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Structural optimization and proposition of pre-sizing parameters for beams in reinforced concrete buildings

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Abstract. The aim of the present paper is to show the application of optimization strategies for the cost of beams in reinforced concrete buildings and to propose pre-sizing parameters. In order for these goals to be met, an optimization software program was developed. The program combines the analysis of structures by the grid model, reinforced concrete sizing, and the simulated annealing optimization heuristic. Sizing is compliant with the NBR 6118 (2007) Brazilian standard, according to which flexural, shearing, torsion, and web reinforcements and serviceability limit states (deflection and crack width limitation) are checked. Besides the dimensions of the situations mentioned above, the influence the cost of each material (steel, concrete and formwork) has on the overall cost of structures was also determined.

Keywords: optimization; beams; reinforced concrete; grid model; simulated annealing

1. Introduction

Usually, pre-sizing is one of the first stages in the development of a structural design. After finding a solution that meets stability and functionality requirements, the designer often ceases his/her quest for alternatively more economical solutions, and sticks to the best result obtained so far. However, the structural sizing process is iterative and relies ultimately on the intuition and experience of the designer, who has to choose from the several available options and be very sensitive to the proposed initial solution, for achieving a satisfactory outcome. Despite the advent of commercial software programs, which automate several design stages, the initial sizing of structural elements requires an engineer's direct and intellectual work, and the solution found by him/her is very unlikely to be the best among the several options compliant with safety and usage requirements. Nevertheless, by using optimization strategies coupled to the structural design, one seeks to find the best solution by means of a systemic search, based on a well-defined mathematical model, with the definition of objective functions, parameters, and constraints. In structural optimization, the smallest weight and the lowest cost are the main goals to be attained.

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Cost minimization of reinforced concrete structures, in compliance with the ultimate and serviceability limit states and with other technical regulations, could mean remarkable cost savings, enabling construction companies and, especially, structural design offices, to stay ahead of their competitors. The lack of resources and the need to lower the consumption of raw materials and thus have a more sustainable development should also be taken into account. Structural optimization plays a key role in this process by allowing for more rational projects in an efficient and relatively quicker fashion. This is a field that has been neglected in practice, but it has large potential for application. To do that, it is necessary to fine-tune structural optimization with the practical characteristics of the project, seeking to describe the actual situations faced by designers and their complexity, as well as classic and trivial examples of optimization of structures, widely reported in the technical literature. As far as reinforced concrete is concerned, structural optimization will be attractive when the examples used are closely related to the conditions commonly found in usual constructions with typical floors and with realistic geometry and loading conditions, and when technical regulations are effectively followed.

There are numerous works in the literature that address optimal sizing of reinforced concrete structures, in which calculations are made using several classical optimization techniques. In most studies, the objective is to minimize costs of concrete section so as to fulfill functional constraints based on calculation standards and to meet the constraints involving strength criteria.

The interest in investigating the optimization of reinforced concrete structures began in the 1950s, with the remarkable work of Heyman (1951), in which linear programming was used in optimal plastic design. However, it was in the 1970s that several seminal works were carried out, resulting in the dissemination of optimization processes worldwide (e.g. Goble and Lapay 1971, Kirsch 1972, Friel 1974).

The aim of the present paper is to show the application of optimization strategies for the cost of beams in reinforced concrete buildings and to propose pre-sizing parameters. In order for these goals to be met, an optimization software program was developed. The program combines the analysis of structures by the grid model, reinforced concrete sizing, and the simulated annealing optimization heuristic. Sizing is compliant with the NBR 6118 (2007) Brazilian standard, according to which flexural, shearing, torsion and web reinforcements, and serviceability limit states (deflection and crack width limitation) are checked. Besides the dimensions of the situations mentioned above, the influence the cost of each material (steel, concrete and formwork) has on the overall cost of structures was also determined.

Although this is not the primary goal of this work, the relations and parameters obtained can be used to verify the provisions included in codes and standards, regarding the span-depth ratios (Bischoff and Scanlon 2009).

2. Simulated annealing method

In structural engineering, optimization techniques are usually applied in an attempt to find the ideal weight or cost for columns, beams, slabs, frames and trusses, models which are commonly used in most studies. The constraints that define the search for optimized structures are determined by the codes that regulate practice in structural design. Optimization has been constantly applied to a wide range of problems, allowing the use of the best sets of material, topology, geometry and/or dimensions of cross-sections in different types of structural systems (Suji *et al.* 2008).

The algorithms used to solve an optimization problem can be deterministic or probabilistic. Deterministic optimization methods, also known as classical methods, in which mathematical programming methods are included, are often based on the calculation of first-order derivatives or of second-order partial derivatives. Conversely, heuristic methods, based on probabilistic algorithms, add stochastic data and parameters to the optimization process, solving the problem from a probabilistic perspective.

Mathematical programming methods have some limitations such as their difficulty in finding global optimal solutions, as they rely on the starting point, difficulty in employing discrete variables, and difficulty in utilizing non-differentiable functions. A *sine qua non* condition for the application of classical methods is that the objective function must be continuous and differentiable in the search space. However, this does not occur in most of practical engineering problems, thus preventing their application.

Heuristic methods do not calculate derivatives, but they directly search for solutions in the feasible space. Nevertheless, these methods require a larger number of evaluations of the objective function value, and are therefore computationally more expensive than mathematical programming methods. Thus, they should not be used injudiciously, but only to solve problems for which mathematical programming techniques show limitations.

Heuristic methods include a large number of algorithms such as genetic algorithms, simulated annealing, ant colony algorithm, bee colony algorithm, harmony search, particle swarm optimization, among others. Genetic algorithms and simulated annealing are the most popular of these methods (Degertekin 2007, Suji *et al.* 2008).

Simulated annealing is a heuristic method based on statistical mechanics which dates back to the annealing process, and was introduced by Kirkpatrick *et al.* (1983). In the physical process of solid hardening, a material is quickly heated and slowly cooled so that its structural flaws can be eliminated. If cooling is sufficiently slow, the final configuration of the material will correspond to the minimum energy state. On the other hand, quick cooling will result in a metal with weak and brittle structure.

In brief, in simulated annealing, a single neighboring state s' of current solution s is randomly generated in each iteration. The difference (Δ_f) between the quality of the new solution *s'* and the quality of the current solution *s* (Eq. (5)) is calculated to assess the acceptance of this new solution *s'*

$$\Delta_f = f(s') - f(s) \tag{5}$$

In a minimization problem, if the value of Δ_f is less than zero, the new solution *s'* is automatically accepted and can substitute *s*. Otherwise, the acceptance of the new solution *s*' depends on the probability established by the Metropolis criterion

$$p = \exp\left(\frac{-\Delta_f}{T}\right) \tag{6}$$

As temperature drops throughout the process, there is a higher probability of acceptance of new solutions in the initial stages, even if this eventually worsens the current solution. This probability decreases throughout the process, reaching the point (when temperature is close to zero) at which only those movements that improve the cost function are accepted.

Several works, published in the past few years, successfully used simulated annealing for structural optimization. Hasançebi and Erbatur (2002) used this heuristics and optimized a 942-

member truss tower, an 18-member truss and a 47-member plane truss tower. In the latter two cases, the geometry of the models was optimized along with the cross-sections. Discrete variables were used. By comparing the results with those of other studies, the proposed simulated annealing algorithm outperformed genetic algorithms.

Park and Ryu (2004) proposed altering the parameters in order to improve the heuristics. They optimized the weight of two structures usually found in structural optimization problems: 10-member plane trusses and 25-member spatial truss. Both discrete and continuous variables were used. The authors concluded that the number of necessary iterations in the new simulated annealing algorithm was significantly smaller than that of the conventional algorithm.

Kripka (2004) optimized plane and spatial trusses to discrete variables, and compared the obtained results with those of different methods. In all cases, the optimal solution provided by simulated annealing was equal to or better than the others.

Dagertekin (2007) optimized the section of steel frames using simulated annealing and genetic algorithms. Three simulations were carried out, using frames with 8, 26 and 84 members. Simulated annealing had a slight advantage in all simulations.

Payá-Zaforteza (2007) optimized reinforced concrete frames used in the construction of buildings following five heuristic methods, including simulated annealing and genetic algorithms. Initially, the different methods were tested using a model made up of two bays and four floors. Of these methods, simulated annealing was more efficient in the search for an optimal solution, showing an intermediate processing time. Later, several models were used to fine-tune the method: two-bay frames with two, four, six and eight heights.

Suji *et al.* (2008) optimized fiber-reinforced concrete beams. The reinforcement of beams was used as rationally as possible, given the costly prices of this material.

González-Vidosa *et al.* (2008) optimized four reinforced concrete frames. The first model consisted of a soil barrier system. The second and third models consisted of frames used in road construction. And the fourth optimized model consisted of a 20-member plane frame commonly used in buildings.

Payá-Zaforteza *et al.* (2010) conducted multiobjective optimization using simulated annealing. In addition to assessing cost minimization, they evaluated three other objectives: maximization of model constructability, minimization of environmental impacts, and maximization of global structural safety. The model used consisted of a 20-member reinforced concrete plane frame of a 4-story building. Their results indicated that, with a small increase in optimal cost, it is possible to have structures with higher constructability, larger sustainability and better global structural safety. In another study, in 2010, the same authors used this method once again to optimize a 20-member reinforced concrete plane frame, but they considered only the costs this time. Their aim was to improve the parameters of this method.

Hasançebi *et al.* (2010) proposed the improvement of the simulated annealing algorithm. To test the alterations, the authors optimized plane and spatial steel frames with 304 and 132 members, respectively. The simulated annealing algorithm had the lowest weight for both structures, compared to the results of two other heuristic methods: harmony search and tabu search.

Finally, Sonmez (2011) optimized truss weight using discrete variables and different methods, including simulated annealing. The best result was obtained for the classical model with a 10-member plane truss with simulated annealing, bee colony algorithm, and ant colony algorithm. The simulated annealing algorithm had the best result for the 25-member spatial truss, compared to other heuristic methods. Particle swarm optimization and genetic algorithms were the other methods used in the analyses.

3. Formulation of the optimization problem for reinforced concrete frames

The aim of this paper is the cost minimization of reinforced concrete beams, taking into account the influence of formwork, of concrete, and of transverse and longitudinal reinforcements. The major optimization problem variable was the cross-sectional height of beams. The cost of each material was obtained by multiplying the respective amounts by the unit costs of each material. Steel was quantified in mass (kg), concrete was expressed in volume (m^3) and formwork as area (m^2). Steel yield stress f_y = 500 MPa was considered for longitudinal reinforcements, and f_y = 600 MPa for transverse ones. Therefore, the objective function is given by Eq. (7), where C_t corresponds to the overall cost of the analyzed structure, P_A, P_{Asw}, A_F, V_C refer to the amounts of material (500 MPa steel, 600 MPa steel, formwork and concrete, respectively), while C_A, C_{Asw}, C_F and C_C stand for the unit costs of each material. The latter ones were calculated based on the compositions and mean values recommended for the southern Brazilian region.

$$C_t = [(P_A + P_{Asw}) \cdot C_A] + (A_F \cdot C_F) + (V_C \cdot C_C) \tag{7}$$

NBR 6118 (2007), Brazilian standard for the design and execution of reinforced concrete structures, was used for the sizing and detailed description of structural elements. The optimization problem constraints are shown in Eqs. (8) through (19).

The first two constraints, Eqs. (8) and (9), refer to the serviceability limit states. The maximum deflection of each element, δ, taking into account long-term effects, should be smaller than the limit deflection $\delta_{\lim}$, and the characteristic crack width w_k should be smaller than the stipulated limits ($w_{k,\lim}$).

$$\delta \le \delta_{\lim} \tag{8}$$

$$w_k \le w_{k,\lim} \tag{9}$$

The following constraints refer to flexural reinforcements: the ratio between the fractions of the bending moment absorbed by the compression ($M_{AS'}$) and tension reinforcements (M_{AS}) should not exceed 30% (Eq. (10)), in order to prevent large concentration of reinforcements, hindering concrete placement; the minimum reinforcement ratio ($\rho_{\min}$) should be larger than the ratios defined in the NBR 6118 (2007) standard, whereas the maximum ratio should be equivalent to at most 4% of the cross-section area, Eq. (11).

$$\frac{M_{As'}}{M_{As}} \le 0{,}30 \tag{10}$$

$$\rho_{\min} \cdot A_C \le A_s + A_s' \le 4\% \cdot A_C \tag{11}$$

To check for shearing, the NBR 6118 (2007) standard determines that the strain concrete should withstand in compressed struts (V_{Rd2}) should be greater than the respective stress (V_{Sd}), as outlined in Eq. (12), and the strength of concrete and of reinforcements in the tensioned struts (V_{Rd3}) should be larger than the working stress (V_{Sd}), as shown in Eq. (13).

$$V_{Sd} \le V_{Rd2} \tag{12}$$

$$V_{Sd} \le V_{Rd3} \tag{13}$$

With respect to torsion, the first constraint (Eq. (14)) defines that the working shearing stress should not exceed 70% of the structural resistance, so that compatibility torsion can be excluded from the analysis, including only the equilibrium torsion. This equation is more restrictive than Eq. (12). Moreover, the working torsional moment must be smaller than the moment of resistance of the compressed concrete diagonals (Eq. (15)) and then the moments supported by stirrups (Eq. (16)) and by longitudinal bars (Eq. (17)).

$$V_{Sd} \leq 0{,}7 \cdot V_{Rd2} \tag{14}$$

$$T_{Sd} \leq T_{Rd2} \tag{15}$$

$$T_{Sd} \leq T_{Rd2} \tag{16}$$

$$T_{Sd} \leq T_{Rd4} \tag{17}$$

For combined shear and torsion, the NBR 6118 (2007) standard establishes that the requirement established in Eq. (18) be met, representing one more problem constraint.

$$\frac{V_{Sd}}{V_{Rd2}} + \frac{T_{Sd}}{T_{Rd2}} \leq 1 \tag{18}$$

The last constraint (Eq. (19)) refers to the minimum reinforcement ratio, taking into account shear and torsion.

$$\rho_{SW_{\min}} = \frac{A_{SW}}{b_w \cdot s} \geq 0{,}2 \cdot \frac{f_{ct,m}}{f_{ywk}} \tag{19}$$

The optimization problem was solved by adding an algorithm to the Fortran 90 compiler. This algorithm analyzes the beams of a floor by applying the grid model and using simulated annealing as an optimization tool. In the grid model, loads are applied perpendicularly to the plane formed by the floor beams. These beams can be simply or elastically supported by columns.

The software program was developed in two stages. The first stage is based on a previous study described in Kripka (2003), in which the use of this software program revealed that costs were reduced with the optimization of structures, similarly to what is proposed in this paper. The second stage, described herein, consisted of the update of the software program in terms of the reinforced concrete standard currently in effect in Brazil (NBR 6118 2007), and of the inclusion of a larger number of constraints which had not been contemplated by the previous study (shearing, torsion, web reinforcement and crack width limitation).

The process used by the software for minimizing the cost of the structure is described in the following steps:

(a) Determination of initial parameter values: initial temperature T, temperature reduction factor ΔT *and* stopping criterion;

(b) Determination of the initial values of design variables (height of the cross-section of each beam or set of beams). Sizing of elements and calculation of the objective function (cost) value for this solution;

(c) Structural analysis by the grid model for determination of internal forces and displacements on floor beams. Calculation of constraints and function penalization for violated constraints (artificial increase in cost due to a predefined factor);

(d) Random generation of a new solution. Repetition of steps (b) and (c) and comparison of the cost of the previous solution to that of the new one. If the new solution is better than the previous one (Δf less than zero in Eq. (5)), it will then become the current solution to the problem. Otherwise, the likelihood of this solution (Eq. (6)) being accepted is determined, replacing the current solution if probability p is greater than a randomly generated number between zero and one. As the new solutions are generated, temperature T is gradually reduced until the stopping criterion is met, corresponding to total cooling.

To determine the major parameters used in the problem, several numerical analyses were performed, yielding the following values: temperature $T_i = 10$, reduction factor $\Delta T = 0.90$. As stopping criterion, the final temperature T_f was set to less than or equal to $0.001T_i$.

4. Computational analysis

4.1 Optimization of simply-supported beams

The first numerical simulations consist of the analysis and sizing optimization of simply-supported beams. The heights that minimize the cost of beams with lengths between 1.5 m and 10 m, with intervals of 0.5 m, were assessed. The following concrete strengths were tested: 20, 25, 30 and 45 MPa. Beam width was set at 0.15 m, an intermediate value between the widths often used in Brazil, which range from 0.12 to 0.20 m. A height of 0.8 m was the initial solution of the beam optimization process. Two loadings (a minimum and a maximum one) were considered, in order to cover a range of load values to which most beams in residential buildings are subjected. The minimum load corresponds to a permanent load of 9.86 kN/m, and to an accidental load of 2 kN/m. The maximum load consists of 16 kN/m and of an overload of 7 kN/m. The self weight is calculated automatically by the software, based on the specific weight of the material (25 KN/m³). In this and in the following example, the deflection was constrained in terms of visual effects (1/250 of the span), following the NBR 6118 (2007) standard. Although the standard establishes a limit for displacements with the aim to avoid damage to nonstructural elements such as masonry, this limit is less restrictive, since it takes into account only the loading share relative to the weight of masonry. Thus, we opted to take into account only the displacement constraint relative to visual effects.

At first, only the costs of materials were used, without taking labor into account. Later, the effect of including labor costs in the results was assessed, and the analyses for the 20 MPa concrete strength were repeated. The unit costs of materials, whether or not including labor, are shown in Table 1.

Table 1 Unit costs of materials

Material	Unit	Cost *w/o* labor (R$)	Cost *w/* labor (R$)
f_y 500 MPa steel	kg	3.97	4.77
f_y 600 MPa steel	kg	3.89	4.69
Formwork	m²	8.68	15.39
f_{ck} 20 MPa concrete	m³	213.07	243.07
f_{ck} 25 MPa concrete	m³	233.55	263.55
f_{ck} 30 MPa concrete	m³	252.70	282.70
f_{ck} 45 MPa concrete	m³	303.71	333.71

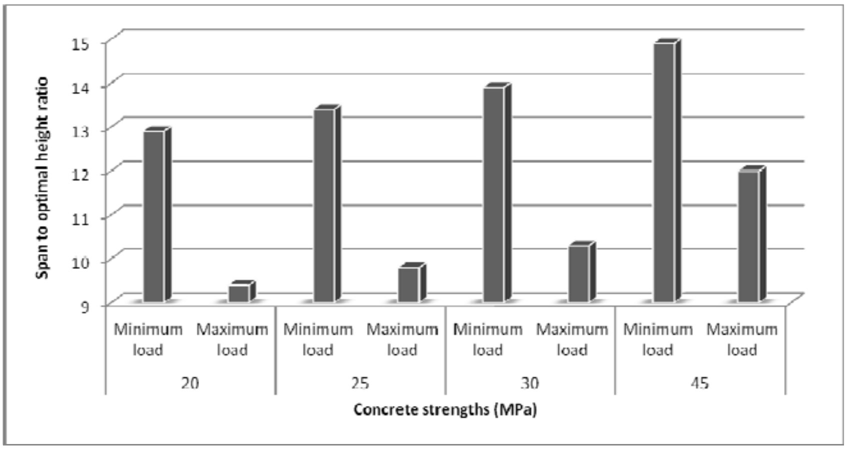

Fig. 1 Optimal span to height ratios for different concrete strengths

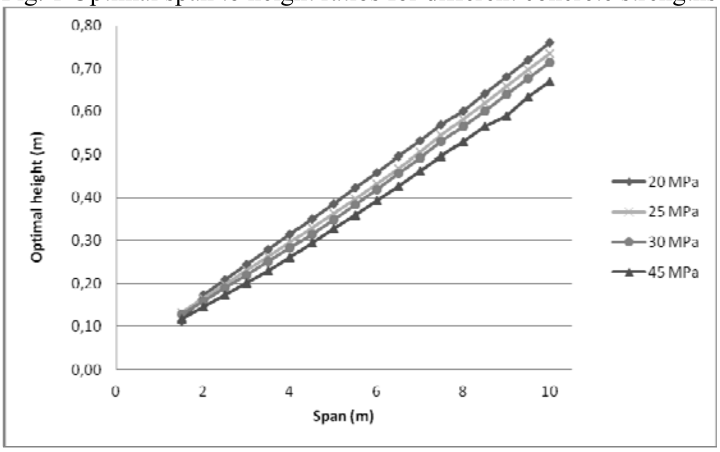

Fig. 2 Optimal height versus increase in span (minimum load)

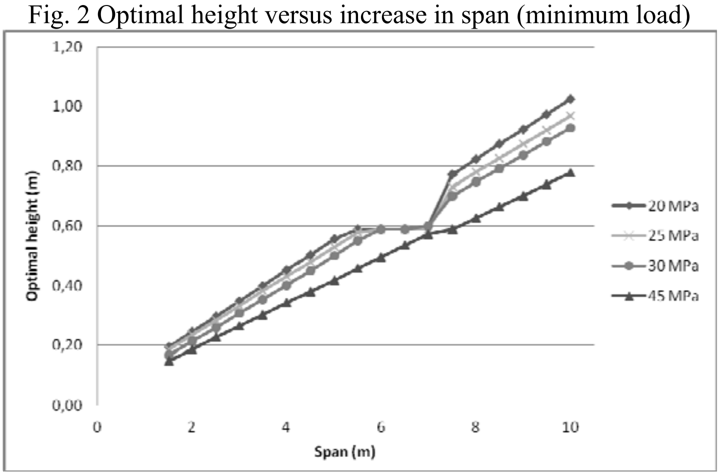

Fig. 3 Optimal height versus increase in span (maximum load)

The results revealed that the optimal span to height averages 13.5 for the minimum load and is close to 10.4 for the maximum load, without including labor costs. Fig. 1 shows the optimal span to height ratio for all concrete strengths, both for minimum and maximum loads. As concrete strength increases, the span to height ratio also increases, due to the reduction in optimal height.

The variation in optimal height tends to increase linearly for all analyzed loads as the span also increases. However, linearity is broken at a given point of the function for maximum load, and the optimal height value stabilizes at around 60 cm for spans between 5 and 7 m. This eventually occurred due to the attempt to refrain from the use of web reinforcement, necessary for heights greater than 0.6 m. This difference in cost function values is more easily noticed in lower strengths as the cost of web reinforcement, which is fixed for a given cross section, has a stronger influence on the overall cost in such cases, owing to the smaller price of the cubic meter of concrete. Fig. 2 illustrates the ratio between optimal height and the increase in span, taking into consideration the four classes of concrete strength for the minimum load. Fig. 3 shows a similar graph for the maximum load. Both graphs show that, in most cases, the lower the strength, the higher the optimal height, as expected.

Also, an important aspect is that deflection is the main optimization problem constraint in simulations, given that it is active, especially for beams submitted to minimum loads. In addition,

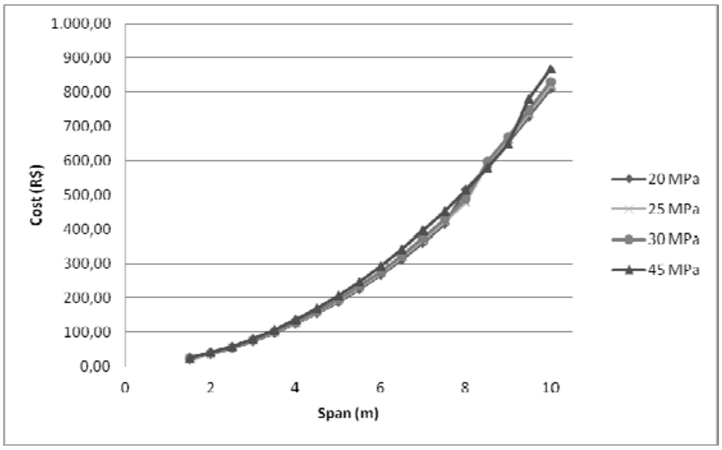

Fig. 4 Optimal cost variation based on f_{ck} (minimum load)

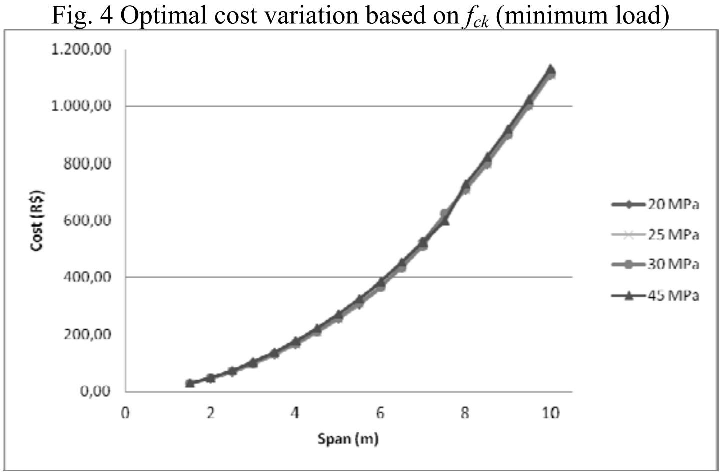

Fig. 5 Optimal cost variation based on f_{ck} (maximum load)

the constraint had a larger influence on the concrete strength of these beams. A higher f_{ck} allows obtaining slender beams, which are then closer to serviceability limit states, chiefly with respect to deflection.

By analyzing the variation in the cost of beams in relation to the increase in span, it is possible to observe that, as optimal height exceeds 60 cm, the slope of the curve steepens, because the cost of the web reinforcement plays a role from this point onward. Fig. 4 shows the span to optimal cost ratio for the four classes of concrete strengths, as far as minimum load is concerned. Fig. 5 shows a similar graph for the maximum load.

Looking at the results shown on both graphs, it should be noted that the overall optimal cost is quite close among the different values of f_{ck}. In general, higher-strength concrete requires structures that are a bit costlier, i.e., the higher strength obtained with a larger f_{ck} is not sufficiently large to make up for the higher unit cost of the material. However, there is some variation in span between 7 and 9 m and, therefore, at some point, the 20 MPa concrete structure turns out to be the least cost-effective. This occurred because, with lower strengths, there is an increase in web reinforcement cost as the span decreases.

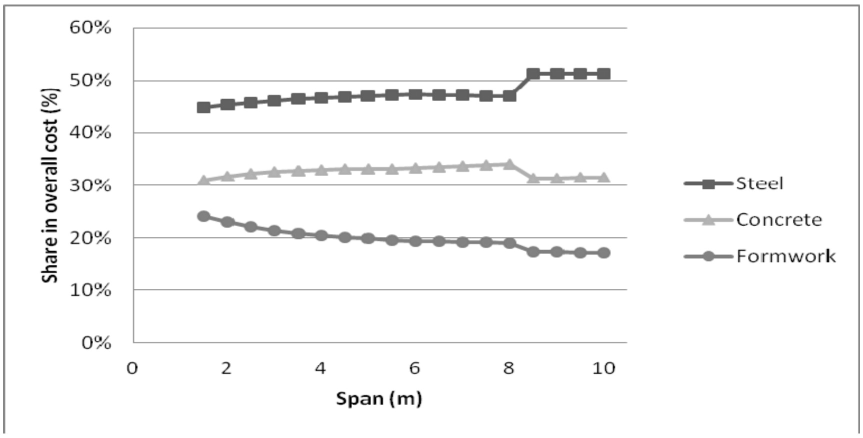

Fig. 6 Influence of each material on cost (f_{ck} 25 MPa and minimum load)

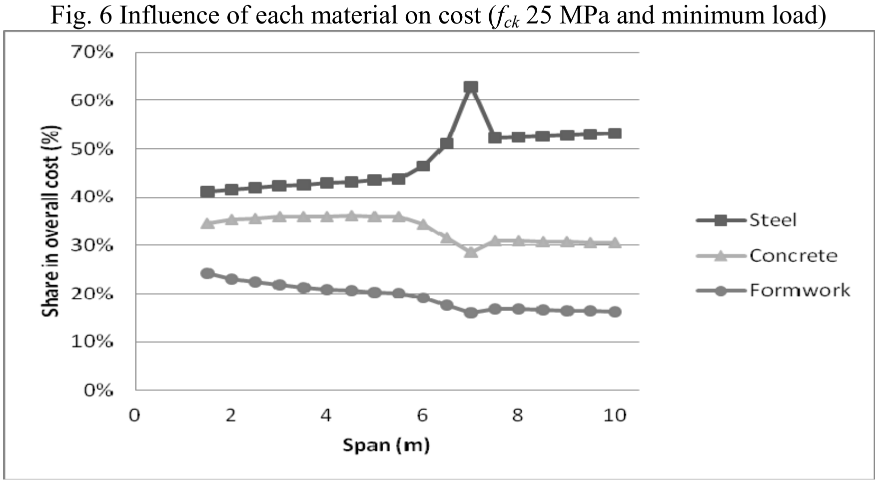

Fig. 7 Influence of each material on cost (f_{ck} 25 MPa and maximum load)

Table 2 Reinforcement ratios and consumption of steel for different f_{ck} values

f_{ck} (MPa)	Load	Kg of steel/m³ of concrete	Reinforcement ratio (%)
20	Minimum	83	0.92
	Maximum	84	0.85
25	Minimum	86	0.96
	Maximum	86	0.89
30	Minimum	91	1.02
	Maximum	90	0.97
45	Minimum	101	1.13
	Maximum	116	1.29
Average	Minimum	90	1.01
	Maximum	94	1.00

The graphs also allow analyzing the effect of the cost of each material on the optimal cost of the simply-supported beam. Figs. 6 and 7 show the results for an f_{ck} of 25 MPa (minimum and maximum loads, respectively).

In general, steel accounts for the highest fraction of the overall cost of beams, followed by concrete and formwork. In terms of minimum load, the effect of percentage values of each material on the overall cost is somewhat constant. Only with larger spans, due to the presence of web reinforcement, steel has a higher impact on the overall cost. In general, steel often accounts for 50% of overall costs, followed by concrete (around 30%) and formwork (20%). On average, these values vary slightly, regardless of the load and of the type of concrete used.

In the case of maximum load, there were instances of higher influence of steel on the overall cost. This was more remarkable when concrete strength was smaller. Not to use web reinforcement, it is necessary to maintain the height within the range of 60 cm. So, as the span increases, the higher the reinforcement ratio to offset the larger influence of deflection and the larger the working moment. When the deflection constraint does not allow for beams smaller than 0.6 m, the percentage of steel decreases in relation to the obtained peak, stabilizing again, without reaching the same level obtained for smaller spans, as web reinforcement influences costs from then on.

The average reinforcement ratio corresponded to 1% for minimum and maximum loads. Increasing ratios were found as concrete strength increased. A larger f_{ck} tends to result in lower structure height, increasing the relative influence of deflection and the need for reinforcement. The average consumption of steel per cubic meter of concrete was 90 kg for the minimum load and 94 kg for the maximum load. The data on reinforcement ratio and consumption of steel per volume of concrete are shown in Table 2 for all classes of concrete strengths.

By including labor costs in the case of 20 MPa strength, optimal heights decreased, and this was more easily perceived under maximum load. Nevertheless, the values of these heights were quite close to those obtained without the inclusion of labor costs. The percentage variation in cost averaged 30% for both loads.

In general, steel accounts for the most significant cost, but the cost of formwork was similar to that of concrete, with a slightly higher influence on smaller spans. With the impact of higher formwork costs on the overall cost, it should be recalled that, in a building with typical floors, formwork will be reused, consequently reducing its impact on the cost of the optimization problem. Again, the increase in reinforcement ratio at smaller spans is closely related to the active deflection constraint.

4.2 Optimization of typical floor with 33 beams

The study also optimized the beams of a quite traditional typical floor, shown in Fig. 8. Concrete strength of 25 MPa was used, and the deflection was also limited to *L*/250.

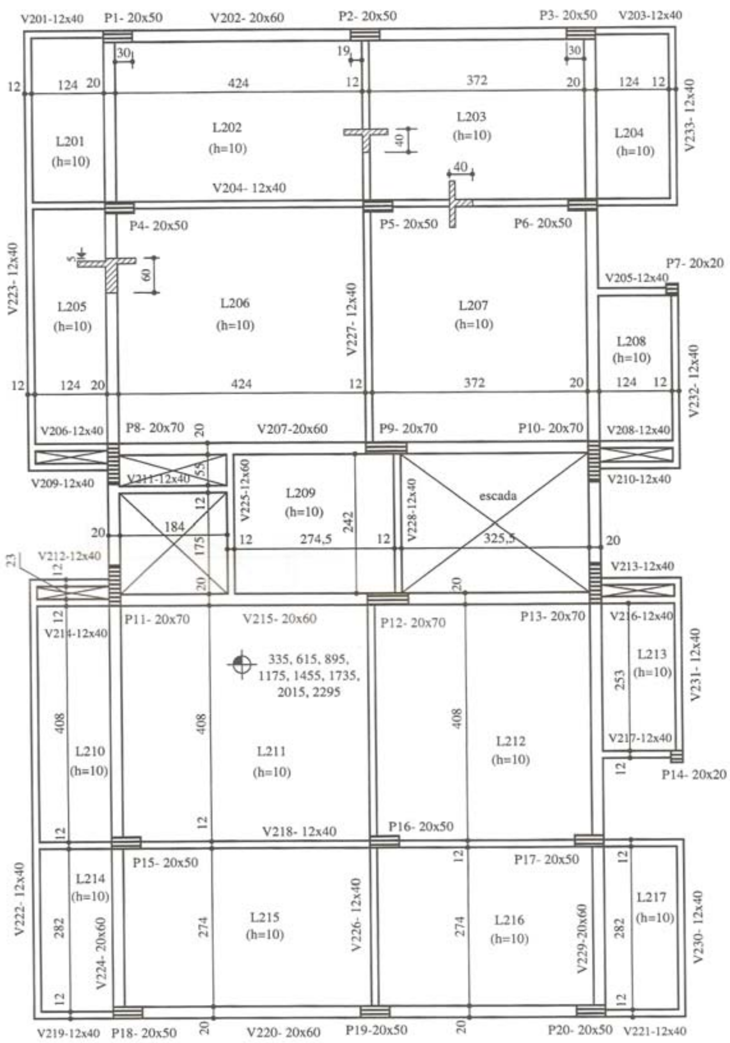

Fig. 8 Formwork for typical floors with 33 beams (dimensions in cm)

At first, the same three original groups of beams of the example were used. Later, the beams were categorized into 10 groups. And finally, the heights of all beams were regarded as distinct variables in the optimization process. Initially, the study was conducted only with the price of inputs. After that, the effect of including labor cost in the simulations was assessed.

The results of the analysis without inclusion of labor costs revealed excellent percentage reductions in the overall costs of structures, as shown in Table 3. This table also shows the monetary values of the overall cost, average percentage variation of optimal heights relative to the originally proposed structure, amount of reinforcement per volume of concrete and flexural reinforcement ratio for each of the three simulations used, as well as for the initial solution with the dimensions given in Fig. 8.

The results obtained for the same three groups of sections of the original structure demonstrate remarkable savings of 23.44% in the cost of the typical floor, which was equal to 27.69% and 32.54% for the other analyzed situations, which represent larger difficulties in terms of construction.

As to optimal heights, in the first simulation, the height of beams measuring 15 x 40 cm varied only 1 cm, indicating that they were highly optimized in the original structure. For beams measuring 20 x 60 cm, and for lift beams (12 x 60 cm), the height could be considerably reduced, as suggested by the software (36 cm and 21 cm, respectively).

While in simulation 1 there was slight variation in the height of beams with an original height of 40 cm, in the other two simulations, the reduction was much larger. Therefore, the average percentage reduction in optimal heights, compared to the original structure, increased as the number of section types (35%, 41% and 43%) also increased, as shown by the third item of Table 3.

Deflection led to active constraint on the optimization process in all simulations. Hence, the larger the number of existing section groups and the more optimized the structure, the higher the number of regions with this active constraint (6 regions for simulation 1, 13 regions for simulation 2 and 20 regions for simulation 3).

Fig. 9 displays the percentage of each type of longitudinal reinforcement for the overall material weight.

Flexural reinforcement accounts for virtually all of the longitudinal reinforcement for the three results obtained, without large variation in the percentage values of longitudinal reinforcement types as the number of section types increased. For the initial solution, web reinforcement corresponds to approximately 30% of the steel weight. As no beam had optimal height less

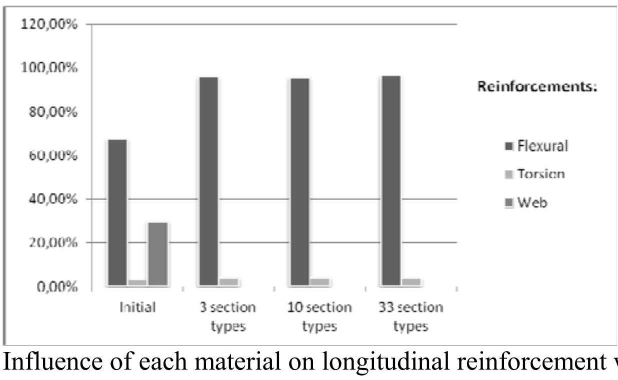

Fig. 9 Influence of each material on longitudinal reinforcement weight

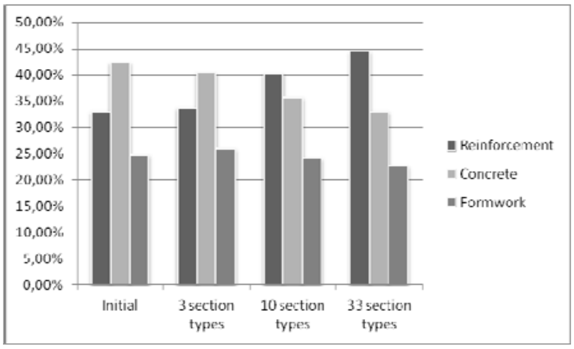

Fig. 10 Influence of materials on the overall cost of the structure (not including labor)

Table 3: Results for typical floors with 33 beams (not including labor)

Assessed item	Initial	3 section types	10 section types	33 section types
Overall cost (R$)	6,266.62	4,798	4,531.46	4,227.69
Variation in optimal overall cost relative to the original structure (%)	-	-23.44	-27.69	-32.54
Average variation in optimal heights relative to the original structure (%)	-	-35	-41	-43
Amount of reinforcement per volume of concrete (kg/m³)	42.03	44.95	60.73	72.43
Flexural reinforcement ratio (%)	0.29	0.44	0.60	0.76

than 60 cm, this type of reinforcement eventually did not influence the total amount of steel.

Fig. 10 shows the influence of costs of each material (steel, concrete and formwork) on the overall structure cost. For the initial result and for simulation 1, concrete was the most significant material in percentage terms as far as overall cost is concerned. For the other two simulations, steel was the most significant material in terms of cost, due to the reduction in cross-sections, and consequent reduction in the amount of concrete and in formwork area, as expected and according to previously optimized simply-supported beams. The reinforcement ratio, as well as the amount of steel per volume of concrete, increased as a larger number of section types was included, as shown by the fifth item of Table 3, being around 1%, considering the ideal theoretical situation according to the first stage of the study (simply-supported beams).

When simulations were repeated, without including labor costs, the optimal heights were virtually the same as those of the study in which only the cost of materials was included. For simulation 1, the result was identical, whereas for simulations 2 and 3, the variation did not exceed 1.9% and 1.1%. For the same reasons, the amounts of material varied slightly in labor cost simulations, and so did the percentage variation in the optimal cost in relation to the original structure. This rate was equivalent to 22.95%, 27.68% and 32.88% for simulations 1, 2 and 3, respectively.

5. Conclusions

The simulated annealing heuristic was quite efficient in minimizing structure costs. The software has been an important tool for the pre-sizing of reinforced concrete building floor grids, and of individual beams, as shown by the tests. The analysis of the typical floor with 33 beams allowed confirming the efficiency of the software program in minimizing the cost of more complex structures, with remarkable savings in comparison with the initial solution. In the study that maintained the same number of cross-section types, which does not represent difficulties in terms of construction, savings amounted to 23%.

Regarding pre-sizing parameters, this study allows us to say that the span to height ratio for low-priced simply-supported beams ranges from 9.5 to 13.5 for strengths between 20 and 45 MPa.

As the span of simply-supported beams increases, the variation in optimal height tends to be linear. However, for beams with spans between 5 and 7 m, optimal heights are kept at slightly less than 60 cm, in a deliberate attempt to refrain from the use of web reinforcement. This is an important piece of information for designers as the obtained results demonstrate that it is always interesting to avoid the use of web reinforcement in order to have cheaper structures, and that it is better to maintain beam height at around 60 cm whenever necessary. This is even more remarkable in the case of concrete with lower strength and higher loads.

Deflection is the major constraint on the optimization of reinforced concrete beams. The analysis of simply-supported beams showed that this occurs mainly under minimum loads and in structures with larger f_{ck}, for which sizing results in more slender structures.

In general, high-strength concrete provides more slender, but more expensive, structures, i.e., the larger strength obtained with a larger f_{ck} is not sufficiently large to make up for the higher unit cost of the material. However, for free spans between 7 and 9 m, simply-supported beams with larger f_{ck} are more economical, as demonstrated by the results of the study.

Steel accounts for the biggest part of the overall cost of beams, followed by concrete and formwork. In the analysis of simply-supported beams, the percentage distribution of cost followed this trend: steel (50%), concrete (30%) and formwork (20%).

The flexural reinforcement ratio is, on average, close to 1% for simply-supported beams optimized individually, and the average consumption of steel ranges from 90 to 94 kg/m³ under minimum and maximum loads.

Crack width limitation did not have a direct effect on the results of simulations.

Finally, it should be noted that the costs associated with both materials and labor vary constantly over time and from one region to another. However, in the present study, the inclusion of labor costs changed the final results of the optimization of beams only slightly, regarding to average consumption.

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Nomenclatures

Lower-case letters:

b_w = beam width
f_{ck} = concrete strength
$f_{ct,m}$ = steel mean characteristic tensile strength
f_y = steel yield stress
f_{ywk} = steel characteristic yield strength
p = probability
s' = new solution
s = current solution, spacing between bars
w_k = characteristic crack width
$w_{k,\ lim}$ = limit crack width

Upper-case letters:

A_c = cross-sectional area of concrete
A_F = total area of formworks
A_s = cross-sectional area of the compression reinforcements
A_s' = cross-sectional area of the tension reinforcements
A_{sw} = cross-sectional area of the transversal reinforcements
C_A = unit cost of 500 MPa steel (R$/kg)
C_{Asw} = unit cost of 600 MPa steel
C_C = unit cost of concrete
C_F = unit cost of formworks
C_t = overall cost
$M_{AS'}$ = bending moment absorbed by the compression reinforcements
M_{AS} = bending moment absorbed by the tension reinforcements
P_A = total weight of longitudinal steel bars
P_{Asw} = total weight of transverse steel
T = temperature
T_f = final temperature
T_i = initial temperature
T_{Rd2} = moment of resistance of the compressed concrete diagonals
T_{Rd3} = moments supported by stirrups
T_{Rd4} = moments supported by longitudinal bars
T_{Sd} = working torsional moment
V_C = total volume of concrete
V_{Rd2} = strength of concrete in compressed struts
V_{Rd3} = strength of concrete and of reinforcements in the tensioned struts
V_{Sd} = working shear stress

Greek letters:

δ = maximum deflection

δ_{lim} = limit deflection
ρ_{min} = minimum reinforcement ratio (longitudinal)
ρsw_{min} = minimum reinforcement ratio (shear and torsion)
Δ_f = difference between the quality of the new solution and the current solution
ΔT = temperature reduction factor