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Analytical investigation of the surface effects on nonlocal vibration behavior of nanosize curved beams

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Abstract. This paper deals with free vibration analysis of nanosize rings and arches with consideration of surface effects. The Gurtin-Murdach model is employed for incorporating the surface effect parameters including surface density, while the small scale effect is taken into consideration based on nonlocal elasticity theory of Eringen. An analytical Navier solution is presented to solve the governing equations of motions. Comparison between results of the present work and those available in the literature shows the accuracy of this method. It is explicitly shown that the vibration characteristics of the curved nanosize beams are significantly influenced by the surface density effects. Moreover, it is shown that by increasing the nonlocal parameter, the influence of surface density reduce to zero, and the natural frequency reaches its classical value. Numerical results are presented to serve as benchmarks for future analyses of nanosize rings and arches.

Keywords: radial vibration analysis; nanosize rings and arches; curved nanobeams; surface effects

1. Introduction

Nano materials are attracting many researchers over the recent years due to their extraordinary properties. Both experimental and atomistic modeling studies show that when the dimensions of structures become very small, the size effect gains important. Due to this fact, the size effect plays an important role on the mechanical behavior of micro- and nanostructures (Şimşek 2014). Among various nano structures, nanobeams have more important applications (Daulton et al. 2010, Hu et al. 2010). A nonlocal beam theory is proposed by Thai (Thai 2012), for bending, buckling, and vibration of nanobeams. Closed-form solutions of deflection, buckling load, and natural frequency obtained for simply supported nanobeams. However, the nonlinear vibration of the piezoelectric nanobeams based on the nonlocal theory and Timoshenko beam theory has been investigated by Ke *et al*. (2012). In addition Murmu and Adhikari (2010), have investigated the nonlocal transverse vibration of double-nanobeam-system. Also Eltaher *et al*. (2012), have presented free vibration analysis of functionally graded (FG) size-dependent nanobeams using finite element method. Because the nanobeams have the high proportion of the surface to volume ratio, the surface stress effects has important role in their mechanics behavior of these structures. Hence Gurtin and Murdach (1978) have considered surface stress effects. In this theory the surface is

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considered as a part of (nonphysical) the two-dimensional with zero thickness (mathematically) which has covered the total volume. This theory has used in many researches about nanobeams.

The nonlinear flexural vibrations of micro and nanobeams in presence of surface effects have been studied within the framework of Euler–Bernoulli beam theory including the von Kármán geometric nonlinearity by Gheshlaghi and Hasheminejad (2011). In this research, exact solution has been obtained for the natural frequencies of a simply-supported nanobeam in terms of the Jacobi elliptic functions by using the free vibration modes of the corresponding linear problem. Nevertheless, nonlinear free vibration of functionally graded nanobeams has been investigated by Sarabiani and Haeri-Yazdi (2013). In this study within the framework of Euler–Bernoulli beam theory including the von Kármán geometric nonlinearity. In addition, Sahmani et al, have investigated Surface energy effects on the free vibration characteristics of post buckled third-order shear deformable nanobeams (Hosseini-Hashemi and Nazemnezhad 2014a) and they have been studied Surface effects on the nonlinear forced vibration response of third-order shear deformable nanobeams (Hosseini-Hashemi and Nazemnezhad 2014b). In these papers they have been used to Gurtin-Murdach elasticity theory, using of third-order shear deformation beam theory. Furthermore, The nonlinear free vibration of nanobeams with considering surface effects (surface elasticity, tension and density) has been studied by Nazemnezhad et al. (2012) within framework Euler–Bernoulli beam model including the von kármán geometric nonlinearity. However, Hosseini-Hashemi and Nazemnezhad (2013) have presented Nonlinear free vibration of simply supported FG nanoscale beams with considering surface effects (surface elasticity, tension and density) and in this research, balance condition between the FG nanobeam bulk and its surfaces has been studied. As well as, Ansari *et al*. (2014) have investigated nonlinear forced vibration characteristics of nanobeams including surface stress effect. In this study, a new formulation of the Timoshenko beam theory has been developed through the Gurtin–Murdoch elasticity theory in which the effect of surface stress has been incorporated. Moreover, The surface and nonlocal effects on the nonlinear flexural free vibrations of elastically supported non-uniform cross section nanobeams have been investigated by Malekzadeh and Shojaee (2013) simultaneously.in this paper, The formulations have been derived based on both Euler–Bernoulli beam theory (EBT) and Timoshenko beam theory (TBT) independently using Hamilton's principle in conjunction with Eringen's nonlocal elasticity theory.

All above articles are implemented on straight beams. In recent years vibration of curved nanobeams and nanorings, have been worked in many empirical experiments and dynamic molecular simulations (Wang and Duan 2008). Hence some researchers are interested in studding of vibration curved nanobeams.

Yan and Jiang (2011) have investigated the electromechanical response of a curved piezoelectric nanobeam with the consideration of surface effects. In addition, a new numerical technique, the differential quadrature method has been developed for dynamic analysis of the nanobeams in the polar coordinate system by Kananipour et al. (2014). Moreover, Khater et al. (2014) have investigated the effect of surface energy and thermal loading on the static stability of nanowires. In this research, curved fixed–fixed Euler-Bernoulli nanobeams has been considered and Gurtin-Murdoch theory has been used to represent surface effects. The model takes into account both von Kármán strain and axial strains. Also Wang and Duan (2008) have surveyed the free vibration problem of nanorings/arches. In this article defects and elastic boundary conditions were investigated. The small length scale effect lowered the vibration frequencies. In addition, investigating surface effects on thermomechanical behavior of embedded circular curved nanosize beams has been presented by Ebrahimi and Daman (2016b). They also presented the radial vibration of embedded double-curved-nanobeam-systems (Ebrahimi and Daman 2016b). Recently Ebrahimi and Barati (2016a, b, c, d, e, f, h) presented vibration and buckling analysis of nanosize FG beams and plates subjected to magneto-electro-thermal loadings. They also investigated wave propagation behavior of FG nanobeams based on classical and higher order shear deformation theories (Ebrahimi and Barati 2016c, g, i, Ebrahimi *et al*. 2016a, b, c, d). Nevertheless, explicit solution has been shown for size and geometry dependent free vibration of curved nanobeams with consideration of surface effects by Assadi and Farshi (2011).

To the best of the author's knowledge, there has been no record or any study reported regarding the vibration behavior of curved nanobeams with surface density and considering nonlocal parameter in the literature. Therefore, there is a strong scientific need to understand the vibration behavior of curved nanobeams with surface density effect. The aim of this research is to survey the effects of surface density, nonlocality and opening angles on vibrations and natural frequencies of curved nanobeams. In this regard, the curved nanobeams have been modeled in the framework of Euler-Bernoulli beam theory. An analytical Navier solution is presented to solve the governing equations of motions. Comparison between results of the present work and those available in the literature shows the accuracy of this method.

2. Problem statements

In-plane free vibration of a curved nanobeam is considered as shown in Fig. 1. The radius curvature and thickness are considered R and h respectively. Additional surface effects are supposed for all the external surfaces. The dynamic equilibrium equations for a curved Euler-Bernoulli beam, are given as

$$\begin{aligned}
&\frac{\partial V}{\partial \theta}+P=\rho A R\,\frac{\partial^2 u_r}{\partial t^2}+Rb\,\rho^s\left(\frac{\partial^2 u_r^+}{\partial t^2}+\frac{\partial^2 u_r^-}{\partial t^2}\right)\\
&\frac{\partial P}{\partial \theta}-V=\rho A R\,\frac{\partial^2 u_\theta}{\partial t^2}+Rb\,\rho^s\left(\frac{\partial^2 u_\theta^+}{\partial t^2}+\frac{\partial^2 u_\theta^-}{\partial t^2}\right);\frac{\partial M}{\partial \theta}+RV=0
\end{aligned}\tag{1}$$

where $F(\theta, t)$ is the shearing force, $P(\theta, t)$ is the tensile force, A is the cross sectional area, ρ is the mass density, ρ^s is the surface density of the nanoring and b is the width of nanoring In Eq. (1). It

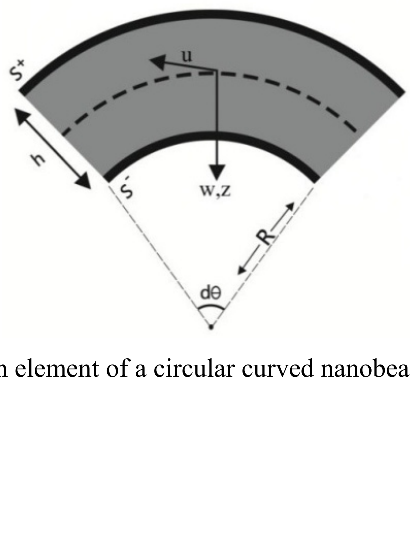


Fig. 1 Geometry of an element of a circular curved nanobeam with surface layers

should be notice the displacement components of the surface property must satisfy the following relations (Rao 2007)

$$\ddot{u}_r^+ = \ddot{u}_r^- = \ddot{u}; \qquad \ddot{u}_\theta^+ + \ddot{u}_\theta^- = 2\ddot{u}_\theta \tag{2}$$

By using Eq. (2) and substituting into Eq. (1) the equilibrium equations can be determined as follows

$$P - \frac{1}{R}\frac{\partial^2 M}{\partial\theta^2} = \left(\rho A R + 2Rb\rho^s\right)\frac{\partial^2 u_r}{\partial t^2}; \qquad \frac{\partial P}{\partial\theta} + \frac{1}{R}\frac{\partial M}{\partial\theta} = \left(\rho A R + 2Rb\rho^s\right)\frac{\partial^2 u_\theta}{\partial t^2} \tag{3}$$

The normal stress resultant P from Eq. (3) should be vanished. Therefore, obtains the relation between radial displacement and bending moment such as Eq. (4)

$$\frac{1}{R}\left(\frac{\partial^2 M}{\partial\theta^2} + \frac{\partial^4 M}{\partial\theta^4}\right) + R\left(\frac{\partial p}{\partial\theta} - \frac{\partial^2 f}{\partial\theta^2}\right) = \left(\rho AR + 2Rb\rho^s\right)\left(\frac{\partial^2 u_r}{\partial t^2} - \frac{\partial^4 u_r}{\partial t^2\partial\theta^2}\right) \tag{4}$$

Using the nonlocal constitutive stress-strain relation, the motion equation of a nonlocal Euler-Bernoulli curved beam can be written as

$$\frac{\mathrm{EI}}{\mathrm{R}^3}\left(\frac{\partial^6 \mathrm{u_r}}{\partial\theta^6} + 2\frac{\partial^4 \mathrm{u_r}}{\partial\theta^4} + \frac{\partial^2 \mathrm{u_r}}{\partial\theta^2}\right) = \left(\rho\mathrm{AR} + 2\mathrm{Rb}\rho^{\mathrm{s}}\right)\left(\frac{\partial^2 \mathrm{u_r}}{\partial \mathrm{t}^2} - \frac{\partial^4 \mathrm{u_r}}{\partial\theta^2\partial \mathrm{t}^2} - \frac{(e_0 a)^2}{R^2}\frac{\partial^4 \mathrm{u_r}}{\partial\theta^2\partial \mathrm{t}^2} + \frac{(e_0 a)^2}{R^2}\frac{\partial^6 \mathrm{u_r}}{\partial\theta^4\partial \mathrm{t}^2}\right) \tag{5}$$

The general solution of Eq. (5) is written as

$$u_r = \overline{u}_r(\theta)e^{i(\omega_n t + \varphi)} \tag{6}$$

which ω_n is the natural frequency, $\overline{u}_r(\theta)$ is the corresponding deformation shape of the nanorings. Substituting the Eq. (6) into Eq. (5) yields

$$\frac{\partial^6 \overline{u}_r}{\partial\theta^6} + 2\frac{\partial^4 \overline{u}_r}{\partial\theta^4} + \frac{\partial^2 \overline{u}_r}{\partial\theta^2} + \beta_n\left(\overline{u}_r - \frac{\partial^2 \overline{u}_r}{\partial\theta^2}\right) = 0; \quad \beta_n^2 = \frac{2\rho AR^4 + 4R^4 b\rho^s}{2EI}\omega_n^2 \tag{7}$$

The nanorings with total central angle α and simply-simply supported, can be solved with analytical method such as

$$u_r = \sin\left(\frac{n\pi}{\alpha}\theta\right)e^{i\omega_n t} \tag{8}$$

Substituting Eq. (8) into Eq. (5), the dimensionless frequencies of the nanorings including surface density effect, can be determined as

$$\Omega_n^2 = \frac{\frac{EI}{R^3}\left(\lambda_n^6 - 2\lambda_n^4 + \lambda_n^2\right)}{\left(\rho AR + 2Rb\rho^s\right)\left(1 + \lambda_n^2 + \frac{(e_0 a)^2}{R^2}\lambda_n^2 + \frac{(e_0 a)^2}{R^2}\lambda_n^4\right)}\frac{(R\alpha)^4\rho A}{EI}; \lambda_n = \frac{n\pi}{\alpha} \tag{9}$$

The dimensionless natural frequency of curved beam without surface density effect, can be written as

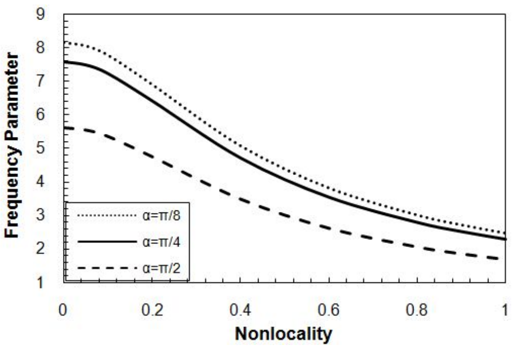
$$\Omega_{n0}^2 = \frac{\mathrm{EI}\left(\lambda_n^6 - 2\lambda_n^4 + \lambda_n^2\right)}{\left(\rho A R^4\right)\left(1 + \lambda_n^2 + \frac{(e_0 a)^2}{R^2}\lambda_n^2 + \frac{(e_0 a)^2}{R^2}\lambda_n^4\right)} \frac{\left(R\alpha\right)^4 \rho A}{EI}; \lambda_n = \frac{n\pi}{\alpha} \tag{10}$$

3. Numerical results

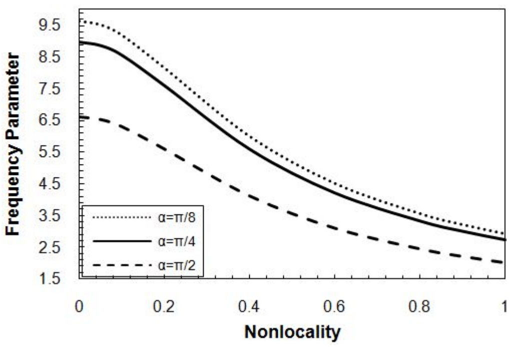
In this section, the dimensionless natural frequency of curved nanobeam (Ω) is compared with the dimensionless natural frequency of straight double nanobeam subjected to different size scale coefficient (μ) in Table 1. In our survey we extremely decrease curvature amplitude for simulating

Table 1 Dimensionless natural frequency of straight double nanobeam system subjected to different size scale coefficients

μ	Present	Murmu and Adhikari 2010
0	9.8696	9.8696
0.1	9.4158	9.4158
0.2	8.3569	8.3569
0.3	7.1823	7.1823
0.4	6.1455	6.1455
0.5	5.3002	5.3002
0.6	4.6253	4.6253
0.7	4.0854	4.0854
0.8	3.6487	3.6487
0.9	3.2909	3.2909
1	2.9935	2.9935



(a) With surface density



(b) Without surface density

Fig. 2 Dimensionless natural frequency of curved nanobeams with and without surface density respect to nonlocality μ for various opening angles to the straight nanobeam like reference (Murmu and Adhikari 2010). As it shown in Table 2, It is observed that the present results agree very well with the given by Ref (Murmu and Adhikari 2010) due to exact solution in this survey that can represent the validity of our research.

3.1 Effect of nonlocal parameter on vibration frequencies with various opening angle

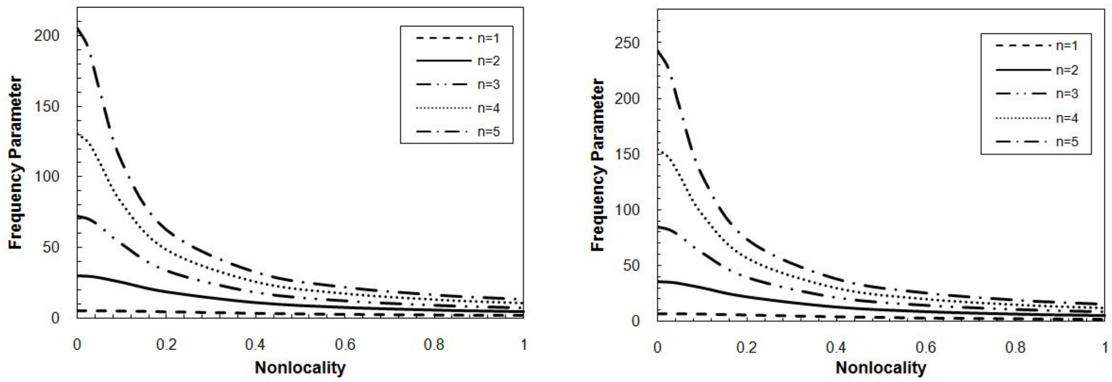
In this section, the bulk elastic properties are E = 177.3 Gpa, ρ = 7000 (Kg/m^3) and ν = 0.27 for which the surface density is given as $\rho^s = 7\times10^{-6}$ (Kg/m^2) (Gurtin and Murdoch 1978). In this section, the effect of nonlocal parameter is investigated on vibration curved nanobeam with and without surface density effect with various opening angles. For this purpose the geometric parameters are selected for the results by the present model in Figs. 2 and 3. Hence the thickness of curved nanobeam is assumed constant and it equal to 5 nm. It can be clearly seen, from Figs. 2 and 3. That the dimensionless natural frequency decrease with the increase of the nonlocal parameter. Figs. 2 and 3 reveal that nonlocality is a prominent parameter in dynamics behavior of curved nanobeam and cannot be neglected from this parameter in dynamics study of size dependent nanobeams. From results of Table 2, increasing the nonlocality parameter yields the reduction in dimensionless frequencies for every mode numbers and opening angles change, which highlights the significance of the nonlocal effect. In addition it can be observed that dimensionless natural frequencies decrease with increase opening angles in total cases of nonlocality and mode numbers. Hence it can be seen that nonlocal effect plays prominent role in natural frequencies of curved nanobeams.

3.2 Effect of thickness on vibration of curved nanobeam with density surface

In this subsection, the effect of the thickness (h) with various opening angles on frequency parameter with surface density effects is examined. The same material and geometric parameters

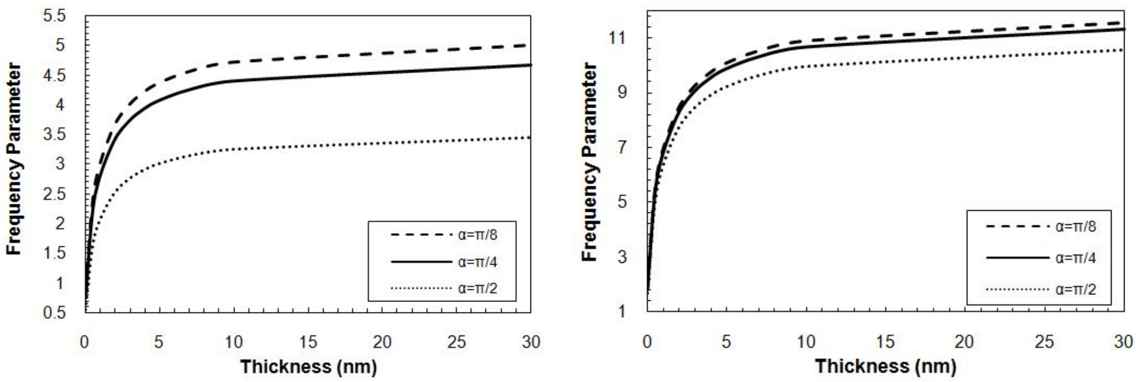
Table 2 Opening angle and nonlocality effect on first three dimensionless frequency of S-S curved nanobeam with surface density (h = 5 nm, $\rho^s = 7\times10^{-6}$ (Kg/m^2))

μ	$n = 1$			$n = 2$			$n = 3$		
	Opening angle			Opening angle			Opening angle		
	$\pi/8$	$\pi/4$	$\pi/2$	$\pi/8$	$\pi/4$	$\pi/2$	$\pi/8$	$\pi/4$	$\pi/2$
0	8.1476	7.5865	5.5955	33.1703	32.5904	30.3461	74.8767	74.2932	71.9936
0.1	7.7730	7.2377	5.3383	28.8664	27.5954	25.6950	54.4898	54.0652	52.3917
0.2	6.8988	6.4238	4.7379	20.6544	20.5933	18.8958	35.091	34.8175	33.7398
0.3	5.9292	5.5209	4.0720	15.5453	15.2735	14.2217	24.9667	24.7721	24.0054
0.4	5.0733	4.7239	3.4842	12.2630	12.0486	11.2189	19.1978	19.0481	18.4586
0.5	4.3755	4.0742	3.0050	10.0610	9.8851	9.2044	15.5432	15.4221	14.9447
0.6	3.8184	3.5554	2.6223	8.5046	8.3559	7.7805	13.0388	12.9372	12.5368
0.7	3.3726	3.1404	2.3162	7.3540	7.2255	6.7279	11.2214	11.1339	10.7893
0.8	3.0121	2.8047	2.0687	6.4722	6.3590	5.9211	9.8446	9.7679	9.4656
0.9	2.7167	2.5296	1.8658	5.7762	5.6752	5.2844	8.7667	8.6984	8.4291
1	2.4713	2.3011	1.6972	5.2136	5.1224	4.7697	7.9003	7.8387	7.5961



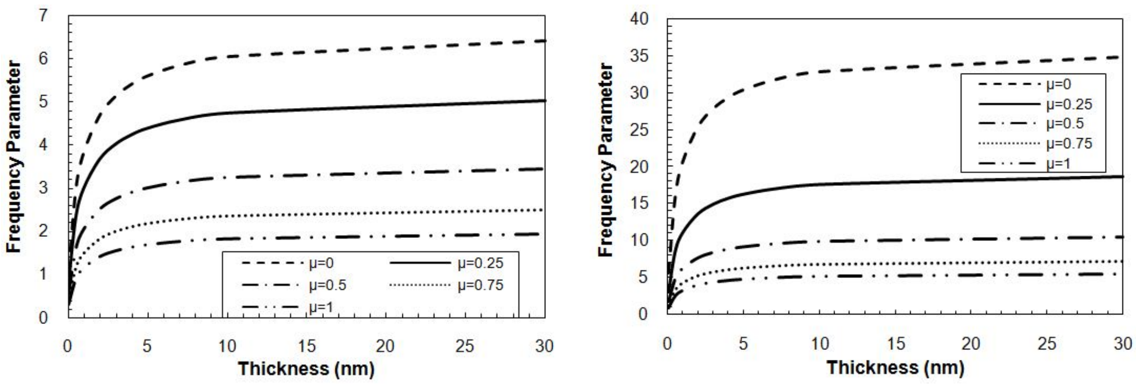
(a) With surface density (b) Without surface density

Fig. 3 Dimensionless natural frequency of curved nanobeams with and without surface density versus nonlocality μ for higher mode numbers



(a) n = 1 (b) n = 2

Fig. 4 First two dimensionless natural frequency versus thickness h for different opening angles



(a) n = 1 (b) n = 2

Fig. 5 First two dimensionless natural frequency respect to thickness h for different nonlocalities are selected is used for the curved nanobeams by the present model in Figs. 4 and 5. In addition the nonlocal parameter is assumed as $\mu = 0.5$. Moreover the opening angle is considered as ($\alpha = \pi$ / 2). To highlight the thickness effect on the vibration of the curved nanobeams, the dispersion curves are presented in Figs. 4. and 5. It is clearly seen that, by increasing thickness h, the dimen

Table 3 Opening angle and thickness effects of first three dimensionless frequency of S-S curved nanobeam with surface density for various nonlocalities ($\rho^s = 7\times10^{-6}$ (Kg/m^2))

μ	Thickness h (nm)	$n = 1$			$n = 2$			$n = 3$		
		Opening angle			Opening angle			Opening angle		
		$\pi/8$	$\pi/4$	$\pi/2$	$\pi/8$	$\pi/4$	$\pi/2$	$\pi/8$	$\pi/4$	$\pi/2$
0	5	8.1476	7.5865	5.5955	33.1703	32.5904	30.3461	74.8767	74.2932	71.9936
	10	8.8004	8.1944	6.0439	35.828	35.2016	32.7775	80.8761	80.2458	77.7620
	15	9.0555	8.4319	6.2191	36.8667	36.2222	33.7278	83.2208	82.5723	80.0165
	20	9.1917	8.5587	6.3126	37.4211	36.7669	34.2350	84.4724	83.8140	81.2198
	25	9.2764	8.6376	6.3708	37.7660	37.1058	34.5505	85.2510	84.5865	81.9684
	30	9.3342	8.6914	6.4105	38.0013	37.3370	34.7658	85.7821	85.1135	82.4791
0.25	5	6.4076	5.9663	4.4005	17.8134	17.5020	16.2967	29.2531	29.0251	28.1267
	10	6.9210	6.4444	4.7531	19.2407	18.9043	17.6025	31.5969	31.3507	30.3803
	15	7.1216	6.6312	4.8909	19.7985	19.4524	18.1128	32.5130	32.2596	31.2611
	20	7.2287	6.7310	4.9645	20.0962	19.7449	18.3852	33.0019	32.7447	31.7312
	25	7.2954	6.7930	5.0102	20.2815	19.9269	18.5546	33.3061	33.0465	32.0237
	30	7.3408	6.8353	5.0415	20.4078	20.0511	18.6703	33.5136	33.2524	32.2232
0.5	5	4.3755	4.0742	3.0050	10.0610	9.8851	9.2044	15.5432	15.4221	14.9447
	10	4.7261	4.4006	3.2457	10.8672	10.6772	9.9419	16.7886	16.6578	16.1422
	15	4.8631	4.5282	3.3398	11.1822	10.9867	10.2301	17.2753	17.1407	16.6101
	20	4.9362	4.5963	3.3901	11.3504	11.1519	10.3840	17.5351	17.3985	16.8599
	25	4.9817	4.6387	3.4213	11.4550	11.2547	10.4797	17.6967	17.5588	17.0153
	30	5.0128	4.6676	3.4426	11.5264	11.3248	10.5450	17.807	17.6682	17.1213
0.75	5	3.1813	2.9639	2.1861	6.8856	6.7652	6.2994	10.4885	10.4067	10.0846
	10	3.4382	3.2014	2.3612	7.4373	7.3073	6.8041	11.3288	11.2405	10.8926
	15	3.5378	3.2942	2.4297	7.6529	7.5191	7.0013	11.6573	11.5664	11.2084
	20	3.5910	3.3438	2.4662	7.7680	7.6322	7.1066	11.8326	11.7407	11.3770
	25	3.6241	3.3746	2.4890	7.8396	7.7026	7.1721	11.9416	11.8486	11.4818
	30	3.6467	3.3956	2.5045	7.8885	7.7506	7.2168	12.0160	11.9224	11.5534
1	5	2.4713	2.3011	1.6972	5.2136	5.1224	4.7697	7.9003	7.8387	7.5961
	10	2.6693	2.4855	1.8332	5.6313	5.5329	5.1519	8.5333	8.4668	8.2047
	15	2.7467	2.5575	1.8863	5.7646	5.6933	5.3012	8.7807	8.7123	8.4426
	20	2.7880	2.5960	1.9147	5.8817	5.7789	5.3809	8.9128	8.8433	8.5696
	25	2.8137	2.6199	1.9324	5.9359	5.8322	5.4305	8.9949	8.9248	8.6486
	30	2.8312	2.6362	1.9444	5.9729	5.8685	5.4644	9.0510	8.9804	8.7024

sionless natural frequencies tend to increase.

Moreover Figs. 4 and 5, reveal that, the values of thickness is effective on dynamics behavior of curved nanobeam with surface density effect with all nonlocal parameter and opening angles. Comparing the frequency values for curved nanobeams with various thicknesses, presented in Table 3. Different nonlocal parameters and opening angles are investigated in Table 3 for the first three dimensionless natural frequencies.

The dimensionless natural frequency increase with increase the values of beam thicknesses. The effect of curvatures in nanobeams plays an important role for design and determining the natural frequencies. Indeed the straight beam in macro and micro size is not exist. Hence we should consider the effect of curvature for determining the natural frequencies such as presented in Table 3.

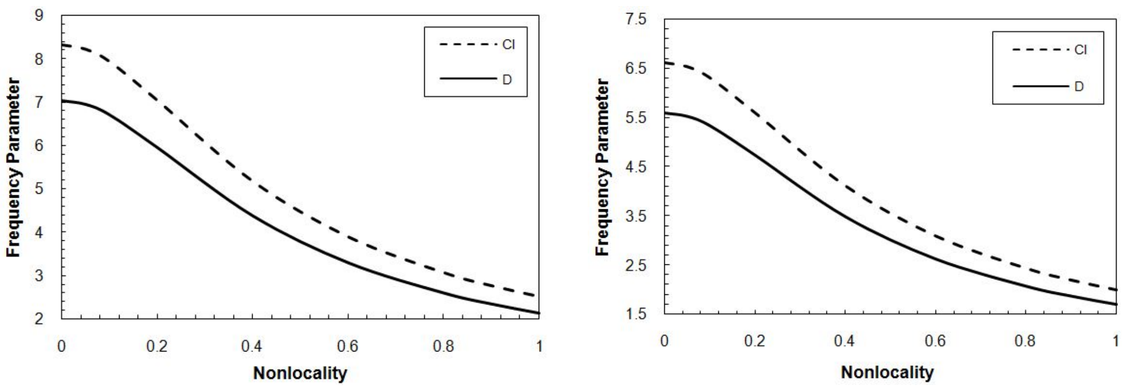
3.3 Comparison of vibration behavior for classical curved nanobeam and curved nanobeam with surface density

To highlight the surface effects, on the vibration frequencies of the curved nanobeams, the dispersion curves are presented in Fig. 6.

Fig. 6 reveals that considering the surface density effect decrease the dimensionless natural frequency of the curved nanobeam. Table 4 is also presented for comparing the effect of surface density on vibration of curved nanobeams. Considering the surface density parameter, will cause different behavior in natural frequencies. Assuming surface density, reduces the amount of frequency parameter. According to Table 4 the frequency parameter will also decreases by considering the surface density.

3.4 Investigation of frequency shift for curved nanobeams with and without surface density

The frequency shift due to surface density, i.e., $\Omega_n - \Omega_n^*$, is illustrated in the diagram of Fig. 7 and Fig. 8 for higher mode numbers. where Ω_n^* is a dimensionless natural frequency without surface density. It is concluded that deviation of the modified results from classical ones become



(a) $\alpha = \pi/3$ (b) $\alpha = \pi/2$

Fig. 6 Comparison of vibration behavior between classical curved nanobeam and curved nanobeam with surface density; Cl: Classic; D: Surface Density

Table 4 Comparison of dimensionless natural frequencies between classical curved nanobeam and curved nanobeam with surface density ($\rho^s = 7\times10^{-6}$ (Kg/m^2))

Opening angle	μ	$n = 1$		$n = 2$		$n = 3$	
		With surface density	Without surface density	With surface density	Without surface density	With surface density	Without surface density
$\pi/8$	0	8.1476	9.6404	33.1703	39.2476	74.8767	88.5954
	0.1	7.7730	9.1972	28.8664	33.2323	54.4898	64.4732
	0.2	6.8988	8.1628	20.6544	24.4386	35.091	41.5202
	0.3	5.9292	7.0156	15.5453	18.3934	24.9667	29.5410
	0.4	5.0733	6.0028	12.2630	14.5098	19.1978	22.7151
	0.5	4.3755	5.1772	10.0610	11.9044	15.5432	18.3910
	0.6	3.8184	4.5180	8.5046	10.628	13.0388	15.4277
	0.7	3.3726	3.9905	7.3540	8.7014	11.2214	13.2773
	0.8	3.0121	3.5640	6.4722	7.6580	9.8446	11.6483
	0.9	2.7167	3.2145	5.7762	6.8345	8.7667	10.3729
	1	2.4713	2.9241	5.2136	6.1688	7.9003	9.3478
$\pi/4$	0	7.5865	8.9765	32.5904	38.5615	74.2932	87.9049
	0.1	7.2377	8.5638	27.5954	32.6513	54.0652	63.9708
	0.2	6.4238	7.6007	20.5933	24.0113	34.8175	41.1966
	0.3	5.5209	6.5324	15.2735	18.0718	24.7721	29.3108
	0.4	4.7239	5.5894	12.0486	14.2561	19.0481	22.5381
	0.5	4.0742	4.8206	9.8851	11.6963	15.4221	18.2477
	0.6	3.5554	4.2068	8.3559	9.8868	12.9372	15.3075
	0.7	3.1404	3.7157	7.2255	8.5493	11.1339	13.1738
	0.8	2.8047	3.3186	6.3590	7.5241	9.7679	11.5575
	0.9	2.5296	2.9931	5.6752	6.7150	8.6984	10.2921
	1	2.3011	2.7227	5.1224	6.0610	7.8387	9.2749
$\pi/2$	0	5.5955	6.6207	30.3461	35.9060	71.9936	85.1840
	0.1	5.3383	6.3164	25.6950	30.4028	52.3917	619907
	0.2	4.7379	5.6060	18.8958	22.3578	33.7398	39.9215
	0.3	4.0720	4.8181	14.2217	16.8273	24.0054	28.4035
	0.4	3.4842	4.1226	11.2189	13.2744	18.4586	21.8405
	0.5	3.0050	3.5555	9.2044	10.8908	14.9447	17.6829
	0.6	2.6223	3.1028	7.7805	9.2060	12.5368	14.8337
	0.7	2.3162	2.7406	6.7279	7.9606	10.7893	12.7660
	0.8	2.0687	2.4477	5.9211	7.0060	9.4656	11.1998
	0.9	1.8658	2.2076	5.2844	6.2526	8.4291	9.9735
	1	1.6972	2.0082	4.7697	5.6436	7.5961	8.9879

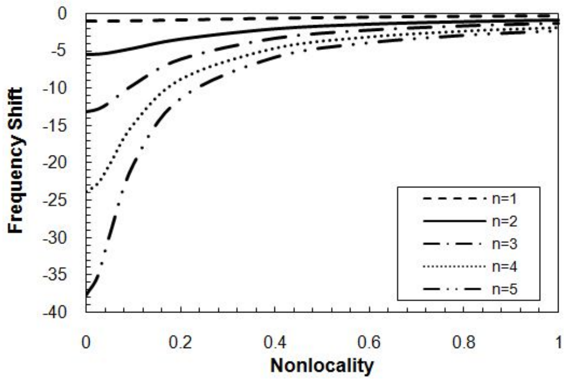


Fig. 7 Dimensionless Frequency shift versus the nonlocality for higher mode numbers

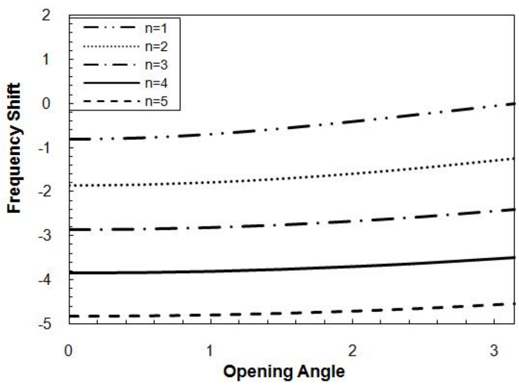


Fig. 8 Dimensionless Frequency shift versus the opening angle for higher mode numbers

higher for higher mode numbers and greater nonlocal parameter of ring elements. Accordingly the effect of surface density must not be vanished for such higher mode numbers. It is observed that, the nonlocal parameter has sensitive effect on frequency parameters. As it shown in Fig. 7 with increasing the nonlocal parameter, surface density effect reduce on dimensionless natural frequency. In addition, it is concluded from Fig. 7 with increasing the opening angle, the effect of surface density is also decrease such as Fig. 6.

4. Conclusions

In the present study, free vibration of circular curved nanobeams including surface density and nonlocality effects with different opening angles is studied within the framework of an Euler-Bernoulli beam theory. The dynamic equilibrium equations is employed to derive the governing equations and related boundary conditions. An analytically exact solution is used to solve governing differential equations for simply-supported boundary conditions. It is indicated that the free vibration of circular curved nanobeams significantly affected by various parameters such as surface density, thickness of curved nanobeam, nonlocal parameter and opening angles. It is observed that by increasing nonlocal parameter, the effect of surface density, tend to vanish. Furthermore it is shown that the size scale and surface density effect, play an important role in dynamics behavior of circular curved nanobeams. The solutions can be used as benchmark for future researches.

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