DOI: http://dx.doi.org/10.12989/cac.2013.11.1.075

Cable layout design of two way prestressed concrete slabs using FEM

Ahmad Ali Khan*[1], K.K.Pathak[2] and N.Dindorkar[1]

[1]Dept. Civil Engg. MANIT, Bhopal (M.P.), India 462051
[2]CS&PM Group, AMPRI (CSIR), Bhopal (M.P.), India 462064

(Received October 12, 2010, Revised March 23, 2012, Accepted April 25, 2012)

Abstract. In this paper, a new approach for cable layout design of pre-stressed concrete slabs is presented. To account the cable profile accurately, it is modelled by B-spline. Using the convex hull property of the B-spline, an efficient algorithm has been developed to obtain the cable layout for pre-stressed concrete slabs. For finite element computations, tendon and concrete are modelled by 3 noded bar and 20 noded brick elements respectively. The cable concrete interactions are precisely accounted using vector calculus formulae. Using the proposed technique a two way prestressed concrete slab has been successfully designed considering several design criteria.

Keywords: pre-stressed concrete; cable layout design; finite element analysis; stress; B-spline.

1. Introduction

The layout of the prestressing cable plays very important role in the stress distribution in prestressed concrete structures. Cable layout design of prestressed concrete structures, as reported in text books, has been worked out on the basis of limiting eccentricities. In these texts, cables are modelled as parabola and their eccentricities are varied to reduce tensile stresses of the concrete. This approach has two major drawbacks -

(1) Cables are not truly parabolic, especially in continuous structures.

(2) It is computationally very expensive since separate parabola has to be defined for each span.

For realistic analysis of prestressed concrete structures, advanced computing techniques such as finite element method are employed. Linear finite element analysis (FEA) of pre-stressed concrete structures has been reported by Pandey *et al.* (1997), Buragohian *et al.* (1993, 1997), Pathak *et al.* (2004). The effect of several factors such as tendon profile (straight or deviated), strength of the concrete, tendon depth, number of deviators on ultimate stress in prestressed concrete beam was studied by Ghallab and Beeby *et al.* (2005). Finite element modeling of continuous reinforced concrete beam with external pre-stressed was carried by Ibrahim and Mubarak *et al.* (2009).

Non linear analysis of the same are reported by Povoas *et al.* (1989), Kang *et al.* (1990), Roca

*****Corresponding author, Ph. D., E-mail: ahmadali_jnp@rediffmail.com

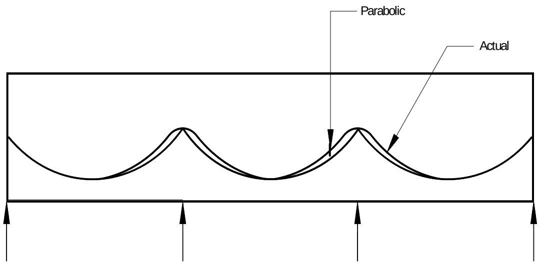


Fig. 1 Actual cable and parabolic profile

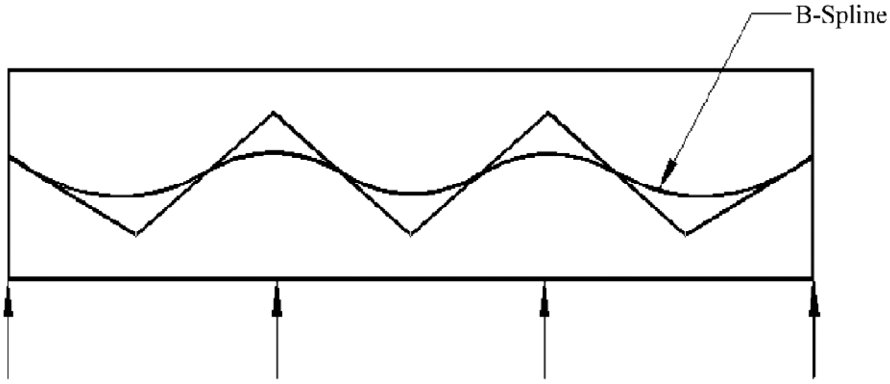


Fig. 2 B-spline profile

et al. (1993), Greunen *et al.* (1983), Figueiras *et al.* (1994), Vanzyl *et al.* (1979), Elwi *et al.* (1987), Diep *et al.* (2002) and Wu *et al.* (2003). Jirousek *et al.* (1979) and Buragohain *et al.* (1993,1997) have considered cable as parabolic and cubic curve in shell and semiloof shell elements, whereas Pandey *et al.* (1997) considered the cable as parabola in 20 node brick element. Pathak *et al.* (2004) considered the cable as cubic spline curve in nine node Lagrangean element. Saleem *et al.* (2008) modelled cable as B-spline for two dimensional finite element analysis. Madhavi *et al.* (2009) presented an efficient method for simulation of the complex behaviour of the prestressed concrete box girders using composite layered approach using non linear finite element analysis.

In advanced attempts of cable layout design, Brandt *et al.* (1989) and Kirsch *et al.* (1973, 1993) have carried out cable layout optimization using mathematical programming methods. Utrilla and Smartin *et al.* (1997), Quiroga *et al.* (1991) obtained optimum cable layout in bridge decks using linear and non-linear programming respectively. Lounis and Arroya *et al.* (1993) carried out multi objective optimization of prestressed concrete beam and bridge girder using Lagrangean algorithm. Kuyucular *et al.* (1991) obtained optimum cable profile of prestressed concrete slabs using elastic theory and finite element method. A generalised approach to the single objective reliability-based optimum design of prestressed concrete beams was presented by Barakat *et al.* (2003). The results consist of initial and final prestressing forces, prestressing losses, deflections and upper and lower bounds on the parabolic tendon profile. The cable profile was assumed to be the combination of parabola, third degree curve, forth degree curve and combination of parabola and third degree curve having common tangent at the junctions. These studies suffer from large degree of complexity in data preparation as well as in computations. To overcome these limitations; in this study, a holistic approach has been proposed for analysis and design of three dimensional prestressed concrete structures. Cable profile is modelled by B-spline which is very suitable for these applications. Cable layout designs of several slabs are successfully carried out using the finite element (FE) based approach in which B-spline ordinates are considered as design variables. The concrete and cable are modelled by twenty noded brick and three noded curved bar elements respectively. In this way, a safe and more slender slab can be obtained. Using the proposed approach, cable layout design of a two way prestressed concrete slab has been successfully carried out.

2. Cable modeling

The cable profile plays very important role during the analysis as pressure on concrete depends on it. To analyse prestressed concrete structures using analytical approach given in the text books (Raju 2000, Lin and Burns 1982), cable profile is modeled by parabola. This brings about constant curvature and simplifies the solution. But profile, thus modeled, becomes discontinuous at supports (Fig. 1). If cable profile, to maintain continuity, is modeled by higher order polynomial, it results in a very zigzag shape. In order to overcome these difficulties, in this study, cable profile is modeled by B-spline. A B-spline is a typical curve of the CAD philosophy (Qing and Liu 1989, Rogers and Adams 1990). It models a smooth curve between the given ordinates (Fig. 2). The theory of the B-spline was first suggested by Schoenberg (1946). A recursive definition useful for numerical computation was independently discovered by Cox and by de Boor (Rogers and Adams 1990). Gordon and Riesenfeld (1974) applied the B-spline basis to curve definition. The brief definition of B-spline curve is given below and detailed account of this can be found in Rogers and Adams (1990).

$$P(t) = \sum P_i \cdot N_{i,k}(t) \qquad 0 < t < n - k + 2,\ 2 \le k < n \tag{1}$$

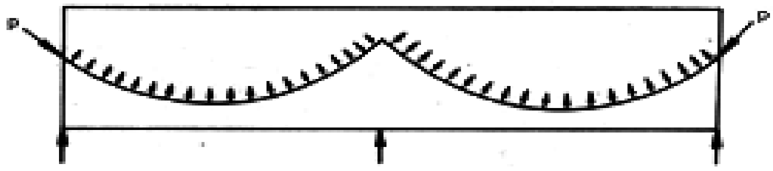
where

$$N_{i,l} = \begin{cases} 1 & \textit{if} \quad x_i \le t \le x_i + 1 \\ 0 & \text{otherwise} \end{cases}$$

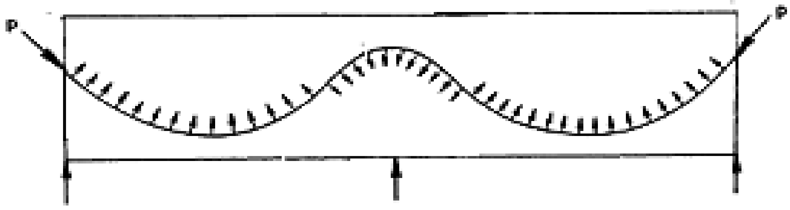
and

$$N_{i,k}(t) = \frac{(t - x_i) N_{i,k-1}(t)}{(x_{i+k-1} - x_i)} + \frac{(x_{i+k} - t) N_{i+1,k-1}(t)}{(x_{i+k} - x_{i+1})} \tag{2}$$

In above equations, $P_{i's}$ are the n+1 defining polygon vertices, k is the order of the B spline and $N_{i,k}(t)$ is called the weighing function. x is the additional knot vector which is used for B- spline curve to account for the inherent added flexibility. A knot vector is simply a series of real integers x_i such that $x_i \le x_i+1$ for all x_i. They are used to indicate the parameter t used to generate a B-spline. The curve generally follows the shape of the defining polygon and the curve is transformed by transforming the defining polygonal vertices. The order of the resulting curve can be changed without changing the number of defining polygon vertices. When a B-spline curve is used, the geometrical regularity is automatically taken into account.



(a) Parabolic profiles



(a) B-spline Profiles

Fig. 3 Equivalent cable force on cable profile

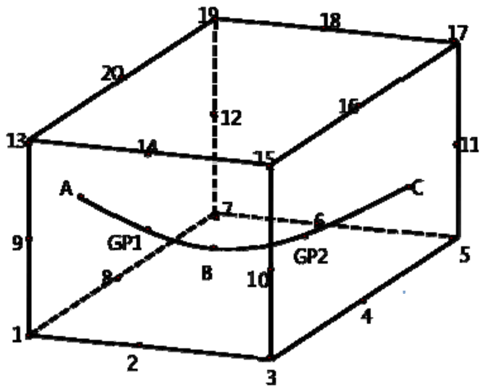


Fig. 4 Brick and curved bar element

3. Finite element modeling

In finite element analysis of pre-stressed concrete structures, modeling of pre-stressing cable is a tedious task. The cable counterbalances the effects of the live and dead loads due to its proper layout of curvature. Hence major task is to obtain curvature along the cable profile. Modeling of the cable as a discrete parabola in different spans does not hold true in continuous span structures as this creates discontinuity at the juncture of different spans. The load transfer on concrete due to parabolic and B-spline profiles are shown in Fig. 3. It can be seen that parabola model is erroneous as it results in opposite forces. Because of the convex hull properties of the B-spline, it is very suitable to represent cable profile in continuous span structures. Braibant and Fleury (1984), Pourazady *et al.* (1996) and Ghoddosian (1998) have used this curve in shape optimization problems and cable geometry is being modeled as B-spline in this study. For finite element analysis, cable is modeled by three noded bar element and concrete by 20 noded brick element (Zienkiewicz and Tayler 1991). In Fig. 4, a 3 node curved bar element is shown embedded in three dimensional 20 node concrete element. The shape functions of a 3 node curved bar elements along local ρ axis are given by

$$N_{c1} = \frac{(\rho-1)\rho}{2}$$
$$N_{c2} = (1-\rho)(1+\rho) \tag{3}$$
$$N_{c3} = \frac{(\rho+1)\rho}{2}$$

The global coordinates on the curved bar can be defined as

$$\overline{X} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum_{i=1}^{3} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} N_{ci} = \sum_{i=1}^{3} X_i N_{ci} \tag{4}$$

The tangent and normal vectors along ρ axis for the cable are given by

$$\overline{T} = \sum_{i=1}^{3} X_i \frac{dN_{ci}}{d\rho} \tag{5}$$

$$\overline{N} = \frac{1}{\overline{\overline{T^2}}} \left(\frac{d^2 X}{d\rho} - \frac{a}{\overline{\overline{T^2}}} \frac{dX}{d\rho} \right) \tag{6}$$

Where, a is the dot product.

$$a = \frac{dX}{d\rho} \cdot \frac{d^2 X}{d\rho^2}$$

The unit tangent and normal vectors are

$$\bar{t} = \frac{\overline{T}}{\left|\overline{T}\right|} \tag{7}$$

$$\bar{n} = \frac{\overline{N}}{\left|\overline{N}\right|} \tag{8}$$

The curvature at any point on the cable is expressed by

$$K = \frac{\left| \frac{dX}{d\rho} \times \frac{d^2 X}{d\rho^2} \right|}{\left\{ \left| \frac{dX}{d\rho} \right|^2 \right\}^{3/2}} \tag{9}$$

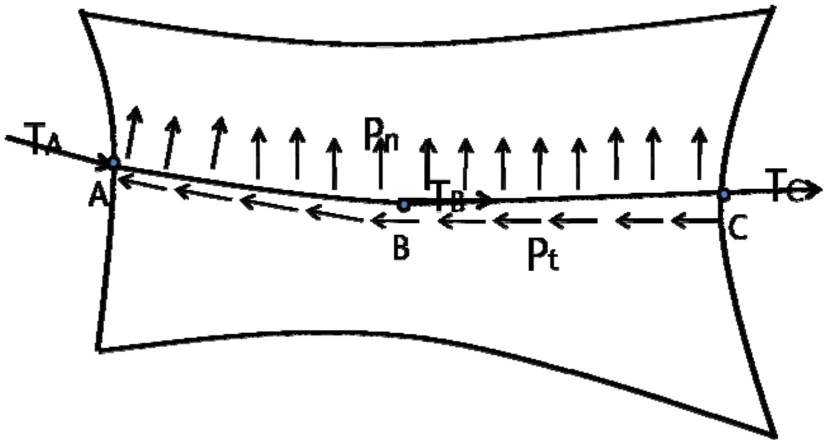


Fig. 5 Forces on concrete due to cable tension

Now radius of curvature can be calculated by

$$R = \frac{1}{K} \tag{10}$$

3.1 Normal and tangential forces

The cable exerts tangential and normal forces on concrete due to interactions between contacting surfaces and curvature of the cable as shown in Fig. 5. Tangential and normal forces can be given by

$$P_t = \frac{dT_n}{dX} = \frac{1}{\overline{\overline{T}}} \frac{dT_n}{d\rho} \tag{11}$$

$$P_n = \frac{T_n}{R} \tag{12}$$

Now the resultant force can be computed by

$$\overline{P} = P_t \bar{t} + P_n \bar{n} \tag{13}$$

Where T_n is the tension in the cable and T is the tangent vector. $\bar{t}$ and $\bar{n}$ are unit tangent and normal vector.

Using the principle of virtual work, these loads can be transferred to the nodes of brick elements. The equivalent nodal force vector for brick element is given by

$$\{P_L\} = \int_{-1}^{1} [N]^T \{\overline{P}\} \left|\overline{T}\right| d\rho \tag{14}$$

Cable reaction acts as concentrated loads on the concrete at the ends, where cable is anchorage. The anchorage end point forces can be calculated by-

$$\{P_A\} = [N]^T \{T_{end}\} \tag{15}$$

Where, Tend is the cable tension at the end points and [N] is the shape functions of 20 node brick elements.

Now total load vector due to interaction of concrete and cable is obtained by

$$\{P_T\} = \{P_L\} + \{P_A\} \tag{16}$$

This nodal load vector is applied on the three dimensional finite element model along with live and dead load vectors to include pre-stressing effects.

Local coordinates of known global coordinates are required for calculation of the anchorage end point forces as described below.

3.2 Local coordinates computation

Evaluation of local co-ordinate corresponding to known global coordinates is an inverse nonlinear problem which can be solved by Newton-Raphson method. The computation is carried out iteratively till the difference of two consecutive values becomes less than the prescribed tolerance. Let (x, y, z) is the global coordinate and (ξ, η, ζ) be the corresponding local co-ordinate. Numerical computations of local co-ordinates can be obtained using following iterative relationship

$$\begin{Bmatrix} \xi \\ \eta \\ \varsigma \end{Bmatrix}_{i+1} = \begin{Bmatrix} \xi \\ \eta \\ \varsigma \end{Bmatrix} + \begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \varsigma} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \varsigma} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \varsigma} \end{pmatrix}^{-1} \begin{Bmatrix} x_{i+1} - x_i \\ y_{i+1} - y_i \\ z_{i+1} - z_i \end{Bmatrix} \tag{17}$$

In above equation inverse matrix is the jacobian matrix. ($x_{i+1}, y_{i+1}, z_{i+1}$) and ($x_i, y_i, z_i$) are the known and computed values of the global co-ordinates. Starting values of ξ, η, ζ are considered as 0, 0, 0. The prescribed tolerance is 0.01.

4. Cable layout design

An algorithm has been developed to find the cable layout so that stresses in the structural element be below the limiting tensile stress. A check on the compressive stresses will be made in order to avoid crushing of the concrete. Based on stress results obtained from finite element analysis, cable profile is changed in iterative manner. In the following sections, these steps are described in detail.

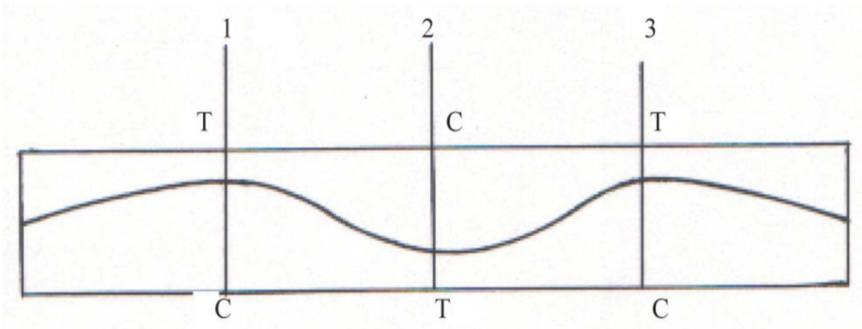


Fig. 6 Required cable profile

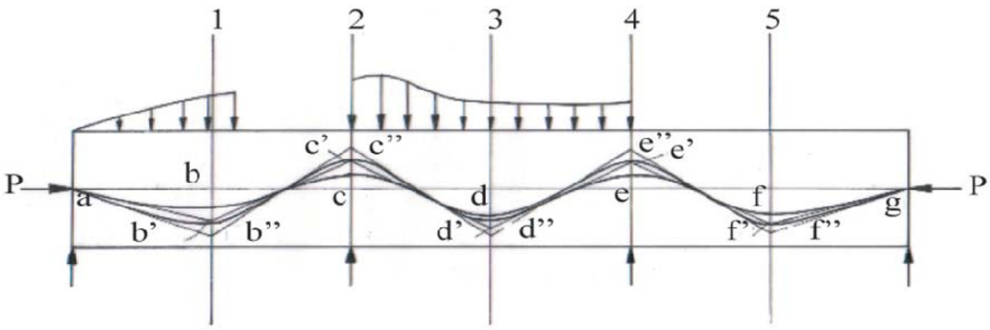


Fig. 7 Variation of cable profile

4.1 Criterion for cable layout design

The convex hull property of the B-spline, described above, ensures the curve to be convex or concave depending upon the ordinates of the polygon. The objective of the prestressed concrete design is to get rid of the tensile stresses produced due to different loading conditions. In Fig. 6, a prestressed concrete beam is shown. It is assumed that top fiber at sections 1 and 3 are in tension and bottom fiber at section 2 is in tension. The shape of the cable to eliminate the tension should be as shown in the same figure. The shape of the cable can be accurately represented by a B-spline (Fig 2). By varying the ordinates of the B-spline, cable shape is changed to get the desired profile. In this way any 3D prestressed concrete structures can be handled to obtain the optimum cable profile. It is also important to know that this would be very difficult if not impossible with any other curve.

4.2 Algorithm for layout design

In Fig. 7 a concrete structure with typical loading is selected for cable layout design. Assume the cable to be straight between anchorage ends and initial prestressing force be *P*. Now finite element analysis of the concrete structure is carried out for external loads and initial prestressing force *P* acting at the ends. Let sections 1, 2, 3, 4 and 5 be such that bottom fibre of 1, 3 and 5 are in tension and top fibre of 2 and 4 are in tension. Assume top and bottom fiber stresses at these sections be σ_{it} and σ_{ib}, where *i* varies from 1 to 5.

Since stresses at bottom fiber at 1, 3 and 5 are in tension, the cable should be concave there, whereas at 2 and 4 it should be convex. The ordinates of B-spline will move downward at 1, 3 and 5 and will move upward at 2 and 4. Let the initial y-coordinates of these points be $y1$, $y2$, $y3$, $y4$ and $y5$. Depending upon top and bottom fibre stresses at a point, following four cases may be detected-

(1) Top fibre is in tension and bottom fibre is in compression.
(2) Bottom fibre is in tension and top fibre is in compression.
(3) Both fibres are in tension.
(4) Both fibres are in compression.

4.2.1 Top fibre is in tension and bottom fibre is in compression

Let σ_{it} and σ_{ib} be top and bottom stresses, then a ratio R is defined as

$$R = \frac{\sigma_{it}}{\sigma_{it} + abs(\sigma_{ib})} \tag{18}$$

Where *abs* is the absolute value. New y-coordinates of the B-spline ordinates are calculated as

$$y_{i+1} = (1 + R)y_i \tag{19}$$

here y_i represents the y-coordinate of the previous iteration.

4.2.2 Bottom fibre is in tension and top fibre is in compression

In this case the stress ratio is defined as

$$R = \frac{\sigma_{ib}}{\sigma_{ib} + abs(\sigma_{it})} \tag{20}$$

Once again abs represents the absolute value. New y-coordinates of the B-spline ordinates are calculated as

$$y_{i+1} = (1 - R)y_i \tag{21}$$

A check on the compressive stresses at the top fibre will be made for it to not cross the limiting value.

4.2.3 Both fibres are in tension

If top and bottom both fibres are in tension, then either the cable force should be increased or the section should be redesigned.

4.2.4 Both fibres are in compression

Check the compressive stresses at the top and bottom fibres for them not to cross the limiting value. If they are below, cable profile is not altered at that section otherwise redesign the section.

These steps are repeated till the cable profile for limiting tensile stresses is obtained. Based on this procedure, ordinate movements of B-spline are shown in Fig. 7. The flowchart of the proposed algorithm of cable layout design is shown in Fig. 8. This algorithm is coded in FORTRAN software named as PRECLA3D which is used for layout design given below.

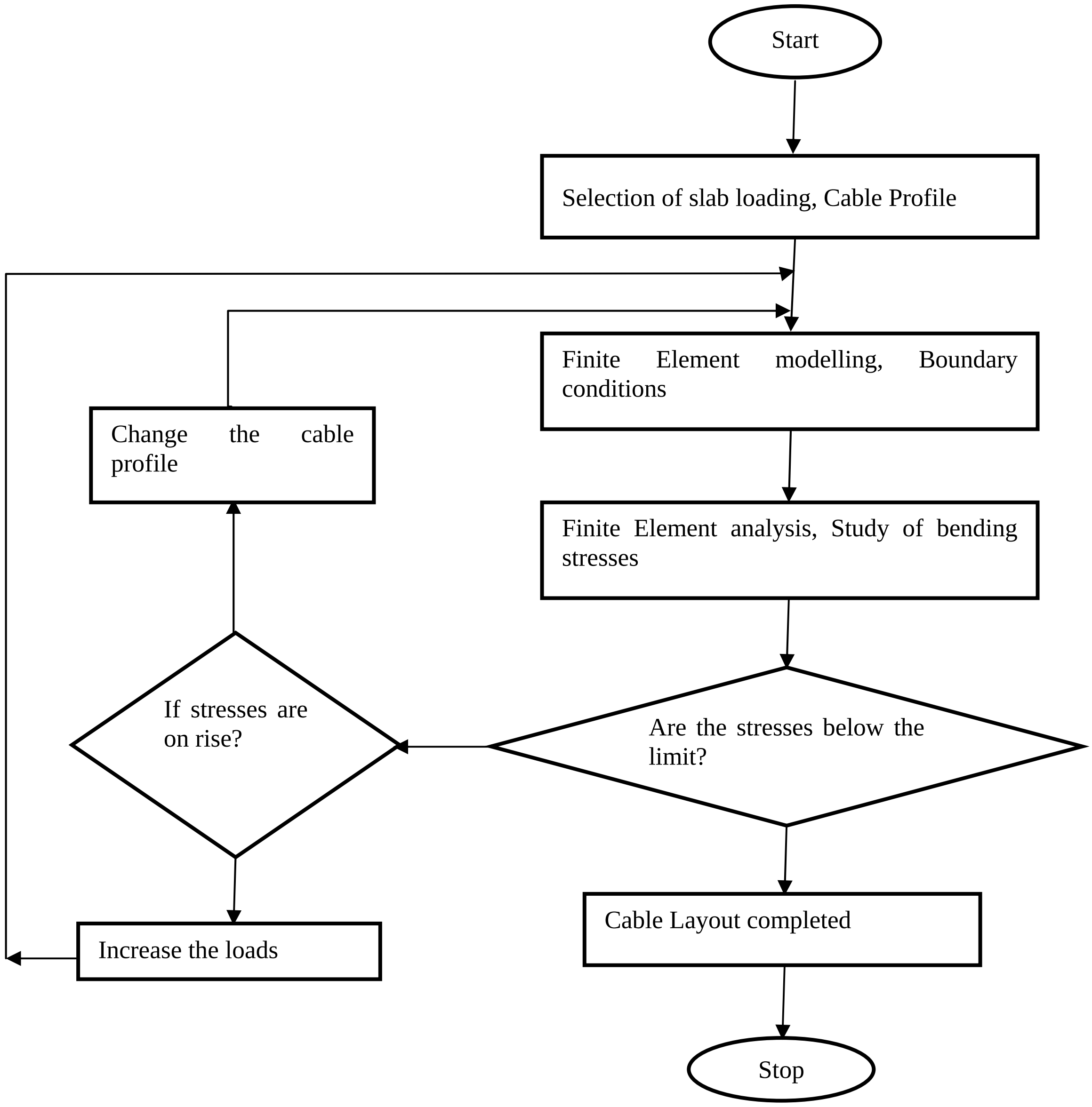


Fig. 8 Flowchart for cable layout design

5. Numerical examples

5.1 Example 1

A two way prestressed concrete slab of 4200 x 4200 x 300 mm is considered for cable layout design. Fourteen prestressing cable, 7 each in length and width directions are used. The slab is discretised into 49 twenty nodded brick elements and 428 nodes. The loading conditions and three

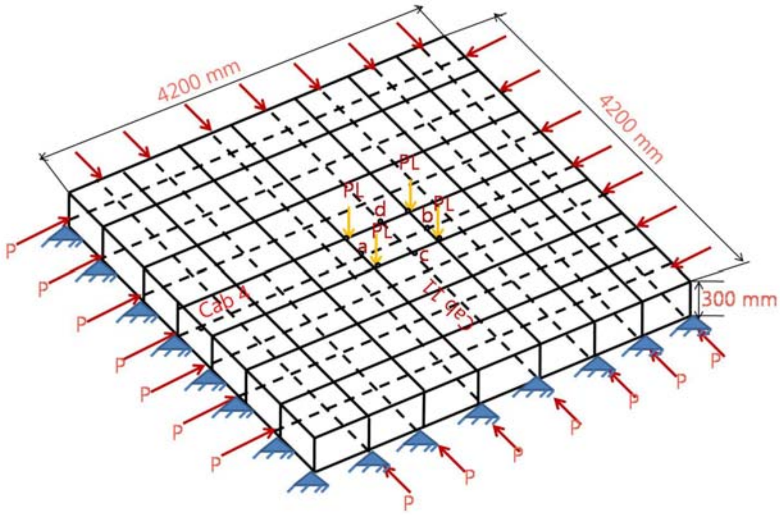


Fig. 9 Two way prestressed concrete slab

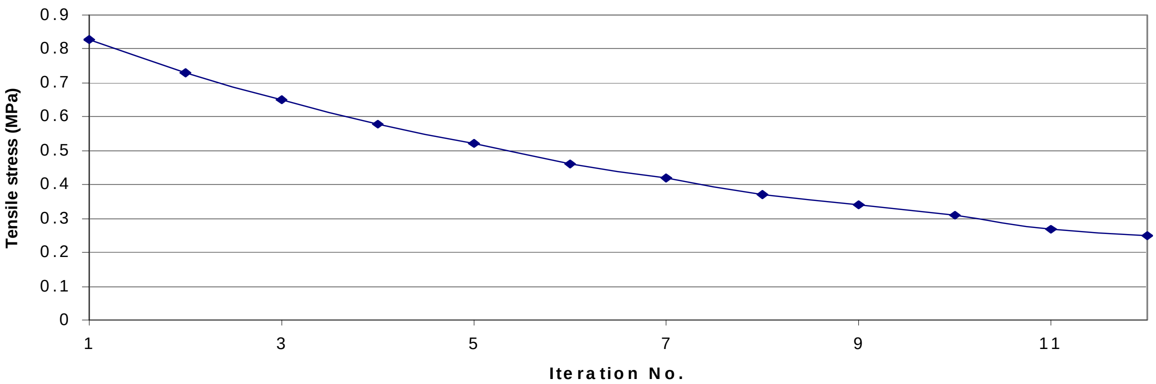


Fig. 10 Tensile stress vs Iterations

dimensional finite element model are shown in Fig. 10. Four concentrated loads, PL, of value 270 kN are applied on the slab as shown in figure. Gravity load is also taken into the account.

Following material properties are adopted:
(1) Young's modulus (concrete) = 2 x10^4 N/mm^2
(2) Compressive strength of concrete = 40 N/mm^2
(3) Poisson's ratio = 0.15
(4) Wobble coeffient = 1x10^{-5}
(5) Tensile strength of concrete = 0.25 N/mm^2
(6) Density of concrete = 2500 Kg/m^3

The cable layout design is taken up for two conditions:

Table 1 Bending stresses (MPa) for constant prestressing force

Iteration No.	Load (kN)	Maximum bending stresses (MPa)		Eccentricity (mm)
		Compressive	Tensile	
1	2000	-17.13	0.83	25
2	2000	-17.03	0.73	27.33
3	2000	-16.94	0.65	29.32
4	2000	-16.86	0.58	31
5	2000	-16.81	0.52	32.48
6	2000	-16.75	0.46	33.76
7	2000	-16.70	0.42	34.89
8	2000	-16.66	0.37	35.88
9	2000	-16.62	0.34	36.75
10	2000	-16.59	0.31	37.52
11	2000	-16.56	0.27	38.21
12	2000	-16.53	**0.25**	38.83

Table 2 Bending stresses (MPa) for variable prestressing load

Iteration No.	Load (kN)	Maximum bending stresses (MPa)		Eccentricity (mm)
		Compressive	Tensile	
1	2000	-17.13	0.83	25
2	2010	-17.16	0.77	25
3	2020	-17.18	0.72	25
4	2030	-17.21	0.67	25
5	2040	-17.23	0.61	25
6	2050	-17.26	0.56	25
7	2060	-17.28	0.51	25
8	2070	-17.31	0.45	25
9	2080	-17.33	0.40	25
10	2090	-17.36	0.35	25
11	2100	-17.38	0.30	25
12	2110	-17.41	0.26	25
13	2115	-17.42	**0.22**	25

5.1.1 Constant prestressing force

A constant prestressing force of 2000 kN is applied on all the fourteen cables. Initially eccentricity of all the cables is kept 25 mm. Maximum compressive and tensile stresses are noted during the design process. It was observed that maximum tensile stresses occur at four locations *a, b, c* and *d* which lie on cable 4 and 11. Hence layout design of these two cable were taken up. It

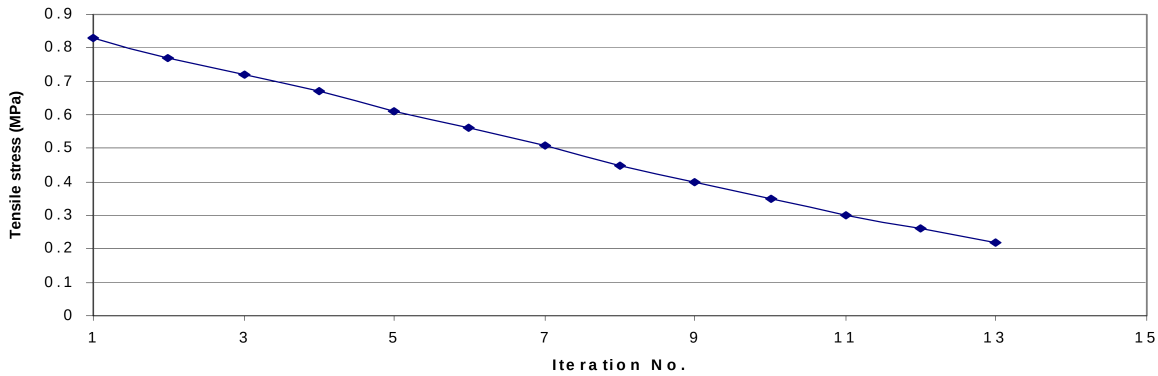


Fig. 11 Tensile stress vs Iterations

took 12 iterations to converge to maximum tensile stress of 0.25 MPa. Maximum compressive and tensile stresses along with eccentricities are given in Table 1. The variation of tensile stresses with respect to iteration is shown in Fig. 10 which shows non-linear trend.

5.1.2 Variable prestressing force

In second case of layout design, eccentricities of all the cables are kept constant 25 mm. based on the stress analysis, cable forces are varied. The initial value of the prestressing force is kept 2000 kN which is varied during the design process. It can be observed that at cable force of 2115 kN maximum tensile stresses are below the limit. The accounts of the compressive and tensile stresses, during the design process, are given in Table 2. It took 13 iterations to bring the tensile stresses below 0.25 MPa. The variation of tensile stresses with respect to iteration is shown in Fig. 11 which shows the linear trend.

5.2 Example 2

A two way prestressed concrete slab of 4900 x 4900 x 300 mm is considered for cable layout design. Fourteen prestressing cable, 7 each in length and width directions are used. The slab is discretised into 49 twenty nodded brick elements and 428 nodes. The loading conditions and three dimensional finite element model are shown in Fig. 10. Four concentrated loads, PL, of value 270 kN are applied on the slab as shown in figure. Gravity load is also taken into the account. Following material properties are adopted:

(1) Young's modulus (concrete) = $2x10^4$ N/mm^2
(2) Compressive strength of concrete = 40 N/mm^2
(3) Poisson's ratio = 0.15
(4) Wobble coeffient = $1x10^{-5}$
(5) Tensile strength of concrete = 0.25 N/mm^2
(6) Density of concrete = 2500 Kg/m^3

The cable layout design is taken up for two conditions:

Table 3 Bending stresses (MPa) for constant prestressing force

Iteration No.	Load (KN)	Maximum bending stresses (MPa)		Eccentricity (mm)
		Compressive	Tensile	
1	2100	-18.15	0.95	25
2	2100	-18.05	0.85	27.33
3	2100	-18.96	0.77	29.32
4	2100	-17.88	0.70	31
5	2100	-17.83	0.64	32.48
6	2100	-17.77	0.58	33.76
7	2100	-17.72	0.54	34.89
8	2100	-17.68	0.49	35.88
9	2100	-17.64	0.45	36.75
10	2100	-17.61	0.41	37.52
11	2100	-17.58	0.37	38.21
12	2100	-17.55	0.33	38.83
13	2100	-17.52	0.30	39.39
14	2100	-17.49	0.27	39.89
15	2100	-17.45	0.25	40.22
16	2100	-17.42	0.24	40.57

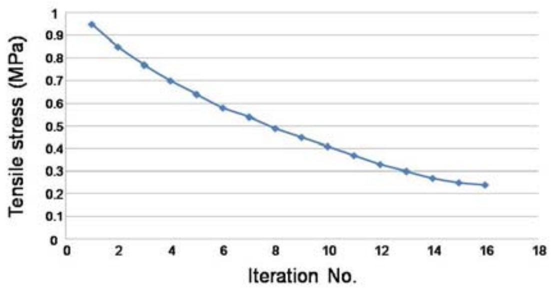


Fig. 12 Tensile stress vs Iterations

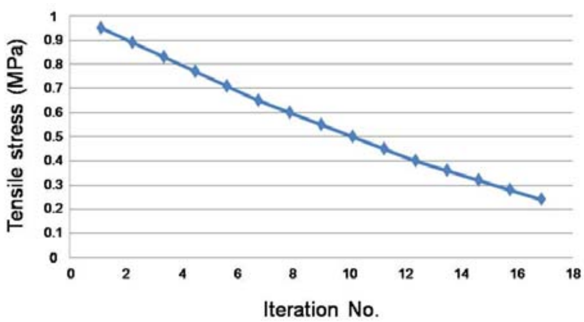


Fig. 13 Tensile stress vs Iterations

Table 4 Bending stresses (MPa) for variable prestressing load

Iteration No.	Load (KN)	Maximum bending stresses (MPa)		Eccentricity (mm)
		Compressive	Tensile	
1	2100	-18.15	0.95	25
2	2110	-18.18	0.89	25
3	2120	-18.21	0.83	25
4	2130	-18.25	0.77	25
5	2140	-18.28	0.71	25
6	2150	-18.31	0.65	25
7	2160	-18.34	0.6	25
8	2170	-18.37	0.55	25
9	2180	-18.40	0.5	25
10	2190	-18.43	0.45	25
11	2200	-18.45	0.4	25
12	2210	-18.47	0.36	25
13	2220	-18.50	0.32	25
14	2230	-18.52	0.28	25
15	2235	-18.54	0.24	25

5.2.1 Constant prestressing force

A constant prestressing force of 2100 kN is applied on all the fourteen cables. Initially eccentricity of all the cables is kept 25 mm. Maximum compressive and tensile stresses are noted during the design process. It was observed that maximum tensile stresses occur at four locations *a*, *b, c* and *d* which lie on cable 4 and 11. Hence layout design of these two cable were taken up. It took 16 iterations to converge to maximum tensile stresses below 0.25 MPa. Maximum compressive and tensile stresses along with eccentricities are given in Table 3. The variation of tensile stresses with respect to iteration is shown in Fig. 12 which shows non-linear trend.

5.2.2 Variable prestressing force

In second case of layout design, eccentricities of all the cables are kept constant 25 mm. based on the stress analysis, cable forces are varied. The initial value of the prestressing force is kept 2100 kN which is varied during the design process. It can be observed that at cable force of 2235 kN maximum tensile stresses are below the limit. The accounts of the compressive and tensile stresses, during the design process, are given in Table 4. It took 15 iterations to bring the tensile stresses below 0.25 MPa. The variation of tensile stresses with respect to iteration is shown in Fig. 13 which shows the linear trend.

6. Conclusions

The proposed methodology of cable layout design is a powerful tool for design of prestressed concrete slabs. It can be observed that for two way slab, at cable force of 2115 kN maximum tensile stresses are below the limit considering constant profile while for variable profile it comes to 2000KN. It means 5.75% more prestressed force is needed to keep the tensile stresses below the permissible limit in case of constant profile. These examples show the efficacy of the proposed methodology of cable layout design in terms of design of cable layout and prestressing force. It is observed that stresses at symmetrical nodes are equal which validate the methodology of the software PRECLAD3D. Out of two preferred cable layout design methods, constant profile is found to require higher prestressing loads.

In this paper a new technique for cable layout design of prestressed concrete slabs is presented. The pre-stressing cables are modelled as B-spline curve whose ordinates are taken as design variables. Stress analysis is carried out using 3D finite element method. Using the proposed methodology, a two way prestressed concrete slab has been successfully designed to satisfy two design criteria. The proposed approach offers a powerful tool for realistic cable layout design of prestressed concrete slabs.

References

Barakat, S., Kallas, N. and Taha, M.Q. (2003), “Single objective reliability-based optimization of prestressed concrete beams”, *Comput. Struct*., **81**(26-27), 2501-2512.

Brandt, A.M. (1989), *Foundations of optimum design in civil engineering*, Nijhoff Publishers.

Braibant, V., Fleury, C. and Beckers, P. (1983), *Shape optimal design: An approach matching CAD and optimization concept*, Report SA-109, Aerospace Laboratory of University of Liege, Belgium.

Buragohian, D.N. and Mukherjee, A. (1993), *PARCS –A pre-stressed and reinforced concrete shell element for analysis of containment structures*, Transactions of 12th SmiRT International Conference, **B**, Stuttgart, Germany.

Buragohian, D.N. and Siddhyae, U.R. (1997), “Finite element analysis of pre stressed concrete box girder bridges”, *J. Struct. Eng. Div*., **24**(3), 135-141.

Diep, B.K. and Umehara, H. (2002), “Non-linear analysis of externally prestressed concrete beam”, *J. Struct. Eng.-ASCE*, **2**, 85-96.

Elwi, A. and Hrudey, T. (1989), “Finite element model for curved cable embedded reinforcement”, *J. Eng. Mech.-ASCE*, **115**(4), 740-754.

Figueiras, J.A. and Povoas, R.H.C.F. (1994), “Modeling of pre-stressing in non -linear analysis of concrete structures”, *Comput. Struct*., **53**(1),173-187.

Ghallab, A. and Beeby, A.W. (2005), “Factors affecting the external prestressing stress in externally strengthened prestressed concrete beams”, *Cement Concrete Comp*., **27**(9-10), 945-957.

Ghoddosian, A. (1998), *Improved zero order techniques for structural shape optimization using FEM*, PhD Dissertation, Applied Mechanics Dept. IIT Delhi.

Gordon, W.J. and Riesenfeld, R.F. (1974), *Computer aided geometric design*, Academic Press, London.

Greunen, J.V. and Scordelis, A.C. (1983), “Nonlinear analysis of pre-stressed concrete slabs”, *J. Struct. Eng. Div.-ASCE*, **109**(7), 1742-1760.

Ibrahim, A.M. and Mubarak, H.M. (2009), “Finite element modelling of continuous reinforced concrete beam with external prestressed”, *Euro. J. Sci. Res*., **30**(1), 177-186.

Jirousek, J., Bouberguig, A. and Saygun, A. (1979), “A macro element analysis of pre-stressed curved box girder bridges”, *Comput. Struct*., **10**(3), 467-482.

Kang, Y.J. and Scordelis, A.C. (1990), “Non-linear analysis of pre-stressed concrete frames”, *J. Struct. Eng. Div.-ASCE*, **106**(2), 445-462.

Kirsch, U. (1973), “Optimized prestressing by linear programming”, *Int. J. Numer. Meth. Eng*., **7**(2), 125-136.
Kirsch, U. (1993), *Structural optimization*, Springer-Verlog, Berlin.
Kuyucular, A. (1991), “Prestressing optimization of concrete slabs”, *J. Struct. Eng. Div.-ASCE*, **117**(1), 235-254.
Lounis, Z. and Cohn, M.Z. (1993), “Multiobjective optimization of prestressed concrete structures”, *J. Struct. Eng. Div.-ASCE*, **119**(3), 794-808.
Lin, T.Y. and Burns, N.H. (1982), *Design of pre-stressed concrete structures*, Third Edition, John Wiley and Sons. Inc., New York.
Madhavi, T.C., Sekar, M. and Paramasivam, V. (2009), “Efficient composite layered finite element modelling for prestressed concrete box girders”, *IE(I) J.–CV*, **90**(MAI),14-21.
Pandey, A.K., Kumar, R. and Trikha, D.N. (1997), “Finite element analysis of pre-stressed concrete containment structures”, *Proceedings of First National Conference on Computer Aided Structural Analysis and Design*, Hyderabad, India, 373-379.
Pathak, K.K. and Sehgal, D.K. (2004), “Analysis of prestress concrete beams using different cable models”, *J. Bridge Struct. Eng*., **33**(2004), 29-40.
Pourazady, M. and Fu, Z. (1996), “An integrated approach to structural shape optimization”, *Comput. Struct*., **60**(2), 279-289.
Povoas, R.H.C.F. and Figueiras, J.A. (1989), “Non linear analysis of curved pre-stressed girder bridges”, *Proceedings of second international conference on computer aided analysis and design*, Austria.
Qing, S.B. and Liu, D.Y. (1989), *Computational geometry- curve and surface modeling*, Academic Press, London.
Quiroga, A.S. and Arroya, M.A.U. (1991), “Optimization of prestressed concrete bridge decks”, *Comput. Struct*., **41**(3), 553-559.
Raju, N.K. (1995), *Pre-stressed concrete*, Third Edition, Tata McGraw Hill, New Delhi, India.
Roca, P. and Mari, A.R. (1993), “Numerical treatment of pre-stressing tendons in the non-linear analysis of pre-stressed concrete structures”, *Comput. Struct*., **46**(5), 905-916.
Rogers, D.F. and Adams, J.A. (1990), *Mathematical element for computer graphics*, Second Edition, McGraw-Hill, New York.
Saleem, A., Pathak, K.K. and Bhaduria, S.S. (2008), “Finite element analysis of pre-stress concrete beams considering realistic cable profile”, *Int. J. Appl. Eng. Res*., **3**(1).
Schoenberg, I.J. (1946), “Contribution to the problem of approximation of equidistant data by analytic functions”, *Q. Appl. Math*., **4**, 45-99.
Utrilla, M.A. and Smartin, A. (1997), “Optimized design of the prestress in continuous bridge decks”, *Comput. Struct*., **64**(1-4), 719-728.
Vanzyl, S.F. and Scordelis, A.C. (1979), “Analysis of curved pre-stressed segmental bridges”, *J. Struct. Eng.-ASCE*, **105**(11), 2399 -2411.
Wu, X.H. and Lu, X. (2003), “Tendon model for nonlinear analysis of externally prestressed concrete structures”, *J. Struct. Eng*., **129**(1), 96-104.
Zienkiewicz, O.C. and Taylor, R.L. (1991), *The finite element method*, **1-2**, Fourth Edition, McGraw-Hill London.

CC