Computers and Concrete, Vol. 1, No. 3 (2004) 285-302
DOI: http://dx.doi.org/10.12989/cac.2004.1.3.285

Direct frequency domain analysis of concrete arch dams based on FE-(FE-HE)-BE technique

Vahid Lotfi†

Civil Engineering Department, Amirkabir University, Tehran, Iran
(Received January 24, 2004, Accepted June 24, 2004)

Abstract. A FE-(FE-HE)-BE procedure is presented for dynamic analysis of concrete arch dams. In this technique, dam body is discretized by solid finite elements, while the reservoir domain is considered by a combination of fluid finite elements and a three-dimensional fluid hyper-element. Furthermore, foundation rock domain is handled by three- dimensional boundary element formulation. Based on this method, a previously developed program is modified, and the response of Morrow Point arch dam is studied for various conditions. Moreover, the effects of canyon shape on response of dam, is also discussed.

Keywords: dynamic analysis; concrete arch dams; boundary element; fluid hyper-element.

1. Introduction

Researches have used different methods to study the dynamic response of concrete arch dam-reservoir-foundation systems, such as the time domain analysis (Chuhan, *et al.* 1995), the hybrid frequency-time method (Camara 2000), or the frequency domain solution. Concentrating on this last alternative, which is also the procedure adopted in this study, one of the most extensive investigations has been carried out by Fok and Chopra (1986). However, the dam-foundation rock interaction was simplified in that study by implementing a massless foundation model. Later on, Tan and Chopra (1995a) improved the initial underlying technique and they presented a procedure, which considered dam-foundation rock interaction completely. This was achieved by employing a two-dimensional (2D) boundary element formulation combined with a series expansion along the canyon axis direction for the foundation rock domain. However, the main limitation of this work, is that the foundation rock geometry must be that of a uniform canyon extending to infinity. An alternative approach was also presented for dynamic analysis of general concrete arch dam-reservoir-foundation rock systems, which was mainly dependent on boundary element formulation (Maeso and Dominguez 1993, Dominguez and Maeso 1993, Maeso, *et al.* 2002).

In this paper, the problem is analyzed by FE-(FE-HE)-BE technique. That means, dam body is discretized by solid finite elements, while the reservoir domain is considered by a combination of fluid finite elements and a three-dimensional fluid hyper-element. Furthermore, the foundation rock domain is represented by utilizing a three-dimensional (3D) boundary element formulation. It should

† Associate Professor

be also mentioned that in this approach, the geometry of canyon could be quite arbitrary and this has become possible due to the fact that a 3D boundary element formulation is applied for foundation rock domain. In the following, the general formulation is presented initially. Based on this method, a previously developed computer program (Lotfi 2001, 2003) is modified, and the response of Morrow Point arch dam is studied as a typical example. The investigation is carried out for various conditions and the effects of canyon shape on the response, is also discussed.

2. Method of analysis

The analysis technique utilized in this study is based on the FE-(FE-HE)-BE method, which is applicable for a general concrete arch dam-reservoir-foundation system. This means, the dam is discretized by solid finite elements, while, the reservoir is divided into two parts, a near field region (usually an irregular shape) in the vicinity of the dam and a far field part (assuming uniform channel), which extends to infinity. The former region is discretized by fluid finite elements and the latter part is modeled by a three-dimensional fluid hyperelement (similar to references, Fok and Chopra 1986, Tan and Chopra 1995a). Furthermore, boundary elements are used for modeling of foundation rock domain (see Figs. 1 and 2 for typical discretizations).

The formulation could be explained much easier, if one concentrates initially on a dam with finite reservoir system (basically the same as a model of dam and reservoir near field), and subsequently add the effects of reservoir far field region and foundation domain for the general case. For this purpose, let us begin with this simpler formulation and then complete the formulation for the more general case on that basis.

2.1. Dam with finite reservoir system

This is the problem, which can be totally modeled by finite element method. It can be easily shown that in this case, the coupled equations of the system may be written as Lotfi (2002):

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{B} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{B}^T \\ \mathbf{0} & \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{MJa}_g \\ -\mathbf{BJa}_g \end{bmatrix} \tag{1}$$

M, **C**, **K** in this relation represent the mass, damping and stiffness matrices of the dam body, while **G**, **L**, **H** are assembled matrices of fluid domain. The unknown vector is composed of **r**, which is the vector of nodal relative displacements and the vector **p** that includes nodal pressures. Meanwhile, **J** is a matrix with each three rows equal to a 3×3 identity matrix (its columns correspond to unit rigid body motion in cross-canyon, stream, and vertical directions) and $\mathbf{a}_g$ denotes the vector of ground accelerations. Furthermore, **B** in the above relation is often referred to as interaction matrix.

For harmonic ground excitations $\mathbf{a}_g(t) = \mathbf{a}_g(\omega)e^{i\omega t}$ with frequency ω, displacements and pressures will all behave harmonic, and the Eq. (1) can be expressed as=

$$\begin{bmatrix} -\omega^2\mathbf{M} + \mathbf{K}(1 + 2\beta_d i) & -\mathbf{B}^T \\ -\omega^2\mathbf{B} & -\omega^2\mathbf{G} + i\omega\mathbf{L} + \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{MJa}_g \\ -\mathbf{BJa}_g \end{bmatrix} \tag{2}$$

In this relation, it is assumed that the damping matrix of the dam is of hysteretic type. This means:

$$\mathbf{C} = \frac{2\beta_d}{\omega}\mathbf{K} \tag{3}$$

Where β_d is the constant hysteretic factor of the dam body. Relation (2) is the coupled equations of a dam with finite reservoir system in frequency domain, which can be made symmetric by multiplying the lower partition matrices by a factor of ω^{-2}.

2.2. Pseudo-symmetric technique

Considering the coupled Eq. (2), it is noticed that unsymmetric terms are due to **B** matrix and its transpose appearing in this relation. This matrix is usually obtained by assemblage of contributing submatrices of interface elements located at fluid-solid contact, or even surfaces where fluid elements are adjacent to rigid or absorbing boundaries. However, to make it more convenient from programming point of view, one can eliminate these interface elements and consider its effect as part of adjacent fluid element matrices. In that case, matrices of the *i*th fluid element which contribute to the corresponding total mass, damping and stiffness matrices of the system would be generally as follows, respectively:

$$\mathbf{Q}_i^M = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{B}_i & \mathbf{G}_i \end{bmatrix},\ \mathbf{Q}_i^C = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_i \end{bmatrix} \text{ and } \mathbf{Q}_i^K = \begin{bmatrix} \mathbf{0} & -\mathbf{B}_i^T \\ \mathbf{0} & \mathbf{H}_i \end{bmatrix} \tag{4}$$

In the present work, interface elements are excluded and their effects are considered as part of fluid element matrices similar to the above explanation. However, everything is made symmetric from the very beginning. This means that fluid element matrices are considered symmetric artificially as below:

$$\mathbf{Q}_i^M = \begin{bmatrix} \mathbf{0} & \mathbf{B}_i^T \\ \mathbf{B}_i & \mathbf{G}_i \end{bmatrix},\ \mathbf{Q}_i^C = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_i \end{bmatrix} \text{ and } \mathbf{Q}_i^K = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_i \end{bmatrix} \tag{5}$$

This presumption makes the method very convenient from programming point of view. However, it would yield to a coupled relation in the frequency domain, which is not really satisfied completely (Lotfi 2002)

$$\begin{bmatrix} -\omega^2\mathbf{M} + \mathbf{K}(1 + 2\beta_d i) & -\omega^2\mathbf{B}^T \\ -\omega^2\mathbf{B} & -\omega^2\mathbf{G} + i\omega\mathbf{L} + \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} \hat{=} \begin{bmatrix} -\mathbf{MJa}_g \\ -\mathbf{BJa}_g \end{bmatrix} \tag{6}$$

It is noticed that a special notation $\hat{=}$ is utilized in this relation. This is to emphasize that equality is slightly damaged due to an extra ω^2 factor appearing in the second term of the upper partition of this relation in comparison to Eq. (2). When this term is corrected, it is noticed that the lower partitions are also required to be multiplied by ω^{-2}, to preserve symmetry. Of course, it must be mentioned that in actual programming, the total dynamic stiffness matrix (i.e., the resulting left hand side matrix of Eq. (6)), could be stored based on symmetric skyline technique and the two above-mentioned steps would be simply performed by multiplying the columns corresponding to pressures degree of freedom by a factor of ω^{-2}, while the same factor is also applied to the lower partition of right hand side vector. In this manner, the final coupled equations of the dam with finite reservoir system in the frequency domain would be:

$$\begin{bmatrix} -\omega^2\mathbf{M} + \mathbf{K}(1 + 2\beta_d i) & -\mathbf{B}^T \\ -\mathbf{B} & \omega^{-2}(-\omega^2\mathbf{G} + i\omega\mathbf{L} + \mathbf{H}) \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{MJa}_g \\ -\omega^{-2}\ \mathbf{BJa}_g \end{bmatrix} \tag{7}$$

The above approach could be visualized as the frequency domain extension of the Pseudo-Symmetric technique originally explained elsewhere for time domain (Lotfi 2002).

In this manner, the usual interface elements are excluded and their effects are considered as part of the adjacent fluid finite element matrices. Meanwhile, all these matrices are made symmetric artificially from the very beginning. Therefore, usual symmetric memory allocation and efficient symmetric skyline solvers could be employed. Of course, slight adjustments are required to be implemented as discussed above, before the actual equations solving routine is started.

This approach is very convenient as a technique for general-purpose finite element programs in regard to their fluid-structure module in frequency domain, since the program would not even feel the slightest non-symmetry even at the element level, while the interface elements are also excluded.

2.3. Reservoir near field boundary conditions

As mentioned in the previous section, the boundary conditions for reservoir near field (except at the water surface which is easily applied), are usually implemented by the help of interface elements. However, these elements could be excluded and their effects could be incorporated in the adjacent fluid elements. On that basis, the fluid element matrices are in general as shown in relations (5), and depending on the type of condition utilized, either one of the matrices $\mathbf{L}_i, \mathbf{B}_i$ or both will be generated. Of course, it is clear that if the fluid element is not adjacent to boundary, there is no need for these matrices and displacement degrees of freedom are excluded for those

Table 1 Conditions for reservoir near field boundary

Type	Relation	Generation of Matrices	
		$\mathbf{L}_i$	$\mathbf{B}_i$
I	$\frac{\partial p}{\partial n} = -\rho \ddot{u}_n$	No	Yes
II	$\frac{\partial p}{\partial n} = -\rho a_g^n - q\frac{\partial p}{\partial t}$	Yes	Yes
III	$\frac{\partial p}{\partial n} = -\frac{1}{c}\frac{\partial p}{\partial t}$	Yes	No

elements. In the case of perimetral fluid elements (adjacent to reservoir near field boundary), there are three types of conditions as listed in Table 1, which could be imposed.

It should be mentioned that in relations of Table 1, the constants ρ, c are mass density and compression wave velocity of water, respectively. Furthermore, n is the reservoir near field outward normal direction, a_g^n, the free field ground acceleration in the n-direction and q is the admittance or a damping coefficient for the corresponding boundary (Fenves and Chopra 1984). The coefficient q is also related to a more meaningful wave reflection coefficient α,

$$\alpha = \frac{1 - qc}{1 + qc} \tag{8}$$

which is defined as the ratio of the amplitude of the reflected hydrodynamic pressure wave to the amplitude of a propagating pressure wave incident on the reservoir boundary in the normal direction.

The condition of type I, is considered for the contact of fluid with flexible solid, such as the dam-reservoir interface or even fluid-foundation interface, if the interaction is going to be treated rigorously. The second type of condition is the so-called approximate boundary condition. This can be imposed at the reservoir bottom for an approximate treatment of fluid-foundation interaction. The last type of condition (III) is referred to as Sommer-feld boundary condition. This is usually applied at the reservoir near field upstream boundary (in cases which far field region is not modeled), as a substitute for a precise transmitting boundary. However, when a fluid hyper-element is utilized, this condition is not required and waves are transmitted exactly through that semi-infinite element.

2.4. Fluid hyper-element

As mentioned, the three-dimensional fluid hyper-element is utilized to model the reservoir far-field region for the more general case. This part of the water domain, is assumed to be as a uniform channel with an arbitrary geometric shape in the vertical plane which includes x, z-axes (see Fig. 1(b) for a typical discretization), and extends to infinity in the upstream direction (negative y-direction). Although, this is a three-dimensional semi-infinite fluid element, its discretization is performed in the vertical plane perpendicular to channel axis, which is referred to as the reference plane ($y = 0$). Therefore, the element consists of several sub-channels, which extend to infinity and all the nodes of the hyper-element are located on that reference plane. The formulation of this element is presented as follows:

Assuming water to be linearly compressible and neglecting its viscosity, the small amplitude, irrotational motion of water due to harmonic excitation is governed by Helmholtz equation;

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} + \frac{\omega^2}{C^2}p = 0 \tag{9}$$

where p is the amplitude of the hydrodynamic pressure (in excess of hydrostatic pressure) and C is the velocity of pressure waves in water.

By seeking solutions of the form e^{ky} in the stream direction, the Eq. (9) becomes,

$$\lambda^2 p + \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = 0 \tag{10}$$

with the following definition of λ.

$$\lambda^2 = k^2 + \frac{\omega^2}{C^2} \tag{11}$$

By applying the variational method on Eq. (10), the following matrix relation is obtained at each sub-element level:

$$[-\lambda^2\mathbf{A}^e + \mathbf{C}^e]\mathbf{P}^e = \mathbf{R}^e \tag{12}$$

where $\mathbf{P}^e$ is the vector of nodal pressure amplitudes for each sub-element nodes located on hyper-element reference plane (i.e., $y = 0$). Furthermore, matrices $\mathbf{A}^e$, $\mathbf{C}^e$ and vector $\mathbf{R}^e$ are defined below:

$$\mathbf{A}^e = \frac{1}{\rho}\int_A \mathbf{N}\mathbf{N}^T\, dA \tag{13a}$$

$$\mathbf{C}^e = \frac{1}{\rho}\int_A (\mathbf{N}_x\mathbf{N}_x^T + \mathbf{N}_z\mathbf{N}_z^T)\, dA \tag{13b}$$

$$\mathbf{R}^e = \frac{1}{\rho}\oint_s \mathbf{N}\frac{\partial p}{\partial n} ds \tag{13c}$$

In the above relations, $\mathbf{N}$ is the vector of shape functions, and $\mathbf{N}_x$, $\mathbf{N}_z$ denote derivatives of this vector with respect to x, z coordinates, respectively.

As for boundary conditions; neglecting gravity waves, one can write the condition

$$p = 0 \tag{14}$$

for the water surface. The condition at reservoir-foundation contact boundaries, can be expressed by the approximate relation,

$$\frac{\partial p}{\partial n} = -\rho a_g^n(\omega) - i\omega q p \tag{15}$$

which allows for refraction of hydrodynamic pressure waves into the reservoir bottom materials or flexible foundation rock. The admittance or damping coefficient q in this relation is defined as,

$$q = \frac{\rho}{\rho_f C_f}$$

where ρ is the fluid density, and ρ_f, C_f are density and pressure wave velocity of the reservoir bottom material, or foundation rock material. The damping coefficient q is also related to a more meaningful wave reflection coefficient α,

$$\alpha = \frac{1 - qC}{1 + qC}$$

which is defined as the ratio of the amplitude of the reflected hydrodynamic pressure wave to the amplitude of a propagating pressure wave incident on the reservoir boundary, in the perpendicular direction.

Imposing condition (15) on relation (13c) for sub-elements adjacent to the foundation contact surface, would yield:

$$\mathbf{R}^e = -(\mathbf{D}^{ex}\mathrm{a}_g^x + \mathbf{D}^{ez}\mathrm{a}_g^z + i\omega q\mathbf{L}_h^e\mathbf{P}^e) \tag{16}$$

with the following definitions:

$$\mathbf{D}^{ex} = \int_s \mathbf{N} n_x \, ds \tag{17a}$$

$$\mathbf{D}^{ez} = \int_s \mathbf{N} n_z \, ds \tag{17b}$$

$$\mathbf{L}_h^e = \frac{1}{\rho}\int_s \mathbf{N}\mathbf{N}^T ds \tag{17c}$$

n_x, n_z are the components of a unit outward normal vector for the fluid sub-element boundary.

Taking into account relations (14), and (16), the corresponding relation (12) for the hyper-element, is obtained by assembling contributions from different sub-elements:

$$[-\lambda^2\mathbf{A} + i\omega q\mathbf{L}_h + \mathbf{C}]\mathbf{P} = -(\mathbf{D}^x a_g^x + \mathbf{D}^z a_g^z) \tag{18}$$

P in this relation, is the vector of nodal pressure amplitudes. It includes all nodes of the fluid hyper-element below the water surface (n-nodes), which are located on the reference plane (i.e., $y = 0$).

Considering homogeneous boundary conditions in Eq. (18) corresponding to zero ground acceleration, it leads to the following eigenvalue problem:

$$[-\lambda_j^2\mathbf{A} + i\omega q\mathbf{L}_h + \mathbf{C}]\mathbf{X}_j = \mathbf{0} \tag{19}$$

λ_j^2, $\mathbf{X}_j$ are the jth eigenvalue and eigenvector of the fluid hyper-element.

There also exists particular solutions for relation (18) which corresponds to uniform unit acceleration of reservoir boundary in the l-direction (x, or z-direction). In these case, the solution is independent of y-direction (k=0), and considering relation (11), it yields:

$$\left[-\frac{\omega^2}{C^2}\mathbf{A} + i\omega q\mathbf{L}_h + \mathbf{C}\right]\mathbf{P}_p^1 = -\mathbf{D}^1 \tag{20}$$

The general solution for the amplitude of hydrodynamic pressures vector at an arbitrary y-coordinate is obtained by combinations of the eigenvectors and the particular solutions calculated from relations (19), (20). Meanwhile, considering the exponential form of the individual solutions in y-direction, one would have:

$$\mathbf{P} = \sum_{j=1}^{n} \gamma_j \mathbf{X}_j e^{k_j y} + \mathbf{P}_p^x a_g^x(\omega) + \mathbf{P}_p^z a_g^z(\omega) \tag{21}$$

In this relation, γ_j is the participation factor for the jth mode, and $a_g^x(\omega)$, $a_g^z(\omega)$ are included because, unit vertical accelerations were assumed initially for calculation of particular solutions.

For the hyperelement reference plane (i.e., $y = 0$) which is denoted by h, the vector of pressure amplitudes (21) becomes:

$$\mathbf{P}_h = \sum_{j=1}^{n} \gamma_j \mathbf{X}_j + \mathbf{P}_p^x a_g^x(\omega) + \mathbf{P}_p^z a_g^z(\omega) \qquad (22)$$

It can also be written in a more convenient matrix form,

$$\mathbf{P}_h = \mathbf{X}_h \Gamma + \mathbf{P}_p \mathbf{a}_g(\omega) \qquad (23)$$

with the help of following definitions for reservoir hyper-element modal matrix, vector of participation factors and a matrix which includes particular solution vectors for different directions.

$$\mathbf{X}_h = [\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n]$$

$$\Gamma = [\gamma_1, \gamma_2, \ldots, \gamma_n]^T$$

$$\mathbf{P}_p = \begin{bmatrix} \mathbf{P}_p^x & \mathbf{0} & \mathbf{P}_p^z \end{bmatrix}$$

Moreover, the fluid y-direction (stream component) accelerations vector for an arbitrary vertical plane parallel to reference plane (constant y-plane), is obtained through differentiation of relation (21):

$$\ddot{\mathbf{U}} = -\frac{1}{\rho} \sum_{j=1}^{n} \gamma_j k_j \mathbf{X}_j e^{k_j y} \qquad (24)$$

For the reference plane, this vector becomes;

$$\ddot{\mathbf{U}}_h = -\frac{1}{\rho} \mathbf{X}_h \mathbf{K}_h \Gamma \qquad (25)$$

where $\mathbf{K}_h$ is a diagonal matrix with the jth diagonal element being equal to k_j.

Solving for the participation vector from Eq. (23) by employing orthogonality condition of modal matrix and substituting in Eq. (25) yields:

$$\ddot{\mathbf{U}}_h = -\frac{1}{\rho} \mathbf{X}_h \mathbf{K}_h \mathbf{X}_h^T \mathbf{A} (\mathbf{P}_h - \mathbf{P}_p \mathbf{a}_g) \qquad (26)$$

Multiplying both sides of this relation by $-\mathbf{A}$, one obtains:

$$\mathbf{R}_h = \mathbf{H}_h \mathbf{P}_h - \mathbf{R}_p \mathbf{a}_g(\omega) \qquad (27)$$

by employing the following definitions:

$$\mathbf{R}_h = -\mathbf{A} \ddot{\mathbf{U}}_h \qquad (28a)$$

$$\mathbf{H}_h = \frac{1}{\rho} \mathbf{A} \mathbf{X}_h \mathbf{K}_h \mathbf{X}_h^T \mathbf{A} \qquad (28b)$$

$$\mathbf{R}_p = \mathbf{H}_h \mathbf{P}_p \tag{28c}$$

In relation (27), $\mathbf{R}_h$ represents a consistent vector equivalent to integration of inward horizontal acceleration (negative of stream component) for the hyper-element, and this vector contains essentially similar quantities as the components of the right hand side vectors of usual fluid finite elements.

2.5. Dam-reservoir system

The formulation for a dam with finite reservoir, was already presented. For the case where the reservoir extends to infinity, a hyper-element must be used along with the fluid finite elements utilized for reservoir near-field. Meanwhile, the governing relation for hyperelement was derived in previous section (relation (27)). Therefore, if the matrices of the hyperelement assemble in conjunction with the fluid finite elements, Eq. (7) would now become:

$$\begin{bmatrix} -\omega^2\mathbf{M} + \mathbf{K}(1+2\beta_d i) & -\mathbf{B}^T \\ -\mathbf{B} & \omega^{-2}((-\omega^2)\mathbf{G} + i\omega\mathbf{L} + \mathbf{H}) + \overline{\mathbf{H}}_h(\omega) \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{MJa}_g \\ \omega^{-2}(-\mathbf{BJa}_g + \overline{\mathbf{R}}_p(\omega)\mathbf{a}_g) \end{bmatrix} \tag{29}$$

Where $\overline{\mathbf{H}}_h(\omega)$ and $\overline{\mathbf{R}}_p(\omega)$ are the expanded form of $\mathbf{H}_h(\omega)$ and $\mathbf{R}_p(\omega)$ matrices which cover the entire fluid domain pressure degrees of freedom. Assuming that the pressure degrees of freedom related to hyperelement are ordered first in the unknown pressure vector, then these matrices would have the following forms:

$$\overline{\mathbf{H}}_h(\omega) = \begin{bmatrix} \mathbf{H}_h(\omega) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{30a}$$

$$\overline{\mathbf{R}}_p(\omega) = \begin{bmatrix} \mathbf{R}_p(\omega) \\ \mathbf{0} \end{bmatrix} \tag{30b}$$

The relation (29) is the equation to be used instead of Eq. (7), when the reservoir is extended to infinity and one is considering the direct approach in frequency domain.

2.6. Dam-reservoir-foundation system

In the previous study (Lotfi 2003), derivation of foundation impedance matrix was presented in details by applying boundary element technique.

Furthermore, it was shown that how the combined matrix equation for the dam-foundation system could be obtained by applying the following two conditions: The equilibrium of interaction forces between the dam and foundation rock, and compatibility of corresponding displacements at the interface.

Following the same steps, it can be easily shown that considering a flexible foundation would modify the dam-reservoir governing matrix relation (29) as follows:

$$\begin{bmatrix} \left[-\omega^2\mathbf{M} + K(1+2\beta_d i) + \bar{\mathbf{S}}_f(\omega)\right] & -\mathbf{B}^T \\ -\mathbf{B} & \omega^{-2}((-\omega^2\mathbf{G} + i\omega\mathbf{L} + \mathbf{H}) + \bar{\mathbf{H}}_h(\omega)) \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} -\mathbf{M}\mathbf{J}\mathbf{a}_g \\ \omega^{-2}(-\mathbf{B}\mathbf{J}\mathbf{a}_g + \bar{\mathbf{R}}_p(\omega)\mathbf{a}_g) \end{bmatrix} \tag{31}$$

in which $\bar{\mathbf{S}}_f(\omega)$ is the expanded form of foundation impedance matrix (Lotfi 2003).

The relation (31) is a system of equations, which can be solved for nodal displacements vector at specified frequencies for different forms of ground accelerations vector $\mathbf{a}_g(\omega)$ corresponding to upstream, vertical or cross-canyon excitations.

It should be also mentioned that in this process, a significant amount of the computational time is spent for calculation of foundation impedance matrix at each frequency. However, this could be remedied by calculating the impedance matrix only at certain frequency points and interpolating this matrix for intermediate frequencies similar to the work of Tan and Chopra (1995b). In the present study, 25 equally spaced frequency points are used and cubic interpolation scheme is implemented (Lotfi 2003).

3. Modeling and basic parameters

A computer program (Lotfi 2002) was enhanced based on the theory presented on the previous section. The program is based on the FE-(FE-HE)-BE concept. This means, the dam is treated by solid finite elements, while the reservoir is divided into two parts, the near-field region in the vicinity of the dam, which is discretized by fluid finite elements, and the far-field part is modeled by a three-dimensional hyper-element. Moreover, boundary element technique is utilized for modeling of foundation rock domain.

3.1. Models

An idealized symmetric model of Morrow Point arch dam is considered. The geometry of the dam may be found in reference (Hall and Chopra 1983).

The dam is discretized by 40 isoparametric 20-node finite elements (Fig. 1(a)). The water domain is divided into two regions (Fig. 1(b)). The near-field part is considered as a region, which extends to a length of 0.2H in upstream direction at dam mid-crest point. H being the dam height or maximum water depth in the reservoir. The far-field region starts from that point and extends to infinity in the upstream direction. Both these regions combined are assumed to form a uniform reservoir shape to be consistent with the work of Tan and Chopra (1995a). The former domain, is discretized by 80 isoparametric 20-node fluid finite elements, while the latter part is modeled by a hyper-element which itself is costructed from 40 isoparametric 8-node sub-elements. Furthermore, the foundation rock is modeled by 178 isoparametric 8-node boundary elements considered at the foundation surface. However, two alternatives are studied as far as the foundation geometry. In all of the initial cases, whenever the foundation rock is considered as flexible, the canyon shape is

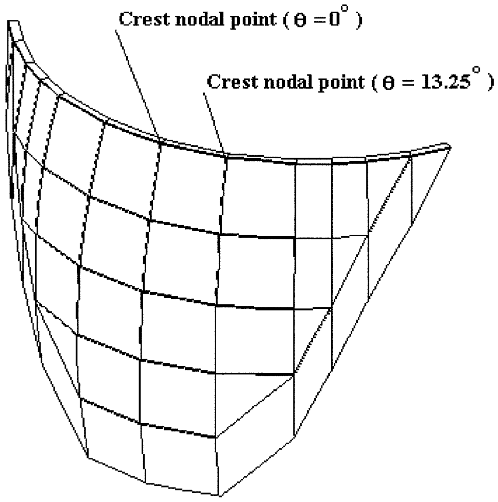

Fig. 1(a) Finite element mesh of the dam body

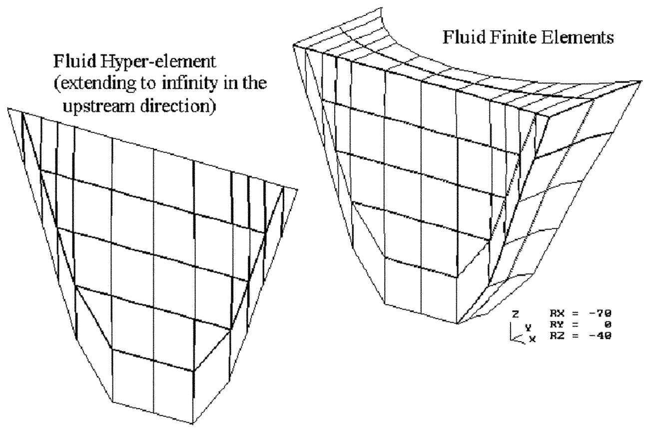

Fig. 1(b) Discretization of water domain (fluid finite elements and the fluid hyper-element)

assumed uniform (Fig. 2(a)) as in the Tan and Chopra study (1995a). Subsequently, in the latter part of the study, a second shape is also employed, which represents a typical non-uniform canyon shape (Fig. 2(b)). This is utilized to study the effects of canyon shape on the response.

3.2. Basic parameters

The dam concrete is assumed to be homogeneous with isotropic linearly viscoelastic behavior and the following main characteristics:

Elastic modulus (E_d) = 27.5 GPa.

Poisson's ratio = 0.2

Unit weight = 24.8 kN/m^3

Hysteretic damping factor (β_d) = 0.05

The impounded water is taken as inviscid, and compressible fluid with unit weight equals 9.81

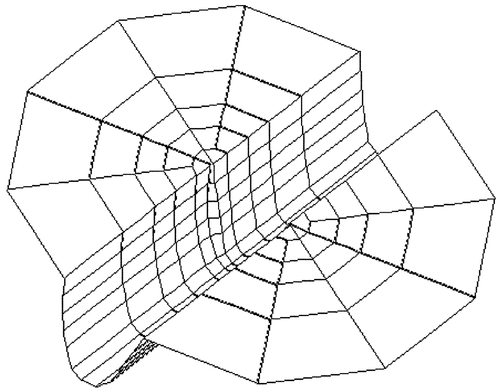

Fig. 2(a) Boundary element discretization of foundation rock (uniform canyon shape)

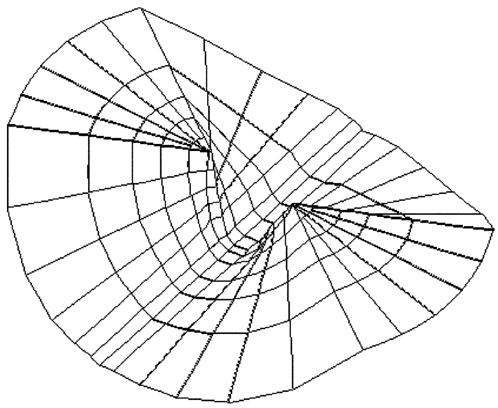

Fig. 2(b) Boundary element discretization of foundation rock (non-uniform canyon shape)

kN/m^3, and pressure wave velocity C=1440 m/sec.

The foundation rock, is idealized by a homogeneous, viscoelastic domain. The basic properties of this region are:

Elastic modulus (E_f) = 27.5 GPa.
Poisson's ratio = 0.2
Unit weight = 26.4 kN/m^3
Hysteretic damping factor (β_f) = 0.05

4. Results

The dam-reservoir model with rigid foundation is considered first (Fig. 1). The responses of dam crest are obtained due to upstream, vertical and cross-stream excitations (Fig. 3). In each case, several values of wave reflection coefficient α are selected to investigate the effects of reservoir boundary absorption. The result for an empty reservoir case is also shown in each graph as a reference.

It should be mentioned that the response quantities plotted are the amplitudes of the complex valued radial accelerations for two points located at dam crest (Fig. 1(a)). This is either the mid-crest point (θ=0°) selected for upstream or vertical excitations or a point located at (θ=13.25°)which is used for the case of cross-stream excitation. This is due to the fact that radial acceleration is diminished at mid-crest for the cross-stream type of ground motion.

In each case, the amplitude of radial acceleration is plotted versus the dimensionless frequency for a significant range. The dimensionless frequency for upstream and vertical excitation is defined as ω/ω_1^S where ω is the excitation frequency and ω_1^S is the fundamental frequency of the dam on rigid foundation with empty reservoir for a symmetric mode. For the cross-stream excitation cases, the dimensionless frequency is defined as ω/ω_1^a, where ω_1^a is the fundamental resonant frequency of the dam on rigid foundation with empty reservoir for an anti-symmetric mode.

In the response to upstream ground motion, it is noted that fundamental resonant frequency of the dam-reservoir system reduces in comparison with an empty reservoir case. The amplitude of the corresponding response at this frequency is increased significantly for the case of fully reflective reservoir boundary (i.e., $\alpha = 1$) relative to the empty reservoir case. However, for absorptive reservoir boundary cases considered, the peaks at the fundamental frequency are lowered. Meanwhile, for all

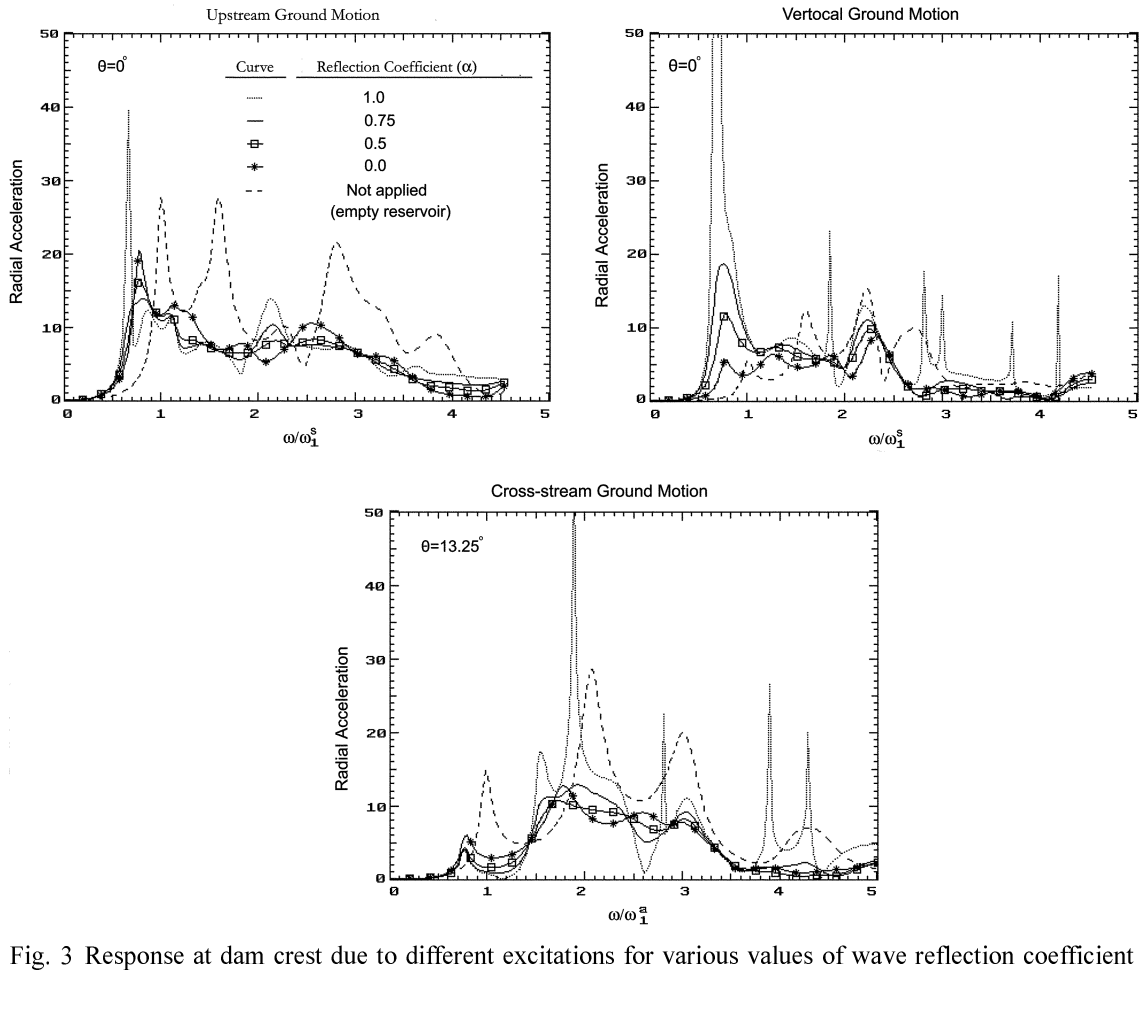

Fig. 3 Response at dam crest due to different excitations for various values of wave reflection coefficient α

cases, the response at higher frequencies is significantly lower than the empty reservoir case due to radiation damping effect. Moreover, it is observed that with decreasing wave reflection coefficient α, the amplitude of the fundamental resonant peak decreases, whereas the second, smaller peak increases, resulting in a single fundamental resonant peak at an intermediate frequency value. This is consistent with what other researchers have reported (Tan and Chopra 1995a).

For vertical and cross-stream excitations, it is noticed that very sharp peaks (actually unbounded) occur at certain frequencies referred to as cut-off frequencies for the rigid reservoir boundary case ($\alpha = 1$). However, reservoir boundary absorption eliminates the unbounded responses at these frequencies for other cases.

In the next stage, the results for the following four systems are compared (Figs. 4-6): dam on rigid foundation rock with empty reservoir, dam on flexible foundation rock with empty reservoir, dam on rigid foundation rock with full reservoir, and dam on flexible foundation rock with full reservoir. It should be mentioned that for the flexible foundation cases of this stage, the canyon shape is assumed uniform. Therefore, the foundation is discretized as shown in Fig. 2(a). Meanwhile, the results of these four cases are compared for two values of wave reflection coefficients $\alpha = 1$, 0.5 due to different excitations. Similar results can be found in the work of Tan and Chopra (1995a) for all three types of excitations presented herein. However, due to space limitations, only their results for the upstream ground motion are illustrated in Fig. 7 for comparison purposes. These results could be compared with the corresponding results of the present study (Fig. 4).

It is noticed that general trend is similar and relatively good agreement exists between the results obtained herein (Fig. 4) and the ones taken from the above-mentioned reference (Fig. 7). The main difference is the amplitude of the peaks, which are slightly higher in the work of Tan and Chopra in comparison with the present study results for the flexible foundation cases. This observation was already reported in the previous study in the case of flexible foundation with empty reservoir (Lotfi 2003). The same trend is noticed here not only for empty reservoir but also for full reservoir cases, whenever the foundation is taken as flexible. This is mainly due to the fact that in the Tan and Chopra study, the foundation impedance matrix is obtained by applying a fine mesh and implementing static condensation to eliminate the extra degrees of freedom (Tan and Chopra 1995a).

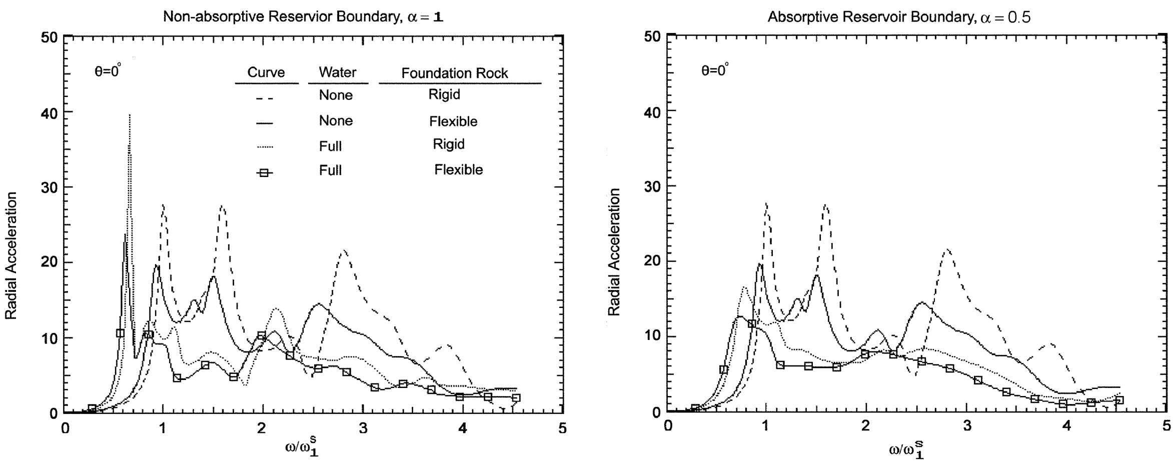

Fig. 4 Response at dam crest due to upstream ground motion under different conditions

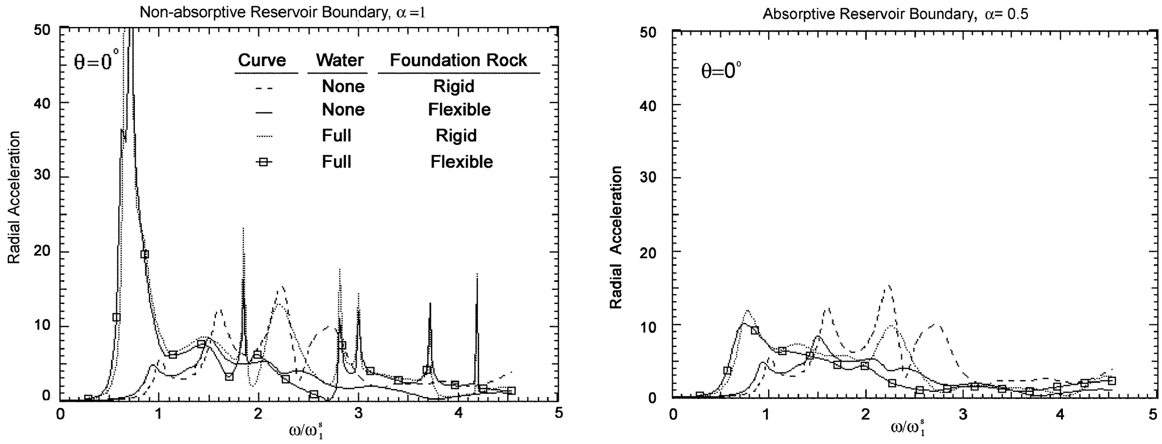

Fig. 5 Response at dam crest due to vertical ground motion under different conditions

Finally, it was decided to show the effects of canyon shape on the response. For this aim, a second foundation rock discretization was introduced (Fig. 2(b)). In this case, it is assumed that the topography at the crest level makes an angle of 60° with the stream direction. This is for both upstream and downstream directions at both left and right banks. At lower elevations, this angle is reduced based on a quadratic function of height, such that it becomes zero at the base of the dam.

The results for uniform and non-uniform canyon shapes are compared in Figs. 8-10 for different types of excitation. It should be also noted that the illustrated responses correspond to full reservoir cases, and for each excitation the comparison is carried out for two different values of wave reflection coefficient. Meanwhile, the response for the rigid foundation case with full reservoir is also shown in each graph as a reference.

It is observed that for upstream excitation, the response is significantly affected due to change in canyon shape (Figs. 8-10). The peaks of the response for the non-uniform canyon shape is lower than for the uniform canyon shape, and the natural frequencies of the system are also reduced. This is because, the foundation domain has become more flexible as a result of thinner abutment, and the

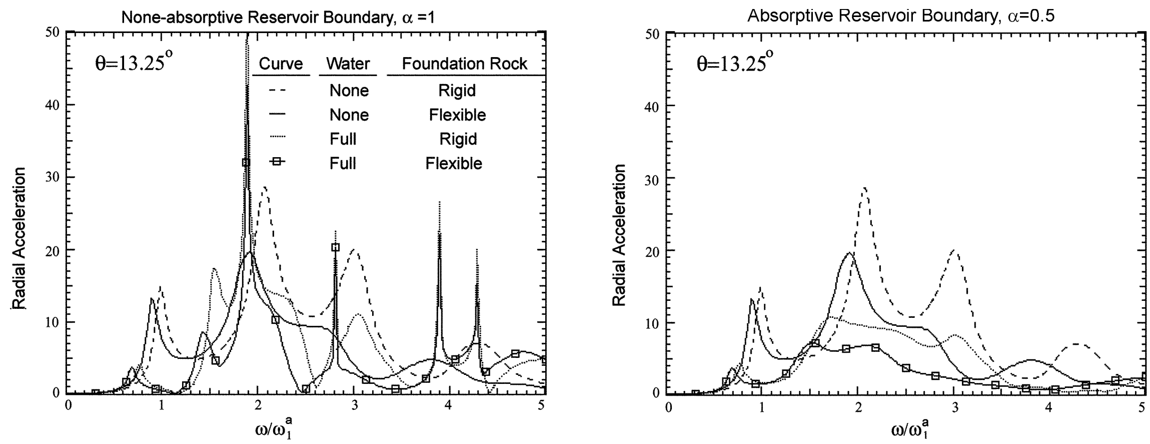

Fig. 6 Response at dam crest due to cross-stream ground motion under different conditions

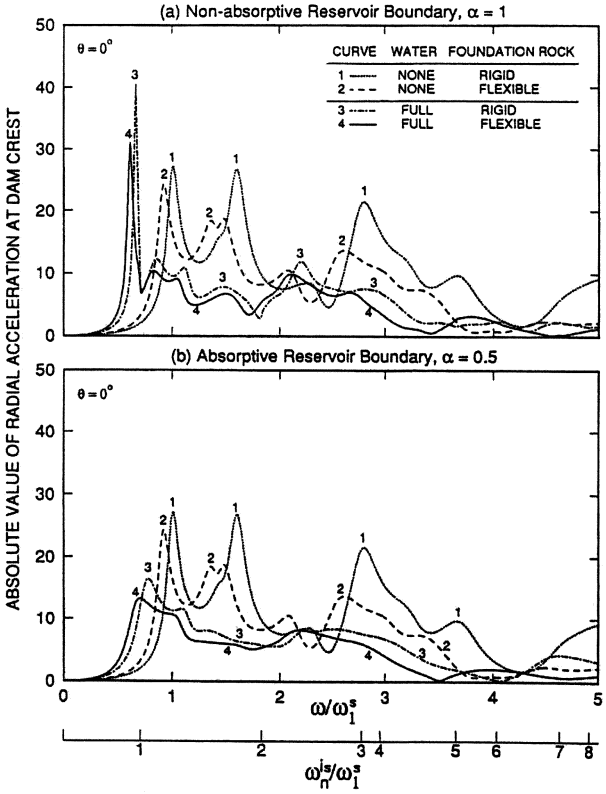

Fig. 7 Response at dam crest due to upstream ground motion under different conditions (result taken from the work of Tan and Chopra (1995a) for comparison)

behavior is similar to reducing the foundation elastic modulus as explained elsewhere under the empty reservoir conditions (Lotfi 2003). Similar observations are noted for the vertical and cross-stream

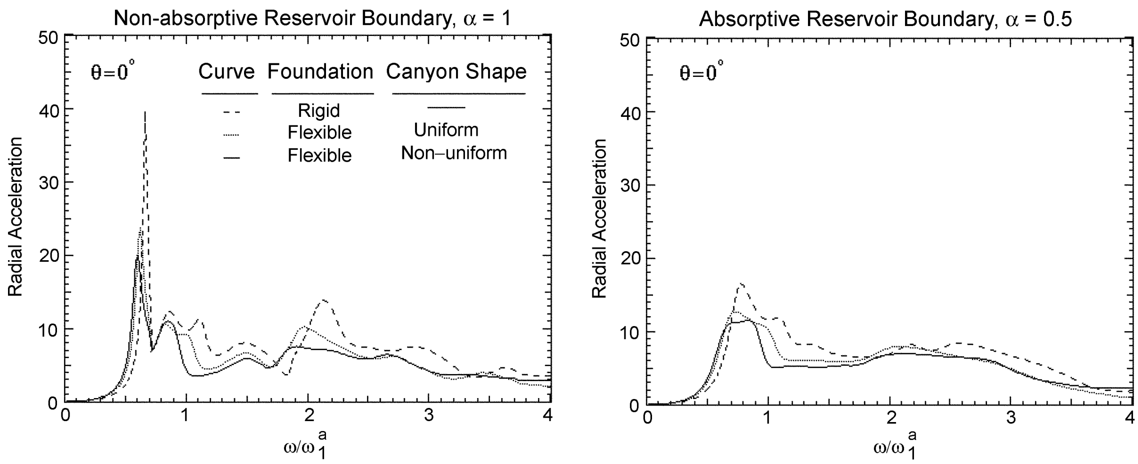

Fig. 8 Influence of canyon shape on the response at dam crest due to upstream excitation

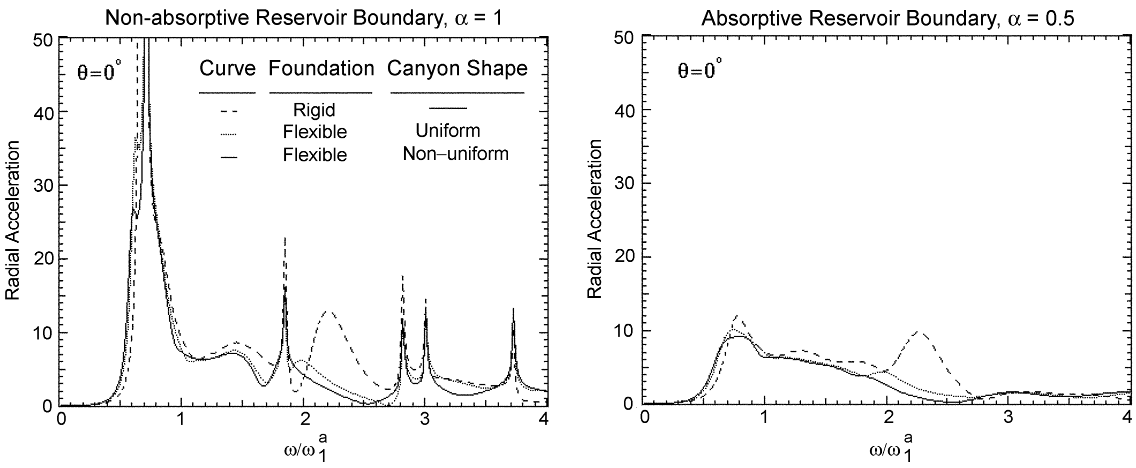

Fig. 9 Influence of canyon shape on the response at dam crest due to vertical excitation

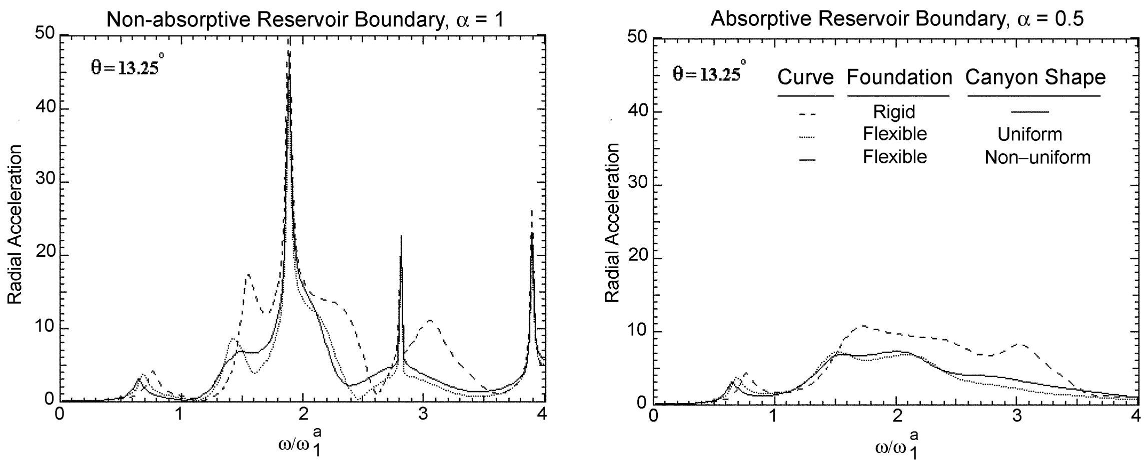

Fig. 10 Influence of canyon shape on the response at dam crest due to cross-stream excitation

excitations. However, in those graphs, sharp peaks are also present in the response for the non-absorptive reservoir boundary condition cases, which correspond to reservoir cut-off frequencies.

5. Conclusions

The formulation based on FE-(FE-HE)-BE procedure for dynamic analysis of concrete arch dam-reservoir-foundation rock systems, was explained. A computer program was prepared based on this methodology and the response of Morrow Point arch dam was studied for various conditions as well as different shapes of canyon. Overall, the main conclusions obtained by the present study can be listed as follows:

- The FE-(FE-HE)-BE technique is proved to be an effective method for dynamic analysis of concrete arch dams.
- The results presented herein are in very good agreement with the work of Tan and Chopra

(1995a) for rigid foundation cases (dam with full reservoir and different assumptions on reservoir boundary condition). However, the amplitude of the peaks obtained at the fundamental frequency due to different excitations are slightly lower in comparison with their results for the flexible foundation cases. This is noticed both for empty and full reservoir conditions, and it is mainly due to the fact that they have computed the foundation impedance matrix at each frequency by applying a fine mesh and implementing static condensation to eliminate the extra degrees of freedom.

- The main advantage of the present technique over the method of Tan and Chopra (1995a) is that there are no restrictions imposed as to the geometric shape of the canyon.
- It is observed that response is significantly affected due to change in canyon shape. This was already noticed and reported in a previous study for empty reservoir condition. However, this effect was controlled even further in the present study under the full reservoir condition and for two different values of reservoir boundary reflection coefficient. It is noted that the peaks of the response for the non-uniform canyon shape is lower than for the uniform canyon shape, and the natural frequencies of the system are also reduced. This is because; the foundation domain considered for the non-uniform canyon shape has become more flexible as a result of thinner abutment.

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