

The external prestressing tendons in prestressed concrete bridges are usually anchored inside the end cross beam. In consequence, large tensile stresses are occurring inside the cross beam. The external prestressing tendons outside the longitudinal beam cross section transfer the prestressing forces exclusively by the anchorage system. In case of a failure in the anchorage block, it can induce an overall collapse of the bridge (Breen *et al.* 1991). Traditional methods cannot determine an appropriate stress distribution because of the complex physical dimensions and the mechanical mechanism inside the anchorage structure. In recent years, structural engineers are frequently applying strut-and-tie models according the reinforcement design, which takes the transmission mechanism of the forces inside the anchorage block into account. Mentioned here are the codes and guidelines of the United States of America and Canada (AASHTO LRFD 2004, ACI 2005,

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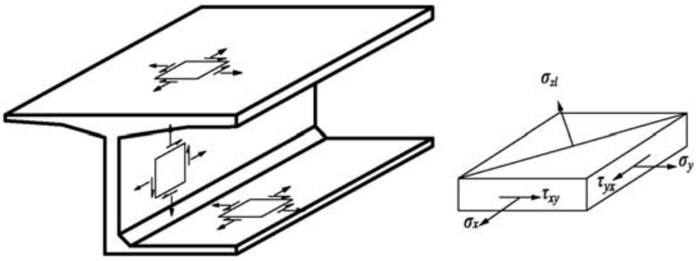


Fig. 1 Tensile stress region for elements under combined shear and bending loading

CSA 2000, Empelmann and Wichers 2009, Victoria *et al.* 2011). These strut-and-tie models can usually determine the total amount of the reinforcement bars. Using the strut-and-tie models, an entire structure needs to be divided into many subparts along all directions (upward, downward, left and right) in order to define and analyze the tension and the compression struts (Bae *et al.* 2011, He and Liu 2010, Li and Wang 2010, Lu *et al.* 2010, Park *et al.* 2010, Perera and Vique 2009, Praveen and Madhavan 2008, Sergio and Micah 2007, Zhao *et al.* 2011). Therefore these approaches are more feasible to analyze regular entire structures; in contrast there are some limitations in the irregular ones. For example, the analysis of an anchorage block with a manhole including the effects of transverse and vertical prestressing is not anymore a regular part of a structure and the strut-and-tie models cannot determine realistically the mechanical mechanism. Their accuracy and applicability according the reinforcement design still needs to be discussed further.

A new theory for the reinforcement design named "tensile stress region method" (we put it abbreviated as TSRM), which is independent on the regularity of a structure, is presented in this paper. In this method, which is based on the stress analysis, the orthogonal components of the tensile stresses of the concrete are represented by lattice reinforcement, where the longitudinal components of the principal tensile stresses are resisted by the longitudinal bars, while the transverse components of those are resisted by the transverse bars (Xu 2008). The basic characteristics of the presented method are as following: crack width at service state is limited by the control of the tensile stress of the ordinary reinforcement bars and at the ultimate state the area of the longitudinal and transverse reinforcement are the control parameters. Using this new method, the stress distribution and the reinforcement design inside an anchorage block of an externally prestressed concrete bridge are discussed.

2. Theory about tensile stress region and reinforcement design

2.1 Tensile stress region of plate structures

For concrete webs under in-plane two-dimensional stress states (principal tensile and principal compressive stresses or in-plane normal and shear stresses), diagonal cracks occur where the principal tensile stress exceeds the tensile strength of concrete. In these regions, principal tensile stresses are distributed regularly and evenly along the thickness of the web. Such regions with a similar distribution of principal tensile stresses are defined as a "tensile stress region". For rectangular and T-beam cross-sections the webs can be considered as a tensile stress region to resist the principal tensile stresses. In case of box girders, the top and bottom plates can also be taken into account as a "tensile stress region", because of a similar characteristic of the stress distribution as compared to the web theory. The tensile stress regions for an element subjected to combined bending and shear stresses is shown in Fig. 1.

An entire structure can be considered as an assembly of several plate elements with distributed in-plane stresses. Assuming a differential element from the plate the in-plane two-dimensional principal stresses can be obtained from the normal stresses σ_x, σ_y and the shear stress τ_{xy}. The principal tensile stress is distributed evenly along the thickness of the differential element and regularly along the length and the width in every plate (top plate, bottom plate and webs). The longitudinal and transverse reinforcement in all plates of the box girder resist the longitudinal and transverse components of the principal tensile stresses. There is always a necessity to design the amount of those steel bars in the design process of the structure, which is dependent strongly on the calculation results. Especially for a web at the midspan of a simply supported box girder bridge with approximately pure bending loading, the main mechanical characteristics are as following:

(1) Out-of-plane shear stresses are much less than the in-plane principle stresses. Therefore principle stresses deviate hardly from the plane of the plate. At the same time, the difference between out-of-plane shear stresses along the thickness of the plate is small and has a minor influence. Principle stresses distribute evenly along the thickness of the plate according to the rule of the stress transmission.

(2) In-plane shear stresses are much less than those in-plane longitudinal normal stresses. The direction of the principle stresses is almost consistent respective to that of the normal stresses. At the same time, the difference between in-plane shear stresses along the depth of the web perpendicular to the principle stresses is small and in consequence the principle stresses change evenly along the depth of the plate. The web can be cut into many slices along the depth. Therefore the orthogonal grid reinforcement can be arranged in every slice to resist the principle tensile stresses.

2.2 Tensile stress region of block structures and divisional basis of slices

It is also possible to define the "tensile stress region" for block structures. When end anchor beams of concrete bridges are loaded by the anchoring forces of the external prestressing tendons, they are subjected to two-directional out-of-plane bending moments. The tensile stress region occur at the inner side of the cross beam. Concurrent to this, the distribution of the in-plane two-dimensional stresses in the transverse tangent plane is regular along the longitudinal direction.

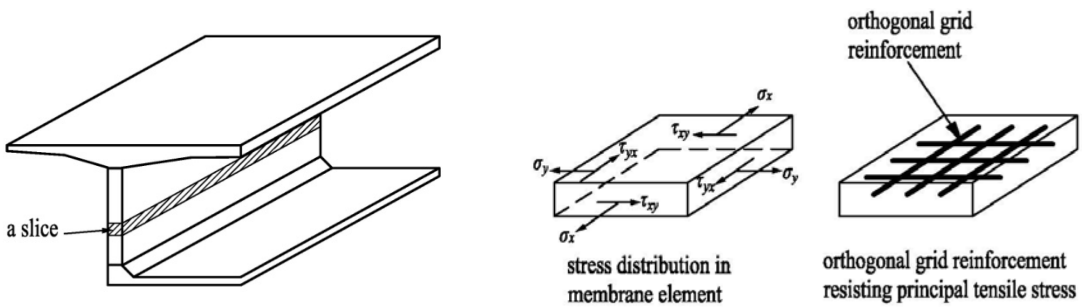


Fig. 2 Shear resistance distribution in membrane element

It is necessary to define many slices with the same thickness along the longitudinal direction from the inner side of the end anchorage beam (see Fig. 4).

A differential element of a slice is shown in Fig. 3(a). There is a certain region with approximately two-directional pure bending moments within the plane of every slice. In this

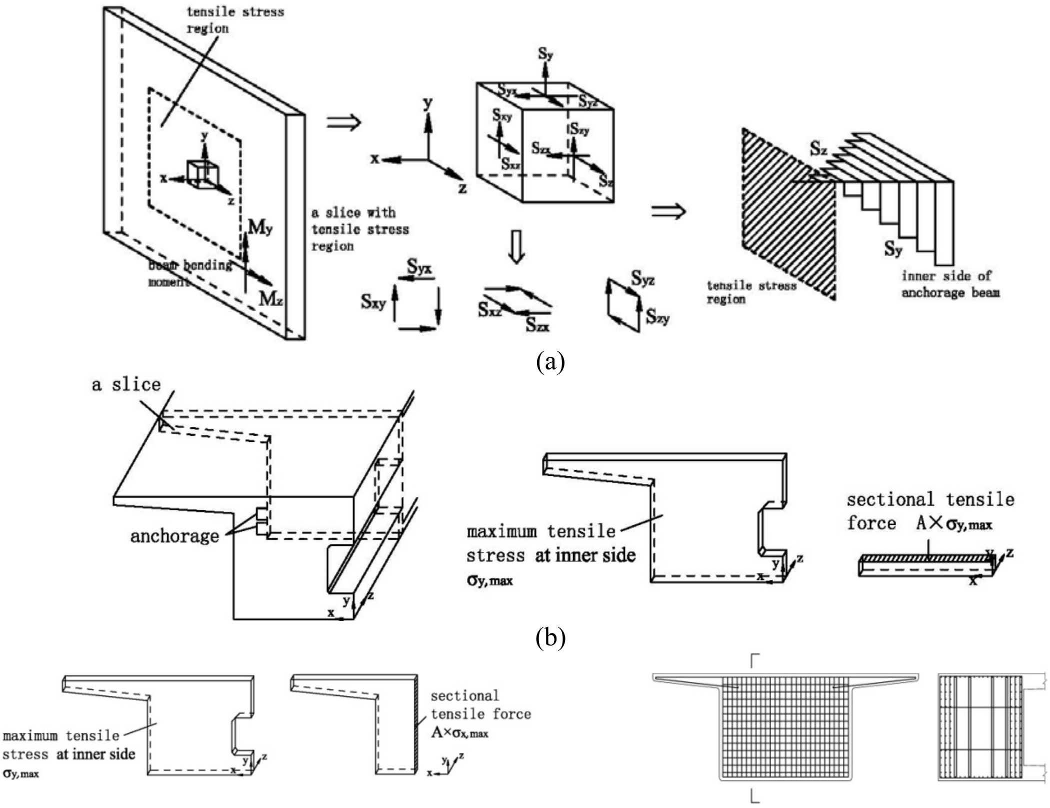


Fig. 3 Tensile stress method reinforcement: (a) tensile stress region of slices, (b) vertical tension force, (c) transverse tension force and (d) layout of reinforcements inside anchor cross beam

Table 1 Ratios of shear stress to principle stress in planes Oxy and Oxz

Numbering	with the manhole				without the manhole			
	$\frac{\lvert S_{xy}\rvert}{\lvert S_y\rvert}$	$\frac{\lvert S_{xz}\rvert}{\lvert S_z\rvert}$	$\frac{\lvert S_{yz}\rvert}{\lvert S_y\rvert}$	$\frac{\lvert S_{zy}\rvert}{\lvert S_z\rvert}$	$\frac{\lvert S_{xy}\rvert}{\lvert S_y\rvert}$	$\frac{\lvert S_{xz}\rvert}{\lvert S_z\rvert}$	$\frac{\lvert S_{yz}\rvert}{\lvert S_y\rvert}$	$\frac{\lvert S_{zy}\rvert}{\lvert S_z\rvert}$
1#	0.16%	1.24%	0.00%	2.86%	0.17%	0.21%	0.00%	2.39%
2#	2.70%	2.61%	0.00%	3.46%	0.62%	2.53%	0.00%	3.14%
3#	5.52%	6.50%	0.00%	3.03%	1.70%	3.48%	0.00%	3.76%
4#	7.49%	7.47%	0.00%	3.47%	6.04%	6.98%	0.00%	3.46%
5#	9.52%	4.96%	0.00%	3.45%	7.17%	5.18%	0.00%	3.06%

region, stresses S_y and S_z are approximately equal to the principle stresses in the tangent plane, if the planes Oxy and Oxz are similar to the webs of the box girder. To investigate the relative ratio of the maximum transverse and vertical shear stresses to the corresponding principle stresses in different tangent planes, an example of a five-span continuous externally prestressed concrete bridge was analyzed (see Section 3.1). Detailed explanations according the divisional method of those slices and the simulation results are presented in Section 4. It should be mentioned here, that the ratios of the shear stresses to the principle stresses in the planes Oxy and Oxz are small (see Table 1).

According to the listed ratios (see Table 1) between the shear stress and the principle stress we can conclude:

(1) Whether or not the manhole of the cross-section is considered in the analysis, the stress ratios in the five slices along the longitudinal direction of the anchor cross beam near the inner side are less than 10%.

(2) Out-of-plane shear stresses S_{yz} and S_{zy} in the planes Oxy and Oxz are much less than the principle stresses S_y and S_z. Hence, the S_y and S_z are deviate hardly from the planes Oxy and Oxz and their distribution are more even in a specific region in the plane of every slice.

(3) In-plane shear stresses S_{xy} and S_{xz} are so much less than the principle stresses S_y and S_z that the S_y and S_z deviate hardly from the planes Oxy and Oxz. The two-dimensional principle stresses in the plane of every slice distribute evenly along the thickness of the plate. This is the divisional basis of every slice along the thickness of the plate.

Furthermore there is a tensile stress region within a certain range of every slice, in which the stresses at each point are similar (see Fig. 3(a)). The region extends to a specific depth along the longitudinal direction of the block structures. Actually, the tensile stress region of every slice in the block structures is similar to those of the web of box girders subjected to pure bending moments. In such girders the reinforcement needs to be arranged mostly for the end anchor beam according to the design requirements. After the tensile stress regions of every slice are determined the mesh of the in-plane orthogonal reinforcement can be arranged in the region with two-dimensional principle stresses to resist transverse and vertical tensile stresses. We assume, that each mesh of the reinforcement inside the cross beam is arranged in the middle of each slice with the thickness 10 cm, i.e., the space between each layer of the reinforcement is 10 cm. For the maximum tensile stress in transverse and vertical direction of every slice, we can assume conservatively that these stresses distributes evenly within their tensile stress region. In the end, the tension forces of each slice in both directions of the box girder can be determined without great effort. The required area of the reinforcement can be computed according to the determined forces. The application above-mentioned concerning “the tensile stress region” is shown in the Figs. 3(b) and (d).

Using this methodology for the reinforcement design the stresses in the slices distribute evenly as the maximum ones, which does not match the expected situation for the cross beams with large sectional area. For a more accurate and feasible design of the reinforcement, it is possible to modify the above-mentioned assumptions. Different zones with varying stress levels are defined therefore. The presented method about the reinforcement design can be use directly to compute numerical results of the stresses in solid elements. It is possible to design the reinforcement according to the required area in each slice. In addition, the layout of the reinforcement can follow the stress distribution inside the cross-section more precisely. Therefore the amount and the layout of the reinforcement can be designed in a more efficient way in comparison to the design codes.

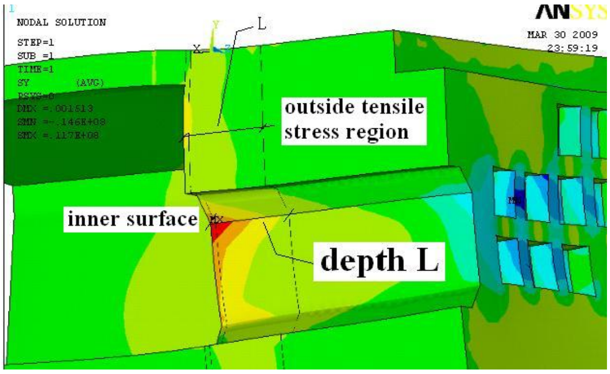


Fig. 4 Depth of tensile stress region

2.3 Depth of the tensile stress region in entire end anchorage beam

The depth of the tensile stress region in an entire structure can be defined according to the regularity of the distribution of the tensile stress region, shown in Fig. 4. The total extended distance *L* of the transverse slice spreads continuously from the inner side to the outside of the anchor cross beam. It can be defined by the depth of tensile stress region which is similar to the height of the sectional tensile zone in flexural concrete beams. We can assume the tensile stress region, when the area for these elements is bounded until the maximum tensile stress in a transverse slice is less than 1.0 MPa. Therefore, the amount of the necessary transverse slices and the longitudinal region for the layout of the reinforcing mesh are determined by the depth *L*. When an anchor cross beam is loaded by the anchor forces, we can assume the cross beam as a four-sided supported elastically beam composed of the top and bottom slabs and webs. The in-plane two-directional tension forces in the cross-section are generated by two-directional bending moments that are caused by the anchor forces. Because the transverse bending moment is usually unequal to the vertical one, the vertical depth of the tensile stress region L_v is usually unequal to the transverse one L_t. For an anchor cross beam, the depth of the tensile stress region is influenced by some parameters such as the anchor forces, the anchor type, local effects of the transverse and vertical prestressing tendons, etc.

For an arbitrary entire structure which can be divided continuously into many slices expressed by the tensile stress region, the maximum tensile stresses in random adjacent sections which are parallel to the slice should be continuous within the depth of the tensile stress region. The function of the maximum tensile stresses $p(x)\,(0 \le x_i \le L_k, k = v, t)$ is shown in Fig. 5. The horizontal axis x is perpendicular to the slices defined by the tensile stress region. The zero point expresses the location of a section with a maximum stress. Total transverse N_t and vertical N_v tensile forces within the depth of the tensile stress region are given by the following equation

$$N_k = \int_{L_k} p(x)dx \approx \sum_i a_i p(x_i), k = v, t \tag{1}$$

where a_i is the thickness of the slice at the location x_i.

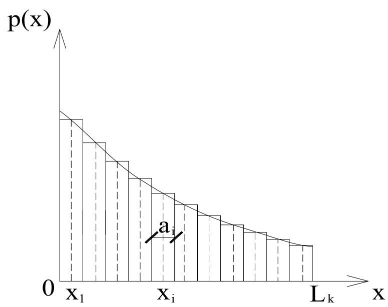


Fig. 5 Function of the depth of tensile stress region

If the maximum stress of the transverse and vertical reinforcement at the service state is limited to 150 MPa, the amount of the required reinforcement is given by the following relation

$$n_k = Int[\frac{N_k}{150 \cdot s}] + 1, k = v, t \tag{2}$$

where n_t and n_v are the amount of transverse, respectively vertical reinforcement within the depth of the tensile stress region. In the above mentioned equation, *s* is the sectional area of a single reinforcement bar.

Applying the TSRM for the design of the reinforcement for an entire structure, the definition of the depth for the tensile stress region is very important and therefore sensitive to the whole design process.

3. Numerical model of end anchorage beam

3.1 Geometry and anchor assembly of externally prestressed bridge

The end anchorage beam is a part of an externally prestressed continuous concrete beam bridge with the span 5×50 m. The bridge is a real existing structure and the design documents are available to compare the presented method with the common design practice. The height and the width of the end transverse beam is 3.0 m and 2.4 m respectively (see Fig. 6). This bridge is composed of the C55 concrete class. The type of the external prestressing tendons is $27\phi^s 15.2$, whose tension stress (1209 MPa) is controlled to 0.65 times the ultimate stress limit. The internal transverse prestressing strands are arranged in the top slab of the end anchorage beam. The tension force of each strand is controlled to 562.5 kN (the sectional area and the tension stress of each strand is equal to 420 mm^2 and 1339 MPa). The space between those strands along the longitudinal direction in the top slab is between 40 and 45 cm. In addition, two rows of fine rolled twisted bars, each is composed of 10 bars, are arranged along the vertical direction of the cross beam. They are 15.0 cm and respectively 28.2 cm away from the inner surface of the cross beam. The tension force of each bar with diameter 32 mm is equal to 747.5 kN (the sectional area and tension stress of each bar equals to 803.8 mm^2 and 930 MPa). The Young's modulus E_s of the transverse strands and the vertical bars is equal to 2.0×10^5 MPa. Because of the extremely large loss of the

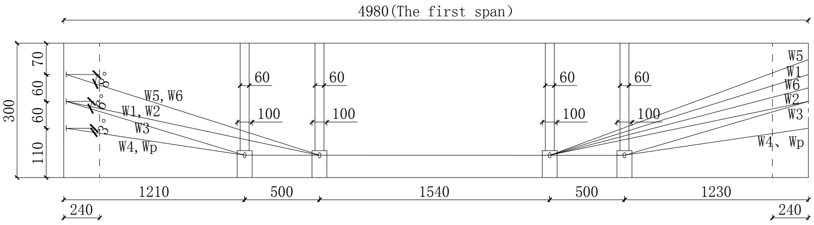
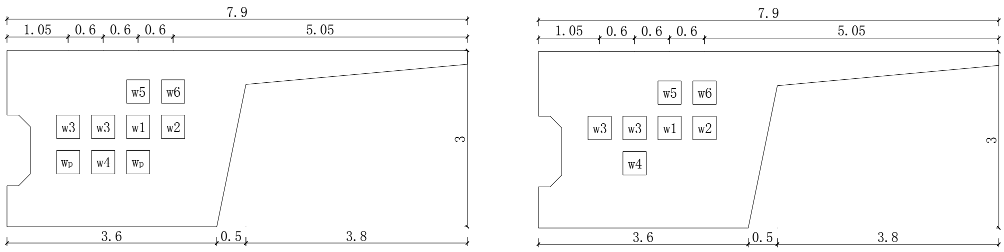
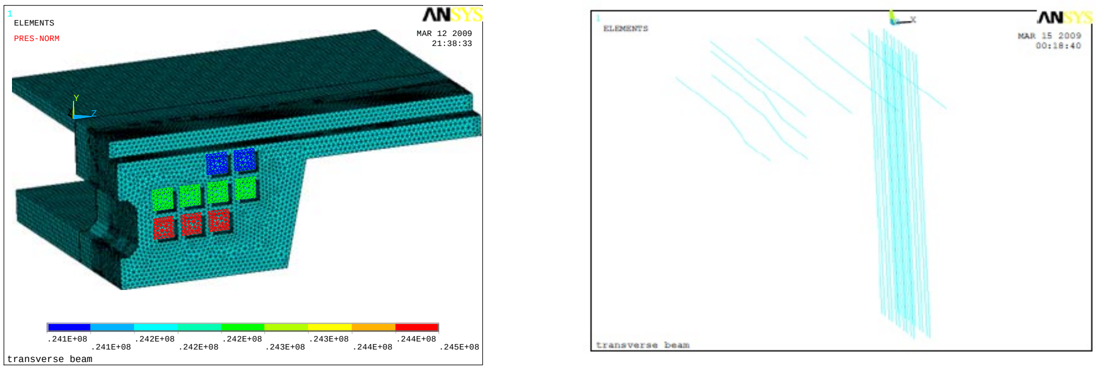


Fig. 6 Layout of external prestressing tendons



(a) (b)

Fig. 7 Anchorage assembly: (a) anchorage assembly 1 and (b) anchorage assembly 2



(a) (b)

Fig. 8 Finite element model: (a) ANSYS model and (b) elements of transverse and vertical prestressing struts/tendons

prestressing force in the vertical prestressing, we assume to multiply their effect by a reduction factor of 0.5 according to experience. Two possible anchor assemblies are considered in the original design, shown in Fig. 7. A local numerical model (FEM) of the end anchorage beam is established using the software ANSYS and is shown in Fig. 8. In the model, the anchoring surface is perpendicular to the bottom slab of the box girder and the effective tension force of the external prestressed tendons is parallel to the bottom slab. The anchorage is instead loaded by uniform plane forces, which are occurring perpendicular to the anchor backing plate reversely. Dimensions of the anchor plate is 430 mm × 430 mm and the equivalent loads in the original design are 4537 kN×cos8°, 4537 kN×cos6° and 4537 kN×cos3° respectively from top to bottom anchor point.

3.2 Finite element model of end anchorage beam for analysis of tensile stress region

An numerical model using the Finite Element Method is established for the anchor cross beam, whose dimensions along the transverse and the longitudinal direction adopt a half of the bridge cross-section width and 6m respectively (using symmetry conditions and Saint-Venant's principle). Studying some pre-simulations with the longitudinal dimensions of 15 m, 10 m and 5 m, the chosen dimension of 6 m shows acceptable results and time efficiency. Axes X, Y and Z express the longitudinal, vertical and transverse direction in the global coordinate system. The stress distribution can be simulated assuming a purely elastic material for the concrete and the steel bars. The strategy for establishing the model is the following: (1) model of a cantilever beam with a constant depth and a length of 6 m. It is established according to the anchor assembly of the end anchorage beam and the layout of the cross-sections. The element type used in the software package ANSYS is SOLID45. The element shape of the SOLID45 is rectangular with eight edge nodes including displacement and rotations degrees of freedom in all directions. Regular parts of the model are meshed by the sweep method, while the irregular ones are meshed manually. The divisional length of an element is 0.1 m. 65515 Nodes and 197669 elements are used in the model neglecting the transverse and vertical prestreesing elements. (2) The surface root of the cantilever is constrained in all directions and rotations and symmetric conditions are applied in the symmetric plane. (3) The effects of transverse and vertical prestressing, whose location is determined by the segmentation of the entire structure, are simulated by the method of common node between the LINK8 bar elements and the block elements. Divisional length of a prestressing element is 0.2 m. The prestressing force is applied to the tendons by initial strains which are derived from the allowable tension stresses, shown in Fig. 8(b).

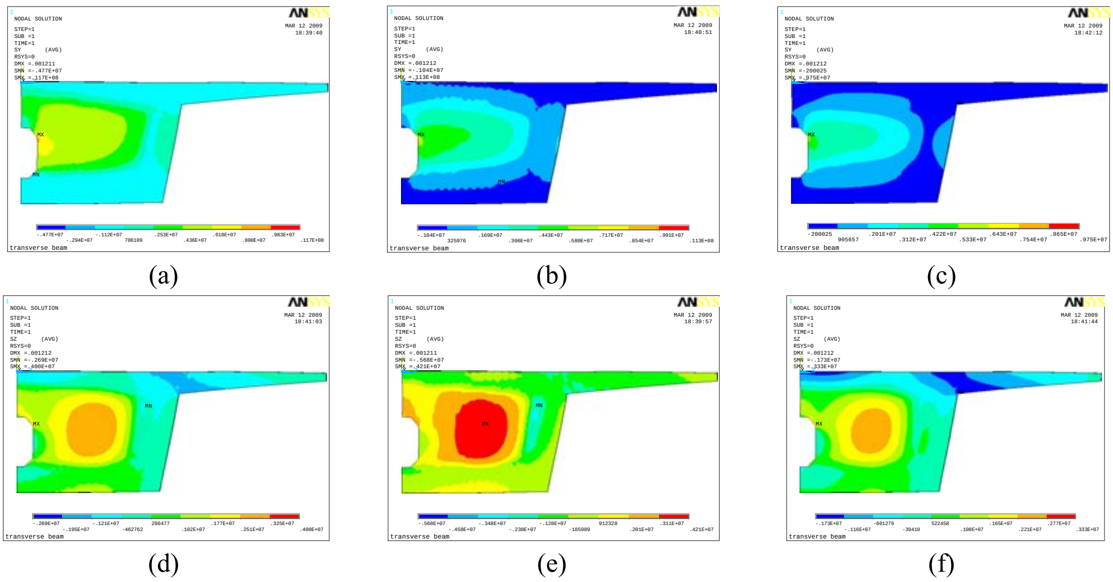


Fig. 9 Stress distribution under the first condition: (a) vertical stress in section 1#, (b) vertical stress in section 2#, (c) vertical stress in section 3#, (d) transverse stress in section 1#, (e) transverse stress in section 2# and (f) transverse stress in section 3#

4. Simulation results of end anchorage beam

4.1 Stress distributions at different slices

We start to cut the first slice along a cross-section, which is 5 cm away from the inner surface of the end anchorage beam. The following slices from the model are cut in consistent 10 cm increments. We analyze the distributions of the transverse and the vertical normal stresses in eight cross-sections, which are 5 cm, 15 cm, 25 cm, 35 cm, 45 cm, 55 cm, 65 cm and 75 cm away from the inner surface of the end anchorage beam. Therefore eight sections are numbered by 1＃, 2＃, 3＃, 4＃, 5＃, 6＃, 7＃ and 8＃. Four different simulation conditions are considered: (1) effect of all transverse and vertical prestressing tendons is neglected, (2) effect of all transverse prestressing tendons is considered, (3) effect of all vertical prestressing tendons is considered and (4) effect of all transverse and vertical prestressing tendons is considered. This paper provides distributing figures of the transverse and the vertical normal stresses for the cross-sections numbered by 1＃, 2＃ and 3＃. Those slices of the end anchorage beam are analyzed for the first and fourth conditions, shown in Figs. 9 and 10.

4.2 Discussion of the stress distribution results

Calculation results considering the two anchor assemblies under the first and fourth conditions are shown in Tables 2 and 3. If the tensile stresses of the concrete are computed to be less than 1.0 MPa the computed required tensile reinforcement is unnecessary for normal strength concrete in bridge structures. However, the constructional reinforcement layers defined by the codes and

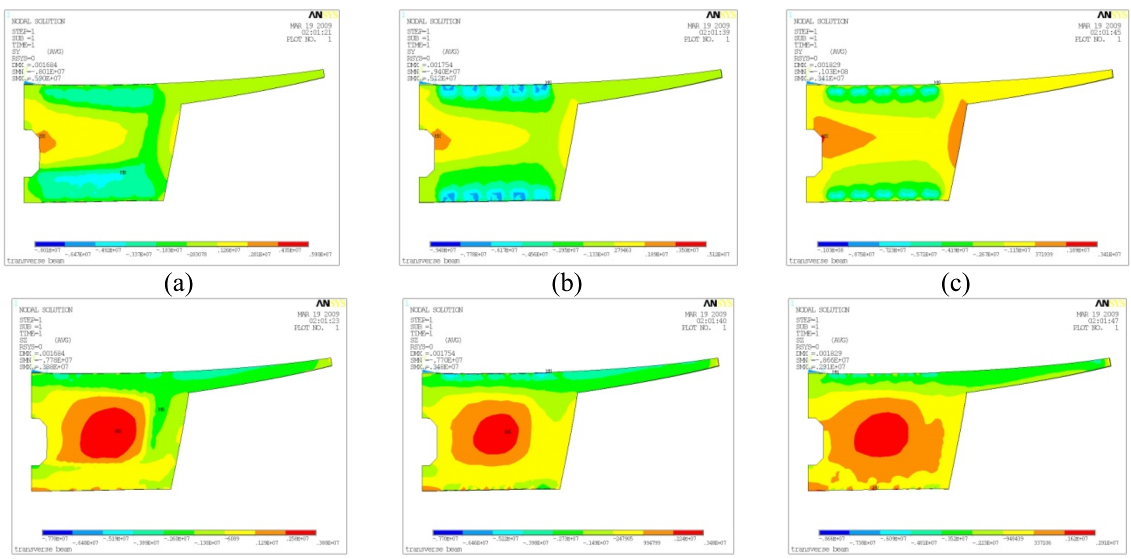


Fig. 10 Stress distribution under the fourth condition: (a) vertical stress in section 1#, (b) vertical stress in section 2#, (c) vertical stress in section 3#, (d) transverse stress in section 1#, (e) transverse stress in section 2# and (f) transverse stress in section 3#

Table 2 Calculation results without the vertical and transverse prestressing tendons (first condition)

Location		Tensile stress region method						Original design	
		Average of maximum Tensile stress [MPa]		Tension force [kN]		Required reinforcement area [mm^2]		Area of reinforcement [mm²]	Number of reinforcement [-]
		Anchor form							
		1	2	1	2	1	2		
1#	Vertical	7.1	6.76	4254	4053	28360	27020	23071	47Ø25
2#	Vertical	6.5	5.05	3891	3027	25940	20180	23071	47Ø25
3#	Vertical	4.8	3.67	2865	2199	19100	14660	23071	47Ø25
4#	Vertical	3.9	3.01	2361	1806	15740	12040	23071	47Ø25
5#	Vertical	3.1	2.33	1845	1398	12300	9320	0	0
6#	Vertical	2.3	1.77	1398	1062	9320	7080	0	0
7#	Vertical	1.7	1.33	1023	795	6820	5300	23071	47Ø25
8#	Vertical	1.6	1.26	948	756	6320	5040	0	0
1#	Transverse	3.7	2.97	732	594	4880	3960	10468	17Ø28
2#	Transverse	2.9	2.26	577	452	3847	3013	10468	17Ø28
3#	Transverse	2.5	1.86	498	372	3320	2480	10468	17Ø28
4#	Transverse	2.1	1.55	412	310	2747	2067	10468	17Ø28
5#	Transverse	1.6	1.48	310	296	2067	1973	201	1Ø16
6#	Transverse	1.3	1.13	259	225	1727	1500	201	1Ø16
7#	Transverse	1.1	0.86	221	171	1473	1140	1005	5Ø16
8#	Transverse	0.9	0.65	169	129	1127	860	201	1Ø16

guidelines should not be neglected. The tensile stress concentration at the sharp corners and at some surfaces in the webs and bottom slabs should not pay attention for the evaluation of the simulation results. Dimensions of the tensile stress region in each slice of the cross-section of the box girder are assumed to have a height of 2 m and a width of 6 m. To limit the influence factors regarding the simulation, the diameters of all reinforcement bars are chosen equally to 20 mm and the allowable stresses of all reinforcement at the service state is 150 MPa. The original design (see Tables 2 and 3) is the outcome from the former design process of the externally prestressed bridge.

The total area of all transverse and vertical reinforcements designed by the TSRM method is shown in Table 4.

Studying the simulation results and the design documents of the bridge (listed in Table 2 to Table 4), it is obvious that the total area of the reinforcement in the original design is much more than that computed by applying the TSRM method. Taking the longitudinal and transverse prestressing tendons into account (fourth condition), leads to a maximum difference of more than 400% for the anchor assembly 2. Therefore, the layout of the reinforcing mesh in the original design is too safe and conservative. Among them, the amount of the vertical reinforcement in the original design in every section is larger under the fourth condition compared to the presented method and for those sections numbered 1# and 2# smaller under the first condition. In the other sections, the amount of the reinforcement of the original design is larger for the first condition that

Table 3 Calculation results with all vertical and transverse prestressing tendons (fourth condition)

Location		Tensile stress region method						Original design	
		Average of maximum Tensile stress [MPa]		Tension force [kN]		Required reinforcement area [mm^2]		Area of reinforcement [mm²]	Number of reinforcement [-]
		Anchor form							
		1	2	1	2	1	2		
1#	Vertical	3.58	1.53	2148	918	14320	6120	23071	47Ø25
2#		2.70	1.31	1617	786	10780	5240	23071	47Ø25
3#		1.13	1.14	678	684	4520	4560	23071	47Ø25
4#		0.84	-	504	0	3360	0	23071	47Ø25
5#		-	-	0	0	0	0	0	0
6#		-	-	0	0	0	0	0	0
7#		-	-	0	0	0	0	23071	47Ø25
8#		-	-	0	0	0	0	0	0
1#	Transverse	3.23	2.56	646	511	4307	3407	10468	17Ø28
2#		2.86	2.18	572	435	3813	2900	10468	17Ø28
3#		2.45	1.87	490	374	3267	2493	10468	17Ø28
4#		1.94	1.45	387	289	2580	1927	10468	17Ø28
5#		1.47	1.11	294	221	1960	1473	201	1Ø16
6#		1.14	0.83	228	166	1520	1107	201	1Ø16
7#		0.85	0.61	169	122	1127	813	1005	5Ø16
8#		3.58	1.53	2148	918	14320	6120	23071	47Ø25

Note: The symbol "–" in the table expresses compression stress.

Table 4 Comparison of reinforcement amount between the anchorage assembly 1 and 2

Condition	Total area of all reinforcements [mm^2]		Reduction [%]
	For anchor assembly 1	For anchor assembly 2	
First condition	145087	117633	18.9%
Second condition	140707	121533	13.6%
Third condition	51427	28953	43.7%
Fourth condition	52367	30627	41.5%
Original design	158835	158835	-

does not match the distribution regularity of stress and is inappropriate and uneconomic. On the other side, the amount of the transverse reinforcement in the range of 0 - 40 cm next to the inner surface of the anchor cross beam is larger, while those in the range of 40 – 80 cm away from the inner surface of the anchor cross beam is slightly smaller, that match the basic design requirements.

In addition, we can also conclude from the simulation results (see Table 4), that the required area of the ordinary steel bars under the second condition is more than that under the third condition. The effect of the vertical prestressing, which resists the vertical tensile stresses, can reduce significantly the required amount of the ordinary steel bars. For the anchor assembly 1 and 2, the required amount of the ordinary steel bar under the third condition are less than those under the first condition by 64.6% and 75.4% respectively. The ones under the second condition are contrary less than those under the first condition by -3.0% and +3.3%. The influence of the transverse prestressing compared to the longitudinal prestressing according the amount of the ordinary steel bars is small. Some large tensile stresses occur additionally next to the manhole and at the outer edge (or bottom edge) of the web due to the effects of the transverse and vertical prestressing (surface stress).

The curve shape of the maximum tensile stress inside the cross beam for all conditions are shown in Fig. 11. Those distributions of maximum tensile stresses under the four conditions are expressed respectively using 1st, 2nd, 3rd and 4th condition.

According to the tensile stress distributions (see Fig. 11), it can be identifying that the change of the maximum transverse tensile stresses is basically the same under the various conditions.

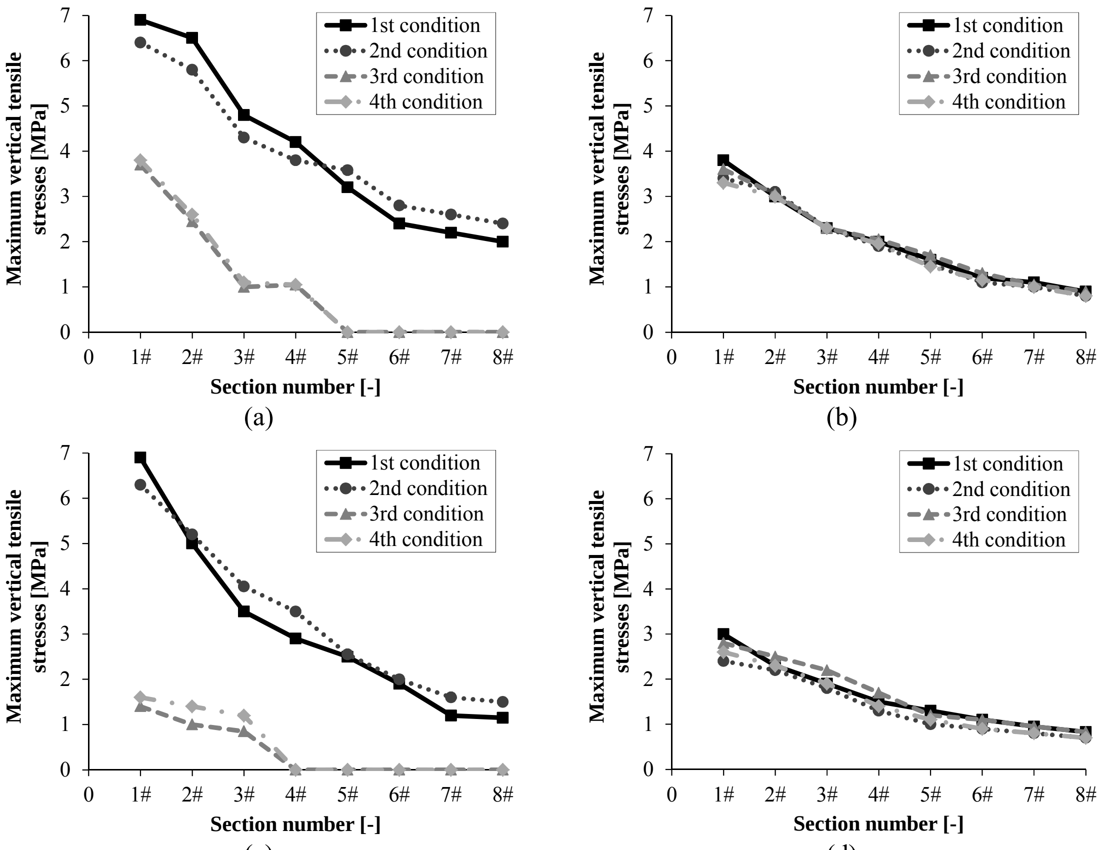


Fig. 11 Distributions of maximum tensile stresses in cross beam: (a) distributions of maximum vertical tensile stresses for anchorage assembly 1, (b) distributions of maximum transverse tensile stresses for anchorage assembly 1, (c) distributions of maximum vertical tensile stresses for anchorage assembly 2 and (d) distributions of maximum transverse tensile stresses for anchorage assembly 2

Table 5 Depth of tensile stress region (unit: cm)

Depth		Conditions			
		1	2	3	4
Anchor assembly 1	L_v	85	90	25	35
	L_t	75	75	75	75
Anchor assembly 2	L_v	75	80	15	25
	L_t	55	55	55	55

Against this statement, the maximum vertical tensile stresses are reduced significantly due to the effects of the vertical prestressing. For the anchor assembly 1 and 2, the average of maximum vertical tensile stresses in various sections is reduced by 3.71 MPa and 5.43 MPa respectively due to the effects of the vertical prestressing. At the same time, an approximation under various conditions for the depth of the vertical and the transverse tensile stress region can be defined regarding the results shown in Fig. 11. The depths of the tensile stress region are listed in Table 5.

The influence of the vertical prestressing concerning the vertical depth L_v of tensile stress region is significant. This depth L_v is for the anchor assembly 1 greater compared to the anchor assembly 2, in numbers 10 cm. The transverse depth L_t of the tensile stress region is equal under various conditions. This depth L_t is for the anchor assembly 1 more than that of the anchor assembly 2, in numbers 20 cm. Therefore, the distributing region of the transverse and the vertical reinforcement can be designed according to the depth of the tensile stress region under various conditions. For example, the range of depth of the tensile stress region, the transverse and the vertical reinforcement can be designed according to the required amount of the reinforcement at each slice.

5. Conclusions

1. In the case of taken the effects of all transverse and vertical prestressing into account, the required amount of the ordinary steel bars in the original design are overestimated and conservative, especially for the vertical ordinary steel bars. We recommend that, the amount of the vertical ordinary steel bars can be reduced and the distribution and the amount of the transverse ordinary steel bars should be adjusted properly according to the range of the depth of the tensile stress region. It should be addressed, that the method in this paper is based on the overall analysis of the anchor block to the local zone, such as the design of the reinforcement bars inside the cross beam. For the design of the reinforcement layers of the local load-bearing anchorage, it is necessary to study an additional local analysis. Finally, the layout of the reinforcement bars should also satisfy the constructional requirements.

2. The effect of the vertical prestressing is a lightly influence according the maximum transverse tensile stresses, while the ones of the transverse prestressing have a lightly influence regarding the maximum vertical tensile stresses. In this paper, the evaluated object of the tensile stress region is the reinforcing mesh inside the box girder of the anchor cross beam. In fact, the surface tensile stresses near the manhole and at the outer edge of web due to the effects of the vertical prestressing can be admitted by the strengthening ribs and the diagonal reinforcement bars.

It can also be identify, that the transverse prestressing can produce large transverse tensile stresses. Their distribution is more continuous in the middle of the cross-section, but the local maximum stresses occur sometimes at the top and the bottom flanges of the cross-section. This should be resisted using the strengthening reinforcement according to the simulation results and the constructional requirements by the design codes and guidelines.

3. The reinforcement design based on the tensile stress region can be applied widely to the reinforcement design of any solid structure. Further research about the presented method is necessary in order to achieve practical design recommendations. The distribution of the reinforcing mesh can be designed according to the stresses in every slice. The depth of the tensile stress region has to resist the principle tensile stresses. The method can be used for the design of completed structures as well as for the construction process of reinforced and prestressed concrete bridges.

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