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Analysis of stress distribution in anchorage zones of pretensioned beams

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Abstract. The stress transmission mechanism in pretensioned concrete beams, though very interesting from an economical point of view, is very complex, integrating various phenomenons such as sliding, bond, bursting. For long the complexity of this mechanism has led engineers to provide a massive rectangular anchorage zone at each end of the beam. The necessity of using such a concrete reinforcement is certainly unquestionable in post-tensioned beams. However in pretensioned elements the stresses induced in concrete in the anchorage zone are smaller than in post-tensioned elements. In this article the stress field in the end zone is calculated numerically and from this analysis the possible reduction of the cross-section of the anchorage block is examined.

Keywords: beams ; prestressing ; pretensioning ; anchorage.

1. Introduction

Prestressing has now become a very popular technique in concrete engineering. Much of the progress that has been made in the field of structural concrete is due to this application and to the important improvements in the knowledge of steel and mainly of concrete material. The calculation, design and construction are presented in various well-known books, where the two transfer techniques, i.e., pretensioning and post-tensioning are also described (Abeles, Bardhan-Roy and Turner 1976, Guyon 1951, 1968, Leonhardt 1964, Lin 1963, Nilson 1987).

Many research studies have been devoted to prestressing, but improvements can still be made in some domains. Among these is the stress transfer in concrete, and particularly in pretensioning.

In post-tensioning the prestressing force is transmitted on a small area of the end section, as the cable is anchored on a metallic plate the area of which is usually less than 10% of that of the concrete end block. These dimensions must however be known precisely, as the ratio of the plate dimensions to the beam dimensions is one of the most important values in determining the bursting force. Therefore the post-tensioning force is close to a point load. The application of such a concentrated load generates an important development of transversal cracking in the anchorage zone, which necessitates a considerable increase of the web thickness with a particularly dense passive reinforcement. This concrete reinforcement seems impossible to avoid in this case, leading

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in most situations to a rectangular section.

The problem is different with pretensioned beams, where the total force to be transmitted is the same, but, nevertheless, the transfer is progressive and therefore takes place in a quite different way. The stress field in the concrete of the anchorage zone is less severe, which means that, compared with post-tensioning, the end block should be less massive and the passive reinforcement noticeably less important. However the transfer mechanism is far more complex in this case, integrating various phenomenons such as sliding, bond and bursting.

Due to this complexity, in European practice, a massive rectangular anchorage zone is usually adopted at each end of the beam, and the stress field is evaluated using a simplified model : the longitudinal load is assumed to be concentrated, but the transmission zone in concrete is longer than in the case of post-tensioning. In the US, end blocks in prestressed I-girders are no longer common practice. However web cracking can sometimes be a problem. The transfer mechanism of prestressing force from strand to concrete must therefore be perfectly understood in order to provide the adequate passive reinforcement.

The transfer of prestress in pretensioning when the tendons are cut loose from the bulkheads is mainly realized by the development of bond stresses between steel and concrete, but other mechanisms take place. A strand is realized from smooth wires, but the external surface of the final product is not smooth, and therefore oblique stresses are transmitted from steel to concrete. On the other hand, after the tendons have been cut loose, the strands have a tendency to expand transversally due to Poisson's effect, and this effect increases when approaching the end section. This transversal expansion is partially prevented by concrete, creating therefore important radial stresses that in addition tend to increase bond.

One of the purposes of this research study was to examine the reduction of the cross-section of the end blocks, i.e., avoiding a rectangular cross section and possibly adopting the same I-shape along the whole beam. To this aim a finite element analysis has been performed. Several degrees of refinement are possible in order to realise such a study. The most elaborated approach consists in discretizing the strand by using finite elements for steel, contact elements at the boundaries, and 3-D elements for the concrete surrounding the strand. This type of approach is highly dependent on the quality of the contact elements that are assumed to represent correctly the bond-slip relation and the damage of concrete surrounding the strand. The stress distribution is then a result of the analysis performed.

This type of modelling has been adopted in Lundgren, *et al.* (2002). In that article a model for the bond between reinforcement and concrete is developed and used in detailed three-dimensional analyses. Both reinforcement bars and concrete are modelled with solid elements. The finite element code used has already interface-elements available, but they require some input data. The article describes the calibration of these input data for the modelling of ribbed bars and strands. Ožbolt, *et al.* (2002) describe the development of a new zero-thickness bond element, which is implemented into a 3-D finite element code. The interaction between tangential and radial stresses is taken into account. The bond element is a two-nodes finite element and it connects a truss/bar finite element (reinforcement) with a three-dimensional solid finite element (concrete). The element displacement field corresponds to a relative movement between the reinforcing bar and the concrete. The basis performance of the model for monotonic and cyclic pull-out load is shown by means of numerical examples.

It has been found that these types of approach were too complicated for the problem envisaged here, i.e., the possible reduction of the cross-section of the anchorage block. Therefore, in this study, the strands have not been represented; only their action on the concrete has been considered, and a linear finite element analysis of the whole beam has been performed.

At the end of the paper, conclusions will be drawn regarding the reduction of the cross-section of the anchorage block and the configuration of reinforcement to be adopted in the transfer zone (Shahawy and Cai 2001).

2. Stress transfer between the strands and the concrete

As already mentioned the stress field developed in concrete is mainly created by bond stresses between the strands and the concrete and by transversal compression. The stress transfer by bond is represented in Fig. 1.

The length l_t is called the prestressing force transmission length and is the required length to transfer by bond the prestressing force from steel to concrete when the tendons are freed from the bulkheads. The distance measured from the end of the beam element corresponds to the point where the tension in the prestressed bar remains constant.

The determination of this prestressing force transmission length and the relation giving the variation of the prestressing force along this length are essential to define a coherent design method of the anchorage zones.

It is very difficult to evaluate this transmission length due to the great number of parameters that have to be considered :

- the external surface characteristic of the bars;
- the type and diameter of the bars;
- the concrete strength;
- the concrete age at the time of release of prestress;
- the position of the bars in the beam.

A physical characteristic that can be measured easily is the displacement g_0 of the bar from the end section inside the anchorage zone after the tension relaxation of the prestressed bars. Most

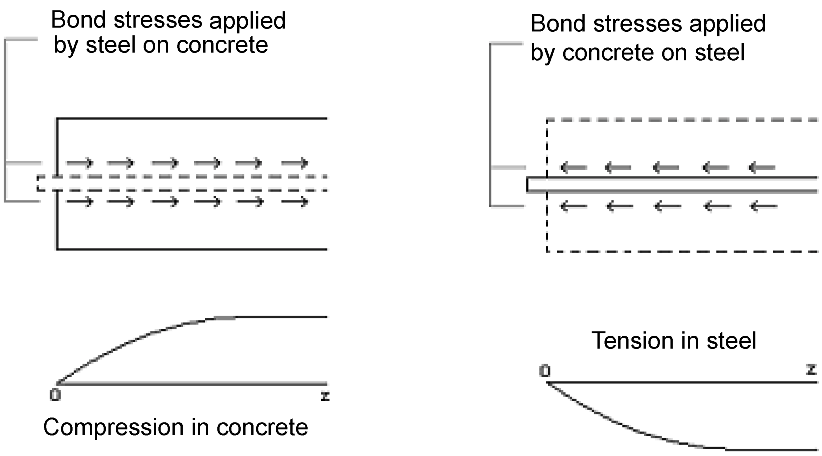

Fig. 1 Stress transfer by bond

authors have tried to evaluate practically the prestressing force transmission length in relation with this characteristic (FIP report 1982, Guyon 1968, Oh and Kim 2000).

The following expression of the prestressing force transmission length can be commonly found:

$$l_t = \frac{2.8E_p}{\sigma_{p0}} g_0 \tag{1}$$

where E_p=elastic modulus of the prestressing steel
σ_{p0}=initial tension in the bar

This expression has been used in this research work as it is generally acknowledged that it is close to the values proposed in most standards. The evaluation of g_0 is made by referring to a standard test giving the displacement of a prestressed bar inside a concrete prismatic sample.

The next step is to determine the distribution of bond stresses developed along this length. Experimental results (Guyon 1951) show that this distribution is close to the one represented in Fig. 2.

The release of the prestressing force creates a dynamic effect and therefore a damaged zone close to the end of the element, in which the concrete strength decreases substantially and does not allow the development of bond between both materials. Beyond this zone, the bond stresses grow very fast, remain approximately constant on some distance and then decrease progressively until they tend to zero at the end of the anchorage zone. The largest bond stresses are found in the beginning of the anchorage zone, and this is due to the radial compression stresses applied by the strand on the concrete (the friction being a function of the compression transversal force).

For the finite element analysis, the bond stresses applied on concrete have been introduced by following the idealized diagram given in Fig. 3.

Three distinct zones can be observed. The first one, developing on a length l_1, is the one in which no bond stresses exist. The second one, developing on a length l_2, is characterized by a constant bond stress (τ_{max}). The third one, developing on a length l_3, corresponds to decreasing values of the bond stresses. An exponential relation has been used to model this decrease.

The bond anchorage of pretensioned tendons in precast elements is examined in Hegger, *et al.* (2002). Based on pull-out test results on HSC, the bond behaviour can be described by separating

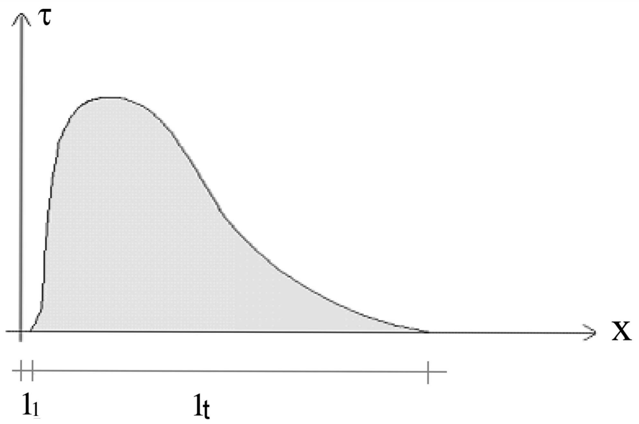

Fig. 2 Distribution of bond stresses along the transmission length

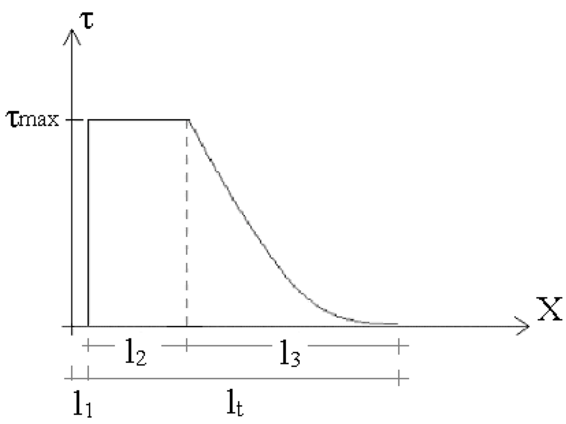

Fig. 3 Idealized distribution of bond stresses

the bonding forces in three parts : a rigid-plastic one, a lateral strain-dependent one and a slip dependent one. The bonding force-slip relation is analysed in connection with these three contributions. The idealized distribution of bond stresses along the transfer length can be derived based on this relation.

The bond stress distribution applied by the strand can therefore be determined if the values of $\tau_{\max}$, l_1, l_2 and l_3 are known.

- Concerning $\tau_{\max}$, it can be shown that, if the bond stresses along the transmission length are assumed to be constant, the expression of the (fictitious) transmission length l_2^* becomes $l_2^* = 2\, E_p g_0/\sigma_{p0}$. Knowing the total prestressing force to be transmitted, the maximum bond stress can thus be determined.
- The value of l_t can be determined using Eq. (1).
- The value of l_1 is given in the standards and depends on the way the prestressing force is released.
- Finally, the value of l_2 and l_3 can be estimated on the basis of the system of the two following equations :

  $l_2 + l_3 = l_t$

  $P_t = P\ (l_2) + P\ (l_3)$

  where P_t = total prestressing force

  $P(l_2)$ = prestressing force transmitted on the length l_2

  $P(l_3)$ = prestressing force transmitted on the length l_3.

The radial stresses can be calculated on the basis of the classical theory of mechanics of materials. A strand has a diameter ϕ before prestressing and ϕ_0 after prestressing. At a distance x from the end section where the prestressing stress $\sigma_p\ (x)$ lies between 0 and σ_{p0}, the strand diameter becomes $\phi(x)$. The radial strain is given by :

$$\frac{\phi(x) - \phi_0}{\phi_0} = \nu_p \varepsilon_p(x) = \nu_p \frac{\Delta\sigma_p(x)}{E_p} \tag{2}$$

with ν_p = transversal contraction (Poisson's) coefficient of the prestressing steel

$\Delta\sigma_p\ (x)$ = stress variation in the strand at the length x from the prestressing stress σ_{p0} to the actual stress state.

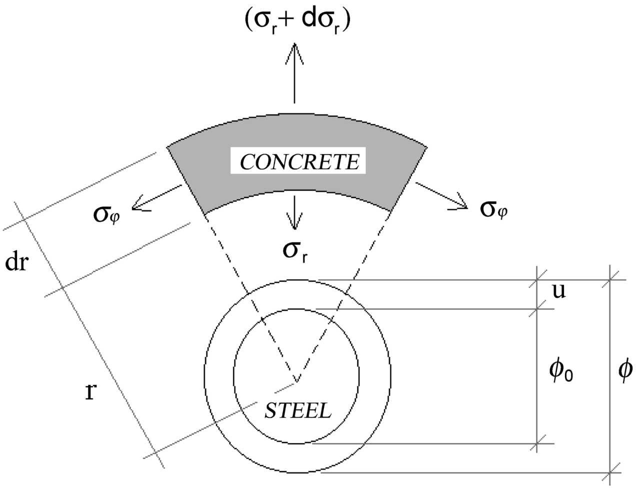

Fig. 4 Determination of radial and circumferencial stresses in concrete

From the expression of the stress strain relationship in polar coordinates, the following expression of the radial and circumferencial stresses can be determined:

$$\sigma_r(x) = -\sigma_\varphi(x) = -\frac{\nu_p}{1+\nu_c}\frac{E_c}{E_p}\Delta\sigma_p(x)\left(\frac{\phi_0}{2r}\right)^2 \tag{3}$$

where σ_r = radial stress
σ_φ = circumferencial stress
ν_c = transversal contraction (Poisson's) coefficient of concrete
E_c = elastic modulus of concrete
r = distance between the point considered and the center of the strand

3. Numerical model used in this study

A linear elastic finite element analysis has been performed by using the program SAPLI 5 developed at the University of Liège. The finite element modelling has been made using a 3 D mesh (see Fig. 5).

In relation with the problem examined here and due to the type of discretization, it is sufficient and even advisable to study a substructure. The dimension of the substructure is governed by the stress regularisation zone, i.e., the length after which the stress distribution in concrete is the one given by the classical beam theory. In pretensioned elements, this regularisation zone is longer than the prestressing force transmission length l_t, but it never exceeds 2 l_t. Therefore the length of the beam substructure L_{mod} has been chosen equal to 2 l_t (cf. Fig. 6).

The definition of the substructure and the modelling of the support conditions are presented in Fig. 6. The distribution of stresses is the same in the two models, but the second one is easier to study.

The prestressing transfer stresses have to be introduced into the finite element model. In order to

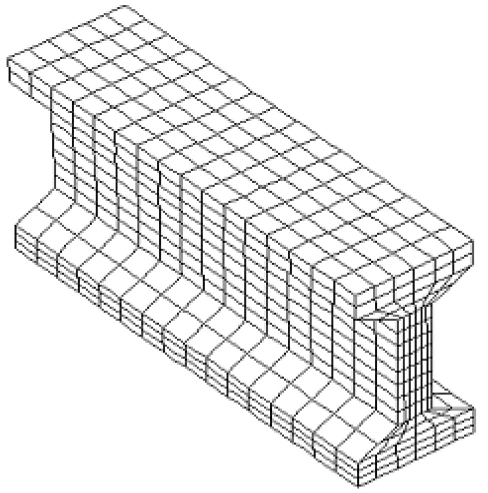

Fig. 5 3 - D mesh used to model the I beam

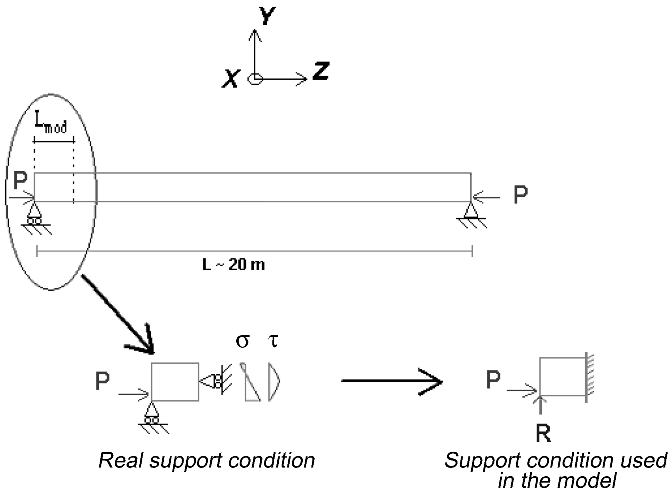

Fig. 6 Definition of the substructure and modelling of the support conditions

simplify the mesh, the strands have not been modelled in this study, which is quite acceptable due to their small size. Their effect on the concrete has been simulated by introducing forces along some load lines corresponding to the edges of the 3D finite elements (see Fig. 7). The length of these load lines is equal to the prestressing force transmission length.

The longitudinal bond stresses can easily be transformed into longitudinal forces by integrating over the perimeter of the cross section of the strand, and these forces are applied along the load line.

Concerning the radial stresses, the situation is more complicated. Indeed, it is impossible to apply radial forces on a load line. In order to solve the problem the radial forces have been applied at the four nodes surrounding perpendicularly the load line and according to the finite element brick discretisation (see Fig. 8). These forces are calculated by integrating the radial stresses over the perimeter of the hatched area and by distributing them proportionally at the four nodes. This

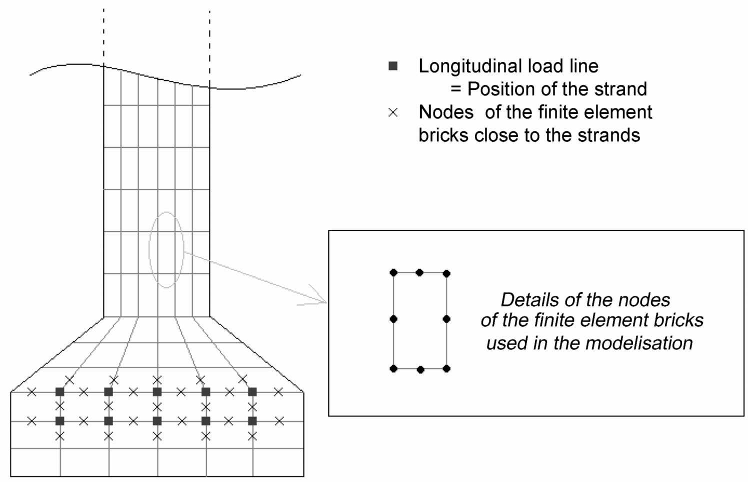

Fig. 7 Discretisation of the cross section and position of the strands

simplified modelling is quite satisfactory for the study of the general radial behaviour of the strands.

4. Practical example

In this research work, four specific beams have been studied. Results are presented here for a I 1200/400 mm beam with a span of 32 m (see Fig. 9).

The distribution of longitudinal stresses in a plane located at mid-width of the cross section of the beam is shown in Fig. 10.

The concrete used in this study has a compression strength of 24 N/mm². As could be expected this strength is widely exceeded here. Sheathing of several strands on some length is essential. Calculations permit to find a configuration of the active reinforcement for which the concrete

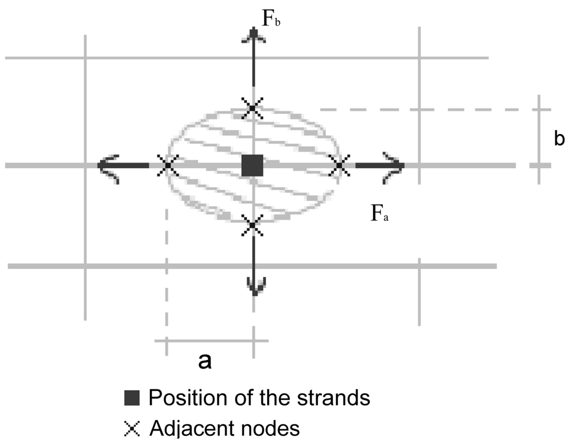

Fig. 8 Simplified modelling adopted for the transmission of radial stresses

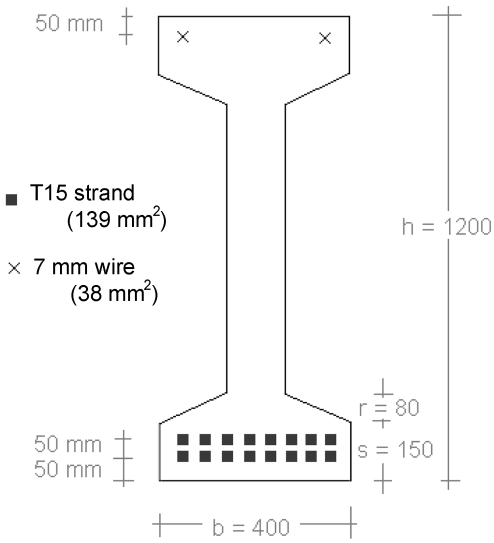

Fig. 9 Cross-section of the I beam analysed here

stresses remain acceptable. In this example, the sheathing of 6 strands on a length of 10 meters has been adopted (see Fig. 11). This figure shows the stress field along the length of the beam. The first area (between 0 and 1.89 m) corresponds to the end block and the second one (between 10 and 11.89 m) corresponds to the anchorage of the 6 sheathed strands. The stress field between these two areas and beyond the second area is not influenced by local effect of pretressing. The stresses can be evaluated easily using the classical beam theory. In this case they are calculated by considering the prestressing force and the self weight of the beam on a span of 32 m.

After determining the configuration of the strands in the anchorage zone in order to respect the longitudinal stress level, the vertical tensile stresses (σ_{yy}) and the shear stresses (τ_{yz}) can be evaluated. Using Mohr's circle, the principal stresses (σ_{s1} and σ_{s2}) can then be calculated (Fig. 12). From this the resultant transversal force per unit length can be evaluated along the length of the

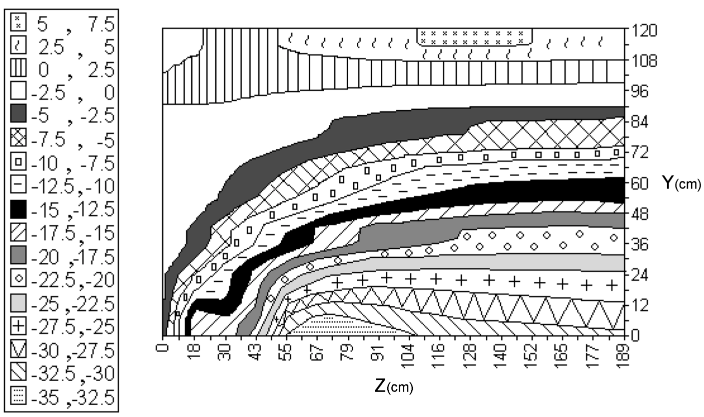

Fig. 10 Longitudinal stresses (N/mm² positive in tension)

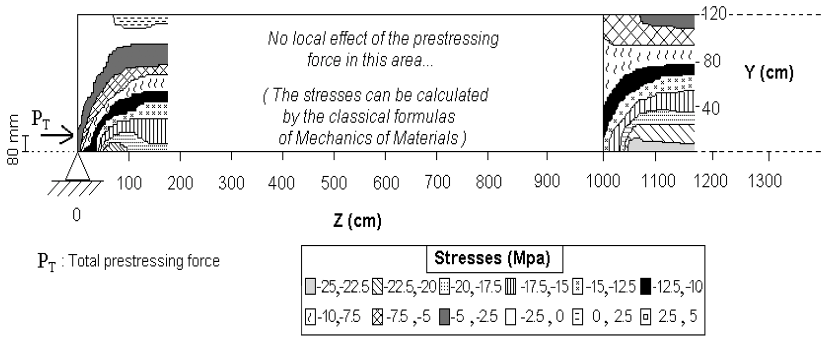

Fig. 11 Longitudinal stresses with sheathing of several strands

anchorage zone (Fig. 13), where tension is considered positive and compression negative. This resultant force is in fact the vertical projection of the principal stresses σ_{s1} multiplied by the width of the beam web. It can be seen that tensile stresses develop in the first part of the anchorage zone. Vertical stirrups can therefore be designed and positioned in this area, considering that concrete carries zero tension and steel all the bursting force. In this case, the total transversal force to be considered in the stirrups is approximately equal to 175 kN for a total prestressing force of 3085.8 kN. As can be observed in Fig. 13 this stress or force distribution can be simplified into a triangle.

Marshall and Mattock (1962) performed an experimental study of the tensile transversal forces in pretensioned beams. Those tests were devoted to non-symmetrical I beams prestressed by tendons of various diameters, a few of them being situated in the upper flange. The tensile stresses were measured by strain-gauges placed on the stirrups.

Experimental results have shown that the tensile stresses decrease more or less linearly from the end surface, which is in agreement with the results shown in Fig. 13. Furthermore they have found that these stresses drop to zero at a distance approximately equal to $h/3$, where h is the total height of the beam. In Fig. 13 this distance is approximately equal to 0.4 h.

These authors also propose a formula for the total transversal force F_t to be considered for the design of the stirrups :

$$F_t = 0.0212P\frac{h}{l_t} \tag{4}$$

By applying this formula to the example studied here, the value $F_t = 136.3$ kN is found. The value obtained by our numerical simulation (175 kN) is therefore on the safe side.

Guyon (1968) has analysed the experimental results obtained by Marshall and Mattock. He found

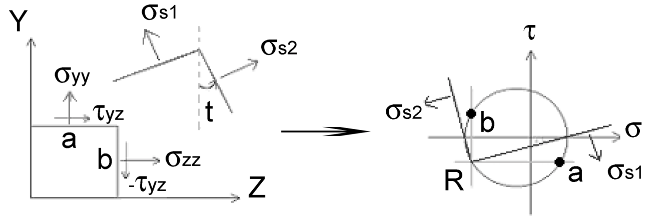

Fig. 12 Determination of the principal stresses

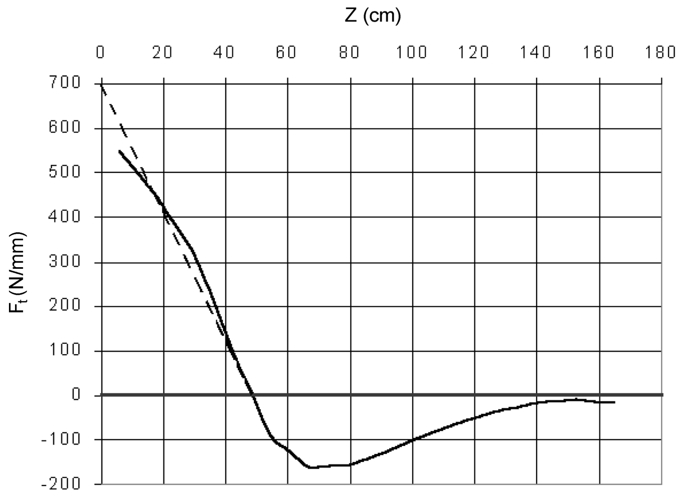

Fig. 13 Resultant transversal force along the anchorage zone

that these results display a very large scatter. Therefore he proposes to place the stirrups along a distance of 0.4 h from the end of the beam and to adopt a transversal force 40 to 50% higher than the one given by Eq. (4), which is in agreement with the results of the numerical simulation.

From the stress distribution obtained from our numerical results, the active and passive reinforcement can be determined. The design is based on a service prestress force with the steel at an allowable stress. In all the passive reinforcements the stress is limited to 60% of its characteristic yield strength, equal to 500 MPa in this case. It can be seen that the rectangular anchorage zone is not needed. Fig. 14 shows the reinforcement arrangement for the I 1200/400 beam.

5. Conclusions

This research work was devoted to the analysis of the stress transfer in the anchorage zone of

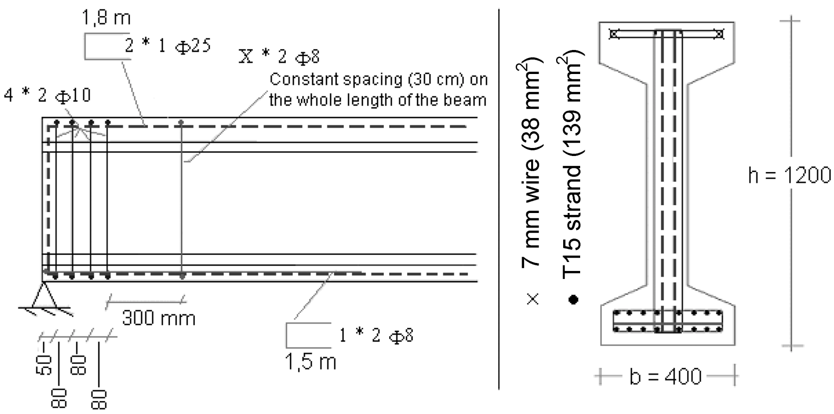

Fig. 14 Reinforcement arrangement

pretensioned beams and to the possible reduction of massivity of the concrete zone.

Simplified relations have been used for the distribution of bond stresses and transversal compression stresses along the prestressing force transmission length of the strands.

From these relations a linear elastic finite element analysis has been performed for the determination of the stress field in the concrete of the anchorage zone and for the design of the stirrups.

On the basis of these calculations a rectangular concrete reinforcement at the end of the beam appears no longer necessary and it seems possible to adopt the same I section along the whole length. It is important to note that sheathing was necessary in the example studied.

This has various advantages, such as decrease of weight, improvement of aesthetics and simplification of formwork, meaning gain in time and money.

Additional research studies will start soon at the University of Liège to refine the analysis. A non linear finite element model should be adopted taking into account non linear behaviour of concrete and cracking.

Furthermore, in relation with the experience gained at the University of Liège in the field of structural analysis under fire conditions, the behaviour of the end zones of pretensioned beams subjected to fire should also be examined.

References

Abeles, P. W., Bardhan-Roy, B. K., Turner, F. H. (1976), *Prestressed Concrete Designer's Handbook* Second edition, Viewpoint Publication Cement and Concrete Association, Wexham Springs.

FIP report (1982), "Report on prestressing steel : Test for the determination of tendon transmission length under static conditions", Cement and Concrete Association, Wexham Springs.

Guyon, Y. (1951), *Béton précontraint, étude théorique et expérimentale - Tome 1*, Eyrolles, Paris.

Guyon, Y. (1968), *Constructions en béton précontraint, classes états limites - Tome 2: étude de la poutre*, Eyrolles, Paris.

Leonhardt, F. (1964), *Prestressed Concrete, Design and Construction,* Second edition, Wilhelm Ernst & Sohn, Berlin Munich.

Lin, T. Y. (1963), *Design of Prestressed Concrete Structures,* Second edition, John Wiley & Sons, New York.

Nilson, A. H. (1987), *Design of Prestressed Concrete,* Second edition, John Wiley & Sons, New York.

Oh, B. H. and Kim, E. S. (2000), "Realistic evaluation of transfer lengths in pretensioned, prestressed concrete members", *ACI Struct. J.*, **97**(6), 821-830.

Lundgren, K., Gustavson, R. and Magnusson, J. (2002), "Finite element modelling as a tool to understand the bond mechanisms", *Bond in Concrete from Research to Standards*, Fib, Budapest, 27-34.

Ožbolt, J., Lettow, S. and Kožar, I. (2002), "Discrete bond element for 3D finite element analysis of reinforced concrete structures", *Bond in Concrete from Research to Standards*, Fib, Budapest, 9-19.

Shahawy, M. and Cai, C. S. (2001), "Enhancement of the performance of prestressed concrete girders using strand anchorage", *PCI Journal*, **46**(5), 82-88.

Hegger, J., Kommer, B. and Nitsch, A. (2002), "Bond anchorage of pretensioned tendons in precast elements made out of high performance concrete (HPC)", *Bond in Concrete from Research to Standards*, Fib, Budapest, 103 - 110.

Marshall, W. T. and Mattock, A. H. (1962), "Control of horizontal cracking in the ends of pretensioned prestressed concrete girders", *PCI Journal*, **7**, 56-74.

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