Advances in Nano Research, *Vol. 4, No. 2 (2016) 65-84*
DOI: http://dx.doi.org/10.12989/anr.2016.4.2.065

An exact solution for buckling analysis of embedded piezo-electro-magnetically actuated nanoscale beams

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(Received December 29, 2015, Revised March 4, 2016, Accepted March 10, 2016)

Abstract. This paper investigates the buckling behavior of shear deformable piezoelectric (FGP) nanoscale beams made of functionally graded (FG) materials embedded in Winkler-Pasternak elastic medium and subjected to an electro-magnetic field. Magneto-electro-elastic (MEE) properties of piezoelectric nanobeam are supposed to be graded continuously in the thickness direction based on power-law model. To consider the small size effects, Eringen's nonlocal elasticity theory is adopted. Employing Hamilton's principle, the nonlocal governing equations of the embedded piezoelectric nanobeams are obtained. A Navier-type analytical solution is applied to anticipate the accurate buckling response of the FGP nanobeams subjected to electro-magnetic fields. To demonstrate the influences of various parameters such as, magnetic potential, external electric voltage, power-law index, nonlocal parameter, elastic foundation and slenderness ratio on the critical buckling loads of the size-dependent MEE-FG nanobeams, several numerical results are provided. Due to the shortage of same results in the literature, it is expected that the results of the present study will be instrumental for design of size-dependent MEE-FG nanobeams.

Keywords: piezoelectric nanobeam; magneto-electro-elastic FG nanobeam; buckling; nonlocal elasticity theory; higher order beam theory

1. Introduction

Magneto-electro-elastic (MEE) materials have encountered to a significant interest for their extensive potential applications, since the first report on a MEE composite including piezo-electric phase and piezo-magnetic phases in 1970s (Van Run *et al*. 1974). MEE materials have the potential to convert magnetic, electric and mechanical energies from one form to the others and this leads to wide application of these materials in sensing and actuating devices, control of structural vibrations and smart structure technology (Milazzo *et al.* 2009). Recently, analyzing the mechanical responses of MEE structural components has received a remarkable attention. A survey in literature shows that, mechanical behavior of MEE structures is studied by several researchers. Among them, Chen *et al*. (2005) studied free vibration of non-homogeneous transversely isotropic magneto-electro-elastic plates. Free vibration of multiphase and layered magneto-electro-elastic beam for $BaTiO_3$-$CoFe_2O_4$ composite is carried out by Annigeri *et al*. (2007). Kumaravel *et al*. (2007) researched linear buckling and free vibration behavior of layered

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http://www.techno-press.com/?journal=anr&subpage=7 ISSN: 2287-237X (Print), 2287-2388 (Online)

and multiphase magneto-electro-elastic (MEE) beam under thermal environment. Transient dynamic response of multiphase magneto-electro-elastic cantilever beam is presented by Daga *et al*. (2009) using finite element method. Also, Liu and Chang (2010) presented a closed form expression for the vibration problem of a transversely isotropic magneto-electro-elastic plate. Razavi and Shooshtari (2015) studied nonlinear free vibration of symmetric magneto-electro-elastic laminated rectangular plates with simply supported boundary condition. They used the first order shear deformation theory considering the von Karman's nonlinear strains to obtain the equations of motion, whereas Maxwell equations for electrostatics and magnetostatics are used to model the electric and magnetic behavior. Recently, Xin and Hu (2015) investigated free vibration of simply supported and multilayered magneto-electro-elastic plates.

Moreover, mechanical analysis of structures made from composition of MEEMs and FGMs have gained notable attentions in the last years. Pan and Han (2005) presented an exact solution for the multilayered rectangular plate made of FG, anisotropic, and linear magneto-electro-elastic materials. In this study, they supposed that the edges of the plate are under simply supported conditions, general mechanical, electric and magnetic boundary conditions can be applied on both the top and bottom surfaces of the plate. Also, Huang *et al*. (2007) studied the plane stress problem of generally anisotropic magneto-electro-elastic beams with the coefficients of elastic compliance, piezoelectricity, dielectric impermeability, piezomagnetism, magnetoelectricity, and magnetic permeability being arbitrary functions of the thickness coordinate. In another study, three-dimensional (3D) static behavior of doubly curved FG MEE shells under the mechanical load, electric displacement and magnetic flux using an asymptotic approach is investigated by Wu and Tsai (2007). Li *et al*. (2008) investigated the problem of a functionally graded, transversely isotropic, magneto-electro-elastic circular plate acted on by a uniform load. Kattimani and Ray (2015) investigated active control of geometrically nonlinear vibrations of FG MEE plates. Sladek *et al*. (2015) analyzed bending of circular magneto-electro-elastic plates with functionally graded material properties using a meshless method.

The significance of size effects motivated the scientific community to explore the behaviors of the nanostructures and nanomaterials much accurately (Alizada and Sofiyev 2011a, b, Alizada *et al*. 2012). By minimizing the size of the structure and becoming comparable to the internal characteristic length scale, the classical continuum mechanics is unable to model such structures in which size-dependent behaviors have been experimentally observed. Due to this reason, various higher order continuum theories such as Eringen's nonlocal elasticity theory are suggested to capture the influence of small size. Ke and Wang (2014) studied the free vibration behavior of magneto-electro-elastic (MEE) nanobeams using nonlocal theory and Timoshenko beam theory. They supposed that the MEE nanobeam is subjected to the external electric potential, magnetic potential and uniform temperature rise. In another study, Ke *et al*. (2014) investigated the free vibration behavior of magneto-electro-elastic (MEE) nanoplates based on the nonlocal theory and Kirchhoff plate theory. Li *et al*. (2014) analyzed buckling and free vibration of magneto-electro-elastic nanoplate resting on Pasternak foundation based on nonlocal Mindlin theory. Ansari *et al*. (2015) studied forced vibration behavior of higher order shear deformable magneto-electro-thermo elastic (METE) nanobeams based on the nonlocal elasticity theory in conjunction with the von Kármán geometric nonlinearity. Wu *et al*. (2015) researched surface effects on anti-plane shear waves propagating in nanoplates made from magneto-electro-elastic materials. As literature shows, there is no study investigating the small scale influence on buckling responses of MEE-FG nanobeams, so it is necessary to investigate the stability of such structures. By ignoring the effects of magnetic and electric fields only a few studies are performed to analyze mechanical behavior of

FG nanobeams. Among them, Şimşek and Yurtcu (2013) presented analytical solutions for bending and buckling of functionally graded nanobeams based on the nonlocal Timoshenko beam theory. Rahmani and Pedram (2014) Analyzed the size effect on vibration of functionally graded nanobeams based on nonlocal Timoshenko beam theory. Rahmani and Jandaghian (2015) studied buckling of functionally graded nanobeams based on a nonlocal third-order shear deformation theory. Also, Ebrahimi and Salari (2015 a, b) studied influences of thermal environments on mechanical behavior of nonlocal temperature-dependent FG nanobeams. Ebrahimi and Barati (2015) presented a nonlocal third order beam theory for vibration analysis of FG nanobeams. Most recently, Zemri *et al*. (2015) analyzed mechanical responses of FG nanobeams using a refined shear deformation theory. Mahmoud *et al*. (2015) investigated bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect. Also, Barati *et al*. (2016) investigated thermo-mechanical buckling analysis of embedded small size FG plates in thermal environments via an inverse cotangential theory.

This paper present a higher order beam model for the buckling analysis of magneto-electro-elastic FG nanobeams resting on two-parameter elastic foundation. Superiority of the present theory is that it consider the influences of shear deformation which is ignored in Euler-Bernoulli beam theory and doesn't require a shear correction factor applied in Timoshenko beam theory. The magneto-electro-elastic material properties of the beam is supposed to be variable in the thickness direction according to the power law distribution. The small size effect is captured using Eringen's nonlocal elasticity theory. Nonlocal governing equations for the buckling of embedded MEE-FG nanobeams have been derived via Hamilton's principle and then solved using Navier type method. Various numerical and illustrative results show the influences of elastic foundation, magnetic potential, external electric voltage, nonlocal parameter, power-law index and slenderness ratio on buckling behavior of MEE-FG nanobeams resting on elastic foundation.

2. Theoretical formulations

2.1 The material properties of MEE-FG nanobeams

Assume a magneto-electro-elastic functionally graded nanobeam composed of $BaTiO_3$ and $CoFe_2O_4$ materials exposed to a magnetic potential $\Upsilon(x,z,t)$ and electric potential $\Phi(x,z,t)$, with length L and uniform thickness h, as shown in Fig. 1. The effective material properties of the MEE-FG nanobeam are supposed to change continuously in the z-axis direction (thickness direction) based on the power-law model. So, the effective material properties, P, can be stated in the following form

$$P = P_2 V_2 + P_1 V_1 \tag{1}$$

In which P_1 and P_2 denote the material properties of the bottom and higher surfaces, respectively. Also V_1 and V_2 are the corresponding volume fractions related by

$$V_2 = (\frac{z}{h} + \frac{1}{2})^{p}, \quad V_1 = 1 - V_2 \tag{2}$$

Therefore, according to Eqs. (1) and (2), the effective magneto-electro-elastic material properties of the FG beam is defined as

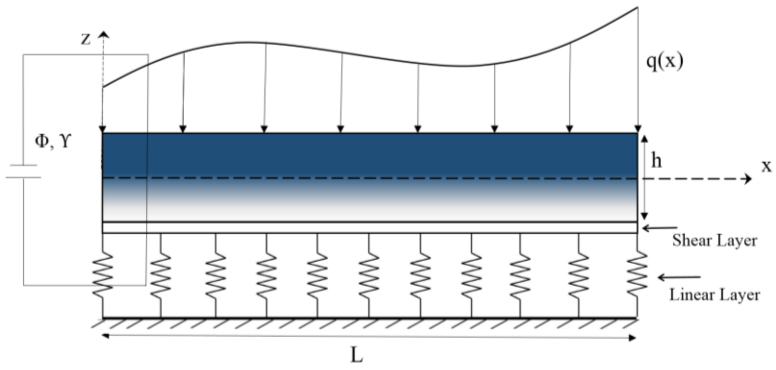


Fig. 1 Configuration of a MEE-FG nanobeam

$$P(z) = (P_2 - P_1)\left(\frac{z}{h} + \frac{1}{2}\right)^p + P_1 \tag{3}$$

where p is power-law exponent which is non-negative and estimates the material distribution through the thickness of the nanobeam and z is the distance from the mid-plane of the graded piezoelectric beam. It must be noted that, the top surface at $z=+h/2$ of FG nanobeam is assumed $CoFe_2O_4$ rich, whereas the bottom surface ($z=-h/2$) is $BaTiO_3$ rich.

2.2 Nonlocal elasticity theory for the magneto-electro-elastic materials

Contrary to the constitutive equation of classical elasticity theory, Eringen's nonlocal theory (Eringen 1972a, b, Eringen 1983) notes that the stress state at a point inside a body is regarded to be function of strains of all points in the neighbor regions. For a nonlocal magneto-electro-elastic solid the basic equations with zero body force may be defined as

$$\sigma_{ij} = \int_V \alpha\left(|x'-x|, \tau\right)\left[C_{ijkl}\varepsilon_{kl}(x') - e_{mij}E_m(x') - q_{nij}H_n(x')\right]dV(x') \tag{4a}$$

$$D_i = \int_V \alpha\left(|x'-x|, \tau\right)\left[e_{ikl}\varepsilon_{kl}(x') + s_{im}E_m(x') + d_{in}H_n(x')\right]dV(x') \tag{4b}$$

$$B_i = \int_V \alpha\left(|x'-x|, \tau\right)\left[q_{ikl}\varepsilon_{kl}(x') + d_{im}E_m(x') + \chi_{in}H_n(x')\right]dV(x') \tag{4c}$$

where σ_{ij}, ε_{ij}, D_i, E_i, B_i and H_i denote the stress, strain, electric displacement, electric field components, magnetic induction and magnetic field and displacement components, respectively; C_{ijkl}, E_{mij}, s_{im}, q_{nij}, d_{ij} and χ_{ij} are the elastic, piezoelectric, dielectric constants, piezomagnetic, magnetoelectric, magnetic constants, respectively; $\alpha(|x'-x|, \tau)$ is the nonlocal kernel function and ($|x'-x|$ is the Euclidean distance. $\tau=e_0a/l$ is defined as scale coefficient, where e_0 is a material constant which is determined experimentally or approximated by matching the dispersion curves of plane waves with those of atomic lattice dynamics; and a and l are the internal and external characteristic length of the nanostructures, respectively. Finally it is possible to represent the integral constitutive relations given by Eq. (4) in an equivalent differential form as

$$\sigma_{ij} - (e_0\, a)^2 \nabla^2 \sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{mij}E_m - q_{nij}H_n \tag{5a}$$

$$D_i - (e_0\, a)^2 \nabla^2 D_i = e_{ikl}\varepsilon_{kl} + s_{im}E_m + d_{in}H_n \tag{5b}$$

$$B_i - (e_0\, a)^2 \nabla^2 B_i = q_{ikl}\varepsilon_{kl} + d_{im}E_m + \chi_{in}H_n \tag{5c}$$

where ∇^2 is the Laplacian operator and e_0a is the nonlocal parameter revealing the size influence on the response of nanostructures.

2.3 Nonlocal magneto-electro-elastic FG nanobeam model

Based on third order beam theory, the displacement field at any point of the beam are supposed to be in the form

$$u_x\left(x,z\right) = u\left(x\right) + z\psi\left(x\right) - \alpha z^3\left(\psi + \frac{\partial w}{\partial x}\right) \tag{6a}$$

$$u_z(x,z) = w(x) \tag{6b}$$

in which α=4/3h^2 and u and w are displacement components in the mid-plane along the coordinates x and z, respectively, while ψ denotes the total bending rotation of the cross-section.

To satisfy Maxwell's equation in the quasi-static approximation, the distribution of electric and magnetic potential along the thickness direction is supposed to change as a combination of a cosine and linear variation as follows

$$\Phi(x,z,t) = -\cos(\xi z)\phi(x,t) + \frac{2z}{h}V \tag{7a}$$

$$\Upsilon(x,z,t) = -\cos(\xi z)\gamma(x,t) + \frac{2z}{h}\Omega \tag{7b}$$

where $\xi=\pi/h$. Also, V and Ω are the initial external electric voltage and magnetic potential applied to the MEE-FG nanobeam. Considering strain-displacement relationships on the basis of parabolic beam theory, the non-zero strains can be stated as

$$\varepsilon_{xx} = \varepsilon_{xx}{}^{(0)} + z\varepsilon_{xx}{}^{(1)} + z^3\varepsilon_{xx}{}^{(3)} \tag{8}$$

$$\gamma_{xz} = \gamma_{xz}{}^{(0)} + z^2\gamma_{xz}{}^{(2)} \tag{9}$$

where

$$\varepsilon_{xx}{}^{(0)} = \frac{\partial u}{\partial x},\ \varepsilon_{xx}{}^{(1)} = \frac{\partial \psi}{\partial x},\ \varepsilon_{xx}{}^{(3)} = -\alpha\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) \tag{10}$$

$$\gamma_{xz}{}^{(0)} = \frac{\partial w}{\partial x} + \psi,\ \gamma_{xz}{}^{(2)} = -\beta\left(\frac{\partial w}{\partial x} + \psi\right) \tag{11}$$

And $\beta = \frac{4}{h^2}$.

According to the Eq. (7), the non-zero components of electric and magnetic field (E_x, E_z, H_x, H_z) can be obtained as

$$E_x = -\Phi_{,x} = \cos(\xi z)\frac{\partial \phi}{\partial x}, \quad E_z = -\Phi_{,z} = -\xi \sin(\xi z)\phi - \frac{2V}{h} \tag{12a}$$

$$H_x = -\Upsilon_{,x} = \cos(\xi z)\frac{\partial \gamma}{\partial x}, \quad H_z = -\Upsilon_{,z} = -\xi \sin(\xi z)\gamma - \frac{2\Omega}{h} \tag{12b}$$

The Hamilton's principle can be stated in the following form to obtain the governing equations of motion

$$\int_0^t \delta(\Pi_S + \Pi_W)\, dt = 0 \tag{13}$$

where Π_S is strain energy and Π_W is work done by external applied forces. The first variation of strain energy Π_S can be calculated as

$$\delta \Pi_S = \int_0^L \int_{-h/2}^{h/2} \left(\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xz}\delta\gamma_{xz} - D_x\delta E_x - D_z\delta E_z - B_x\delta H_x - B_z\delta H_z\right) dzdx \tag{14}$$

Substituting Eqs. (8) and (9) into Eq. (14) yields

$$\begin{aligned}\delta \Pi_S &== \int_0^L (N\delta\varepsilon_{\mathrm{xx}}{}^{(0)} + M\delta\varepsilon_{\mathrm{xx}}{}^{(1)} + P\delta\varepsilon_{\mathrm{xx}}{}^{(3)} + Q\delta\gamma_{xz}{}^{(0)} + R\delta\gamma_{xz}{}^{(2)})\, dx \\ &+ \int_0^L \int_{-h/2}^{h/2} \left(-D_x\cos(\xi z)\,\delta\left(\frac{\partial\phi}{\partial x}\right) + D_z\xi\sin(\xi z)\,\delta\,\phi - B_x\cos(\xi z)\,\delta\left(\frac{\partial\gamma}{\partial x}\right) + B_z\xi\sin(\xi z)\,\delta\,\gamma\right) dzdx\end{aligned} \tag{15}$$

in which N, M and Q are the axial force, bending moment and shear force resultants, respectively. Relations between the stress resultants and stress component used in Eq. (15) are defined as

$$\begin{aligned} &N = \int_A \sigma_{xx}\, dA,\; M = \int_A \sigma_{xx} z\, dA,\; P = \int_A \sigma_{xx} z^3 dA \\ &Q = \int_A \sigma_{xz}\, dA,\; R = \int_A \sigma_{xz} z^2\, dA \end{aligned} \tag{16}$$

The work done due to external electric voltage, Π_W, can be written in the form

$$\begin{aligned}\Pi_W &= \int_0^L (N_H + N_E + N_b)\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x} + q\,\delta w + f\,\delta u - N\delta\varepsilon_{\mathrm{xx}}{}^{(0)} - \hat{M}\frac{\partial\delta\psi}{\partial x} \\ &+ \alpha P\frac{\partial^2\delta w}{\partial x^2} - \hat{Q}\delta\gamma_{xz}{}^{(0)} - k_w\delta w + k_p\frac{\partial^2\delta w}{\partial x^2})dx\end{aligned} \tag{17}$$

where $\hat{M} = M - \alpha P$, $\hat{Q} = Q - \beta R$ and $q(x)$ and $f(x)$ are the transverse and axial distributed loads and k_w and k_w are foundation parameters and also N_b, N_B and N_E are the buckling load and normal forces induced by magnetic potential and external electric voltage, respectively which are defined as

$$N_E = -\int_{-h/2}^{h/2} e_{31}\frac{2V}{h}dz,\; N_H = -\int_{-h/2}^{h/2} q_{31}\frac{2\Omega}{h}dz \tag{18}$$

For a magneto-electro-elastic FGM nanobeam in the one dimensional case, the nonlocal constitutive relations (5a)-(5c) may be rewritten as

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = c_{11}\varepsilon_{xx} - e_{31} E_z - q_{31} H_z \tag{19}$$

$$\sigma_{xz} - (e_0 a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = c_{55}\gamma_{xz} - e_{15} E_x - q_{15} H_x \tag{20}$$

$$D_x - (e_0 a)^2 \frac{\partial^2 D_x}{\partial x^2} = e_{15}\gamma_{xz} + s_{11} E_x + d_{11} H_x \tag{21}$$

$$D_z - (e_0 a)^2 \frac{\partial^2 D_z}{\partial x^2} = e_{31}\varepsilon_{xx} + s_{33} E_z + d_{33} H_z \tag{22}$$

$$B_x - (e_0 a)^2 \frac{\partial^2 B_x}{\partial x^2} = q_{15}\gamma_{xz} + d_{11} E_x + \chi_{11} H_x \tag{23}$$

$$B_z - (e_0 a)^2 \frac{\partial^2 B_z}{\partial x^2} = q_{31}\varepsilon_{xx} + d_{33} E_z + \chi_{33} H_z \tag{24}$$

Inserting Eqs. (15) and (17) in Eq. (13) and integrating by parts, and gathering the coefficients of δu, δw, $\delta\psi$, $\delta\phi$ and $\delta\gamma$ the following governing equations are obtained

$$\frac{\partial N}{\partial x} + f = 0 \tag{25}$$

$$\frac{\partial \hat{M}}{\partial x} - \hat{Q} = 0 \tag{26}$$

$$\frac{\partial \hat{Q}}{\partial x} + q - (N_H + N_E + N_b)\frac{\partial^2 w}{\partial x^2} + \alpha \frac{\partial^2 P}{\partial x^2} - k_w w + k_p \frac{\partial^2 w}{\partial x^2} = 0 \tag{27}$$

$$\int_{-h/2}^{h/2} \left(\cos(\xi z)\frac{\partial D_x}{\partial x} + \xi \sin(\xi z) D_z \right) dz = 0 \tag{28}$$

$$\int_{-h/2}^{h/2} \left(\cos(\xi z)\frac{\partial B_x}{\partial x} + \xi \sin(\xi z) B_z \right) dz = 0 \tag{29}$$

By integrating Eqs. (19)-(24), over the beam's cross-section area, the force-strain and the moment-strain of the nonlocal third order Reddy FG beam theory can be obtained as follows

$$N - \mu \frac{\partial^2 N}{\partial x^2} = A_{xx}\frac{\partial u}{\partial x} + B_{xx}\frac{\partial \psi}{\partial x} - \alpha E_{xx}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + A_{31}^e \phi + A_{31}^m \gamma - N_E - N_H \tag{30}$$

$$M - \mu \frac{\partial^2 M}{\partial x^2} = B_{xx}\frac{\partial u}{\partial x} + D_{xx}\frac{\partial \psi}{\partial x} - \alpha F_{xx}\left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + E_{31}^e \phi + E_{31}^m \gamma \tag{31}$$

$$P-\mu\frac{\partial^2 P}{\partial x^2}=E_{xx}\frac{\partial u}{\partial x}+F_{xx}\frac{\partial \psi}{\partial x}-\alpha H_{xx}(\frac{\partial \psi}{\partial x}+\frac{\partial^2 w}{\partial x^2})+F_{31}^{e}\phi+F_{31}^{m}\gamma \tag{32}$$

$$Q-\mu\frac{\partial^2 Q}{\partial x^2}=(A_{xz}-\beta D_{xz})(\frac{\partial w}{\partial x}+\psi)-E_{15}^{e}\frac{\partial \phi}{\partial x}-E_{15}^{m}\frac{\partial \gamma}{\partial x} \tag{33}$$

$$R-\mu\frac{\partial^2 R}{\partial x^2}=(D_{xz}-\beta F_{xz})(\frac{\partial w}{\partial x}+\psi)-F_{15}^{m}\frac{\partial \phi}{\partial x}-F_{15}^{m}\frac{\partial \gamma}{\partial x} \tag{34}$$

$$\int_{-h/2}^{h/2}\left\{D_x-\mu\frac{\partial^2 D_x}{\partial x^2}\right\}\cos(\xi z)dz=(E_{15}^{e}-\beta F_{15}^{e})(\frac{\partial w}{\partial x}+\psi)+F_{11}^{e}\frac{\partial \phi}{\partial x}+F_{11}^{m}\frac{\partial \gamma}{\partial x} \tag{35}$$

$$\int_{-h/2}^{h/2}\left\{D_z-\mu\frac{\partial^2 D_z}{\partial x^2}\right\}\xi\sin(\xi z)dz=A_{31}^{e}\frac{\partial u}{\partial x}+(E_{31}^{e}-\alpha F_{31}^{e})\frac{\partial \psi}{\partial x}-\alpha F_{31}^{e}\frac{\partial^2 w}{\partial x^2}-F_{33}^{e}\phi-F_{33}^{m}\gamma \tag{36}$$

$$\int_{-h/2}^{h/2}\left\{B_x-\mu\frac{\partial^2 B_x}{\partial x^2}\right\}\cos(\xi z)dz=(E_{15}^{m}-\beta F_{15}^{m})(\frac{\partial w}{\partial x}+\psi)+F_{11}^{m}\frac{\partial \phi}{\partial x}+X_{11}^{m}\frac{\partial \gamma}{\partial x} \tag{37}$$

$$\int_{-h/2}^{h/2}\left\{B_z-\mu\frac{\partial^2 B_z}{\partial x^2}\right\}\xi\sin(\xi z)dz=A_{31}^{m}\frac{\partial u}{\partial x}+(E_{31}^{m}-\alpha F_{31}^{m})\frac{\partial \psi}{\partial x}-\alpha F_{31}^{m}\frac{\partial^2 w}{\partial x^2}-F_{33}^{m}\phi-X_{33}^{m}\gamma \tag{38}$$

where $\mu=(e_0a)^2$ and quantities used in above equations are defined as

$$\left\{A_{xx},B_{xx},D_{xx},E_{xx},F_{xx},H_{xx}\right\}=\int_{-h/2}^{h/2}c_{11}\left\{1,z,z^2,z^3,z^4,z^6\right\}dz \tag{39}$$

$$\left\{A_{xz},D_{xz},F_{xz}\right\}=\int_{-h/2}^{h/2}c_{55}\left\{1,z^2,z^4\right\}dz \tag{40}$$

$$\left\{A_{31}^{e},E_{31}^{e},F_{31}^{e}\right\}=\int_{-h/2}^{h/2}e_{31}\left\{\xi\sin(\xi z),z\xi\sin(\xi z),z^3\xi\sin(\xi z)\right\}dz \tag{41}$$

$$\left\{E_{15}^{e},F_{15}^{e}\right\}=\int_{-h/2}^{h/2}e_{15}\left\{\cos(\xi z),z^2\cos(\xi z)\right\}dz \tag{42}$$

$$\left\{F_{11}^{e},F_{33}^{e}\right\}=\int_{-h/2}^{h/2}\left\{s_{11}\cos^2(\xi z),s_{33}\xi^2\sin^2(\xi z)\right\}dz \tag{43}$$

$$\left\{A_{31}^{m},E_{31}^{m},F_{31}^{m}\right\}=\int_{-h/2}^{h/2}q_{31}\left\{\xi\sin(\xi z),z\xi\sin(\xi z),z^3\xi\sin(\xi z)\right\}dz \tag{44}$$

$$\left\{E_{15}^{m},F_{15}^{m}\right\}=\int_{-h/2}^{h/2}q_{15}\left\{\cos(\xi z),z^2\cos(\xi z)\right\}dz \tag{45}$$

$$\left\{F_{11}^{m}, F_{33}^{m}\right\} = \int_{-h/2}^{h/2} \left\{d_{11}\cos^2(\xi z), d_{33}\,\xi^2 \sin^2(\xi z)\right\} dz \tag{46}$$

$$\left\{X_{11}^{m}, X_{33}^{m}\right\} = \int_{-h/2}^{h/2} \left\{\chi_{11}\cos^2(\xi z), \chi_{33}\,\xi^2 \sin^2(\xi z)\right\} dz \tag{47}$$

The explicit relation of the nonlocal normal force can be derived by substituting for the second derivative of N from Eq.(30) into Eq.(25) as follows

$$N_x = A_{xx}\frac{\partial u}{\partial x} + K_{xx}\frac{\partial \psi}{\partial x} - \alpha E_{xx}\frac{\partial^2 w}{\partial x^2} + A_{31}^{e}\phi + A_{31}^{m}\gamma - N_E - N_H + \mu(-\frac{\partial f}{\partial x}) \tag{48}$$

Omitting $\hat{Q}$ from Eqs. (26) and (27), we obtain the following equation

$$\frac{\partial^2 \hat{M}}{\partial x^2} = -\alpha\frac{\partial^2 P}{\partial x^2} - q + (N_E + N_H + N_b)\frac{\partial^2 w}{\partial x^2} + k_w\, w - k_p\frac{\partial^2 w}{\partial x^2} \tag{49}$$

Also the explicit relation of the nonlocal bending moment can be derived by substituting for the second derivative of $\hat{M}$ from Eq. (31) into Eq. (26) and using Eqs. (31) and (32) as follows

$$\begin{aligned}\hat{M} = {} & K_{xx}\frac{\partial u}{\partial x} + I_{xx}\frac{\partial \psi}{\partial x} - \alpha J_{xx}(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2}) + (E_{31}^{e} - \alpha F_{31}^{e})\phi + (E_{31}^{m} - \alpha F_{31}^{m})\gamma \\ & + \mu(-\alpha\frac{\partial^2 P}{\partial x^2} - q + \frac{\partial}{\partial x}(N_E + N_H + N_b)\frac{\partial w}{\partial x}) + k_w\, w - k_p\frac{\partial^2 w}{\partial x^2})\end{aligned} \tag{50}$$

where

$$K_{xx} = B_{xx} - \alpha E_{xx},\; I_{xx} = D_{xx} - \alpha F_{xx},\; J_{xx} = F_{xx} - \alpha H_{xx} \tag{51}$$

By substituting for the second derivative of $\hat{Q}$ from Eq. (33) into Eq. (27), and using Eqs. (33) and (34) the following expression for the nonlocal shear force is derived

$$\begin{aligned}\hat{Q} = {} & \overline{A}_{xz}(\frac{\partial w}{\partial x} + \psi) - (E_{15}^{e} - \beta F_{15}^{e})\frac{\partial \phi}{\partial x} + \mu((N_H + N_E + N_b)\frac{\partial^3 w}{\partial x^3} - \alpha\frac{\partial^3 P}{\partial x^3} - \frac{\partial q}{\partial x} \\ & + k_w\frac{\partial w}{\partial x} - k_p\frac{\partial^3 w}{\partial x^3} - (E_{15}^{m} - \beta F_{15}^{m})\frac{\partial \gamma}{\partial x})\end{aligned} \tag{52}$$

where

$$\overline{A}_{xz} = A^{*}_{xz} - \beta I^{*}_{xz},\; A^{*}_{xz} = A_{xz} - \beta D_{xz},\, I^{*}_{xz} = D_{xz} - \beta F_{xz} \tag{53}$$

Now we use $\hat{M}$ and $\hat{Q}$ from Eqs. (53) and (55) and the identity

$$\alpha\frac{\partial^2}{\partial x^2}(P - \mu\frac{\partial^2 P}{\partial x^2}) = \alpha(E_{xx}\frac{\partial^3 u}{\partial x^3} + F_{xx}\frac{\partial^3 \psi}{\partial x^3} - \alpha H_{xx}(\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^4 w}{\partial x^4}) + F_{31}^{e}\frac{\partial^2 \phi}{\partial x^2} + F_{31}^{m}\frac{\partial^2 \gamma}{\partial x^2}) \tag{54}$$

It must be cited that inserting Eqs. (28) and (29) into Eqs. (35)-(38), does not provide an explicit expressions for D_x and D_z. To overcome this problem, by using Eqs. (35)-(38), Eqs. (28)

and (29) can be re-expressed in terms of u, w, ψ and ϕ. Finally, based on third-order beam theory, the nonlocal equations of motion for a magneto-electro-elastic FG nanobeam can be obtained by substituting for N, $\hat{M}$ and $\hat{Q}$ from Eqs. (48), (50) and (52) into Eqs. (30)-(33) as follows

$$A_{xx}\frac{\partial^2 u}{\partial x^2}+K_{xx}\frac{\partial^2 \psi}{\partial x^2}-\alpha E_{xx}\frac{\partial^3 w}{\partial x^3}+A_{31}^{e}\frac{\partial \phi}{\partial x}+A_{31}^{m}\frac{\partial \Upsilon}{\partial x}+\mu(-\frac{\partial^2 f}{\partial x^2})+f=0 \tag{55}$$

$$\begin{aligned}&K_{xx}\frac{\partial^2 u}{\partial x^2}+I_{xx}\frac{\partial^2 \psi}{\partial x^2}-\alpha J_{xx}(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^3 w}{\partial x^3})-\bar{A}_{xz}\left(\psi+\frac{\partial w}{\partial x}\right)+(E_{31}^{e}-\alpha F_{31}^{e})\frac{\partial \phi}{\partial x}+(E_{31}^{m}-\alpha F_{31}^{m})\frac{\partial \gamma}{\partial x}\\&+(E_{15}^{e}-\beta F_{15}^{e})\frac{\partial \phi}{\partial x}+(E_{15}^{m}-\beta F_{15}^{m})\frac{\partial \gamma}{\partial x}=0\end{aligned} \tag{56}$$

$$\begin{aligned}&\bar{A}_{xz}\left(\frac{\partial \psi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+\mu((N_E+N_H+N_b)\frac{\partial^4 w}{\partial x^4}-\frac{\partial^2 q}{\partial x^2}+k_w\frac{\partial^2 w}{\partial x^2}-k_p\frac{\partial^4 w}{\partial x^4})-(N_E+N_H+N_b)\frac{\partial^2 w}{\partial x^2}-(E_{15}^{e}-\beta F_{15}^{e})\frac{\partial^2 \phi}{\partial x^2}\\&-(E_{15}^{m}-\beta F_{15}^{m})\frac{\partial^2 \gamma}{\partial x^2}+\alpha(E_{xx}\frac{\partial^3 u}{\partial x^3}+J_{xx}\frac{\partial^3 \psi}{\partial x^3}-\alpha H_{xx}\frac{\partial^4 w}{\partial x^4}+F_{31}^{e}\frac{\partial^2 \phi}{\partial x^2}+F_{31}^{m}\frac{\partial^2 \gamma}{\partial x^2})-k_w w+k_p\frac{\partial^2 w}{\partial x^2}+q=0\end{aligned} \tag{57}$$

$$\begin{aligned}&(E_{15}^{e}-\beta F_{15}^{e})(\frac{\partial^2 w}{\partial x^2}+\frac{\partial \psi}{\partial x})+F_{11}^{e}\frac{\partial^2 \phi}{\partial x^2}+F_{11}^{m}\frac{\partial^2 \gamma}{\partial x^2}+A_{31}^{e}\frac{\partial u}{\partial x}+(E_{31}^{e}-\alpha F_{31}^{e})\frac{\partial \psi}{\partial x}-\alpha F_{31}^{e}\frac{\partial^2 w}{\partial x^2}\\&-F_{33}^{e}\phi-F_{33}^{m}\gamma=0\end{aligned} \tag{58}$$

$$\begin{aligned}&(E_{15}^{m}-\beta F_{15}^{m})(\frac{\partial^2 w}{\partial x^2}+\frac{\partial \psi}{\partial x})+F_{11}^{m}\frac{\partial^2 \phi}{\partial x^2}+X_{11}^{m}\frac{\partial^2 \gamma}{\partial x^2}+A_{31}^{m}\frac{\partial u}{\partial x}+(E_{31}^{m}-\alpha F_{31}^{m})\frac{\partial \psi}{\partial x}-\alpha F_{31}^{m}\frac{\partial^2 w}{\partial x^2}\\&-F_{33}^{m}\phi-X_{33}^{m}\gamma=0\end{aligned} \tag{59}$$

3. Solution procedure

Here, on the basis the Navier method, an analytical solution of the governing equations for buckling of a simply supported magneto-electro-elastic FG nanobeam is presented. To satisfy governing equations of motion, the displacement variables are adopted to be of the form

$$u(x,t)=\sum_{n=1}^{\infty}U_n\cos(\frac{n\pi}{L}x)e^{i\omega_n t} \tag{60}$$

$$w(x,t)=\sum_{n=1}^{\infty}W_n\sin(\frac{n\pi}{L}x)e^{i\omega_n t} \tag{61}$$

$$\psi(x,t)=\sum_{n=1}^{\infty}\Psi_n\cos(\frac{n\pi}{L}x)e^{i\omega_n t} \tag{62}$$

$$\phi(x,t)=\sum_{n=1}^{\infty}\Phi_n\sin(\frac{n\pi}{L}x)e^{i\omega_n t} \tag{63}$$

$$\gamma(x,t) = \sum_{n=1}^{\infty} \Upsilon_n \sin(\frac{n\pi}{L} x) e^{i\omega_n t} \tag{64}$$

where U_n, W_n, Ψ_n, Φ_n and Υ_n are the unknown Fourier coefficients to be determined for each n value. Using Eqs. (60)-(64) the analytical solution can be obtained from the following equations

$$\left\{ \begin{pmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} \\ k_{3,1} & k_{3,2} & k_{3,3} & k_{3,4} & k_{3,5} \\ k_{4,1} & k_{4,2} & k_{4,3} & k_{4,4} & k_{4,5} \\ k_{5,1} & k_{5,2} & k_{5,3} & k_{5,4} & k_{5,5} \end{pmatrix} \right\} \begin{Bmatrix} U_n \\ \Psi_n \\ W_n \\ \Phi_n \\ \Upsilon_n \end{Bmatrix} = 0 \tag{68}$$

where

$$k_{1,1} = -A_{xx}(\frac{n\pi}{L})^2 \ , \ \ k_{1,2} = -K_{xx}(\frac{n\pi}{L})^2 , \ k_{1,3} = \alpha E_{xx}(\frac{n\pi}{L})^3 , \ k_{1,4} = -A_{31}^e(\frac{n\pi}{L}), k_{1,5} = -A_{31}^m(\frac{n\pi}{L})$$

$$k_{2,2} = -I_{xx}(\frac{n\pi}{L})^2 + \alpha J_{xx}(\frac{n\pi}{L})^2 - \bar{A}_{xz} , \ \ k_{2,3} = -\bar{A}_{xz}(\frac{n\pi}{L}) + J_{xx}(\frac{n\pi}{L})^3 , \ k_{2,4} = -((E_{15}^e - \beta F_{15}^e) + (E_{31}^e - \alpha F_{31}^e))(\frac{n\pi}{L}),$$

$$k_{2,5} = -((E_{15}^m - \beta F_{15}^m) + (E_{31}^m - \alpha F_{31}^m))(\frac{n\pi}{L}), \ k_{3,5} = -((E_{15}^m - \beta F_{15}^m) - \alpha F_{31}^m)(\frac{n\pi}{L})^2$$

$$k_{3,3} = (N_H + N_E + N_b)(\frac{n\pi}{L})^2(1+\mu(\frac{n\pi}{L})^2) - \bar{A}_{xz}(\frac{n\pi}{L})^2 - \alpha^2 H_{xx}(\frac{n\pi}{L})^4 + K_p(\frac{n\pi}{L})^2(1+\mu(\frac{n\pi}{L})^2) - K_w(1+\mu(\frac{n\pi}{l})^2),$$

$$k_{3,4} = -((E_{15}^e - \beta F_{15}^e) - \alpha F_{31}^e)(\frac{n\pi}{L})^2 , \ k_{4,4} = -(F_{11}^e(\frac{n\pi}{L})^2 + F_{33}^e), k_{4,4} = -(F_{11}^m(\frac{n\pi}{L})^2 + F_{33}^m)$$

4. Results and discussion

This section provides some numerical examples for the buckling characteristics of MEE-FG nanobeams. To achieve this goal, the nonlocal FG beam made of $BaTiO_3$ and $CoFe_2O_4$, with magneto-electro-elastic material properties listed in Table 1, is assumed. The beam geometry has the following dimensions: L (length)=10 nm and h (thickness)=varied. Also, the following relation is described to calculate the non-dimensional buckling loads as well as foundation parameters

$$N_{bcr} = N_b \frac{L^2}{(c_{11}I)_{\mathrm{CoFe_2O_4}}} , K_w = k_w \frac{L^4}{(c_{11}I)_{\mathrm{CoFe_2O_4}}} , \ K_p = k_p \frac{L^2}{(c_{11}I)_{\mathrm{CoFe_2O_4}}} \tag{69}$$

In which $I=h^3/12$ is the moment of inertia of the cross section of the beam. To evaluate correctness of the present model, the buckling results are compared with those of nonlocal FGM Reddy beams, due to the absence of numerical results for the buckling of MEE-FG nanobeams based on the nonlocal elasticity theory, as provided in Table 2. In this paper, the material selection is carried out as follows: E_m=70 GPa, v_m=0.3, kgm^{-3} for Steel and E_c=390 GPa, v_c=0.3, for Alumina. Tables 3-5, present the influences of magnetic potential (Ω), electric voltage (V), elastic foundation parameters (K_w, K_p), nonlocal parameter (μ), gradient index (p) and slenderness ratio (L/h) on the non-dimensional buckling load of the S-S MEE-FG nanobeams.

Table 1 Magneto-electro-elastic coefficients of material properties (Pan and Han 2005)

Properties	$BaTiO_3$	$CoFe_2O_4$
c_{11} (GPa)	166	286
c_{55}	43	45.3
e_{31} (Cm^{-2})	-4.4	0
e_{15}	11.6	0
q_{31} (N/Am)	0	580.3
q_{15}	0	550
s_{11} (10^{-9} $C^2m^{-2}N^{-1}$)	11.2	0.08
s_{33}	12.6	0.093
χ_{11}(10^{-6} $Ns^2C^{-2}/2$)	5	-590
χ_{33}	10	157
$d_{11}=d_{33}$	0	0

Table 2 Comparison of the non-dimensional buckling load for a S-S FG nanobeam with various power-law index (L/h=20)

	Nonlocal parameter							
p	μ=1		μ=2		μ=3		μ=4	
	RBT (Rahmani and Jandaghian 2015)	Present	RBT (Rahmani and Jandaghian 2015)	Present	RBT (Rahmani and Jandaghian 2015)	Present	RBT (Rahmani and Jandaghian 2015)	Present
0	8.9258	8.925759	8.1900	8.190046	7.5663	7.566381	7.0309	7.030978
0.1	9.7778	9.777865	8.9719	8.971916	8.2887	8.288712	7.7021	7.702196
0.2	10.3898	10.389845	9.5334	9.533453	8.8074	8.807489	8.1842	8.184264
0.5	11.4944	11.494448	10.5470	10.547009	9.7438	9.743863	9.0543	9.054379
1	12.3709	12.370918	11.3512	11.351234	10.4869	10.486847	9.7447	9.744790
2	13.1748	13.174885	12.0889	12.088934	11.1683	11.168372	10.3781	10.378089
5	14.2363	14.236343	13.0629	13.062900	12.0682	12.068171	11.2142	11.214218

It is obvious that for all values of magnetic potential and electric voltage nonlocal parameter weakens the structure of nanobeam by showing a significant reducing influence on the non-dimensional buckling loads. Also, it is observed that elastic foundation enhances rigidity of the beam and leads to increasing the dimensionless buckling loads. Another observation is that the buckling load results are strongly dependent on the magnitude and sign of magnetic potential and electric voltage. For all values of Winkler and Pasternak foundation parameters, the negative voltages provide higher buckling loads, while negative magnetic potentials produce lower buckling loads.

The influences of magnetic potential and electric voltage on the variations of the non-dimensional buckling load of the simply supported FG nanobeams versus power-law index at L/h=20 are plotted in Figs. 2 and 3, respectively. As one can see the non-dimensional buckling load decreases when the gradient index rises, especially for lower values of gradient index. This

Table 3 Variation of the dimensionless buckling load of embedded S-S FG nanobeam for various nonlocal parameter, magnetic potentials and electric voltages (*L*/*h*=15)

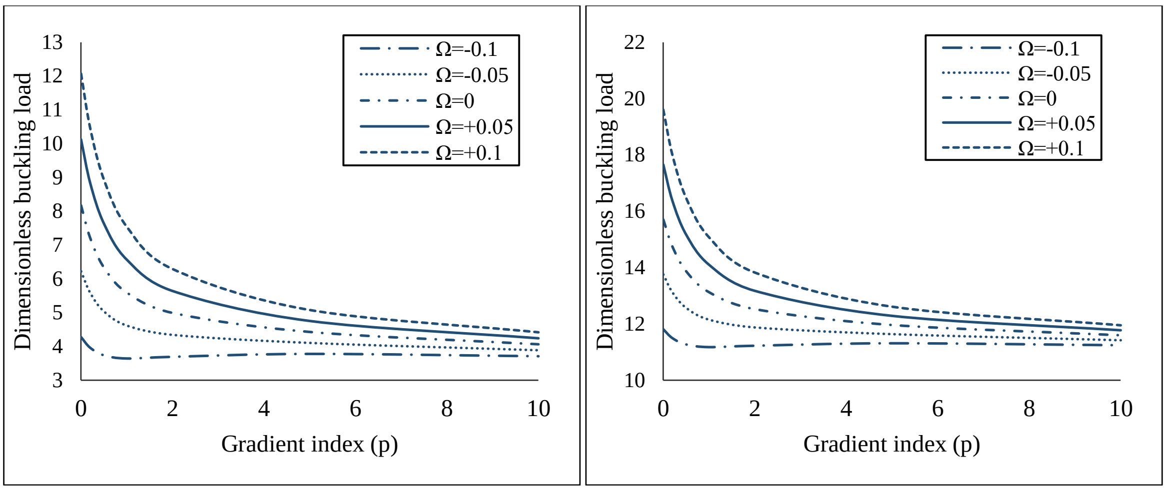
(K_w, K_p)	μ		Ω=-0.05			Ω=0			Ω=+0.05		
			p=0.2	*p*=1	*p*=5	*p*=0.2	*p*=1	*p*=5	*p*=0.2	*p*=1	*p*=5
(0,0)	0	V=-5	8.24863	7.41208	7.10613	8.93342	7.82296	7.24309	9.61822	8.23384	7.38004
		V=0	8.14478	7.10055	6.58690	8.82958	7.51142	6.72385	9.51437	7.92230	6.86081
		V=+5	8.04093	6.78901	6.06766	8.72573	7.19988	6.20462	9.41052	7.61076	6.34158
	1	V=-5	7.45546	6.73733	6.50212	8.14026	7.14821	6.63908	8.82505	7.55909	6.77604
		V=0	7.35162	6.42579	5.98289	8.03641	6.83667	6.11985	8.72121	7.24755	6.25681
		V=+5	7.24777	6.11425	5.46366	7.93257	6.52513	5.60062	8.61736	6.93601	5.73758
	2	V=-5	6.79306	6.17381	5.99769	7.47785	6.58469	6.13465	8.16265	6.99557	6.27161
		V=0	6.68921	5.86227	5.47846	7.37401	6.27315	5.61542	8.0588	6.68403	5.75237
		V=+5	6.58536	5.55074	4.95923	7.27016	5.96161	5.09618	7.95495	6.37249	5.23314
(25,5)	0	V=-5	15.7817	14.9451	14.6392	16.4665	15.3560	14.7761	17.1512	15.7669	14.9131
		V=0	15.6778	14.6336	14.1199	16.3626	15.0445	14.2569	17.0474	15.4553	13.2854
		V=+5	15.5740	14.3220	13.6007	16.2588	14.7329	13.7377	16.9436	15.1438	13.8746
	1	V=-5	14.9885	14.2704	14.0352	15.6733	14.6812	14.1721	16.3581	15.0921	14.3091
		V=0	14.8846	13.9588	13.5159	15.5694	14.3697	13.6529	16.2542	14.7806	13.7898
		V=+5	14.7808	13.6473	12.9967	15.4656	14.0582	13.1336	16.1504	14.469	13.2706
	2	V=-5	14.3261	13.7068	13.5307	15.0109	14.1177	13.6677	15.6957	14.5286	13.8046
		V=0	14.2222	13.3953	13.0115	14.9070	13.8062	13.1484	15.5918	14.2171	13.2854
		V=+5	14.1184	13.0838	12.4923	14.8032	13.4946	12.6292	15.488	13.9055	12.7662

Table 4 Variation of the dimensionless buckling load of embedded S-S FG nanobeam for various nonlocal parameter, magnetic potentials and electric voltages (*L*/*h*=20).

(K_w, K_p)	μ		Ω=-0.05			Ω=0			Ω=+0.05		
			p=0.2	*p*=1	*p*=5	*p*=0.2	*p*=1	*p*=5	*p*=0.2	*p*=1	*p*=5
(0,0)	0	V=-5	7.55085	7.34664	7.68559	9.17407	8.32057	8.01024	10.7973	9.29450	8.33488
		V=0	7.30469	6.60818	6.45483	8.92791	7.58211	6.77947	10.5511	8.55604	7.10411
		V=+5	7.05854	5.86972	5.22406	8.68176	6.84365	5.54870	10.3050	7.81758	5.87334
	1	V=-5	6.74885	6.66554	7.07659	8.37207	7.63947	7.40124	9.99529	8.61340	7.72588
		V=0	6.50270	5.92708	5.84582	8.12592	6.90101	6.17047	9.74913	7.87494	6.49511
		V=+5	6.25655	5.18862	4.61506	7.87976	6.16255	4.93970	9.50298	7.13648	5.26434
	2	V=-5	6.07907	6.09672	6.56799	7.70228	7.07065	6.89263	9.32550	8.04458	7.21727
		V=0	5.83291	5.35826	5.33722	7.45613	6.33219	5.66186	9.07935	7.30612	5.98651
		V=+5	5.58676	4.61980	4.10645	7.20998	5.59373	4.43109	8.83319	6.56766	4.75574
(25,5)	0	V=-5	15.0839	14.8797	15.2186	16.7071	15.8536	15.5433	18.3303	16.8275	15.8679
		V=0	14.8377	14.1412	13.9879	16.4609	15.1151	14.3125	18.0842	16.0891	14.6371
		V=+5	14.5916	13.4027	12.7571	16.2148	14.3767	13.0817	17.8380	15.3506	13.4064
	1	V=-5	14.2819	14.1986	14.6096	15.9051	15.1725	14.9343	17.5283	16.1464	15.2589
		V=0	14.0357	13.4601	13.3789	15.6589	14.4340	13.7035	17.2822	15.4080	14.0281
		V=+5	13.7896	12.7216	12.1481	15.4128	13.6956	12.4727	17.0360	14.6695	12.7974
	2	V=-5	13.6121	13.6297	14.1010	15.2353	14.6037	14.4257	16.8585	15.5776	14.7503
		V=0	13.3659	12.8913	12.8702	14.9892	13.8652	13.1949	16.6124	14.8391	13.5195
		V=+5	13.1198	12.1528	11.6395	14.7430	13.1268	11.9641	16.3662	14.1007	12.2888

Table 5 Variation of the dimensionless buckling load of embedded S-S FG nanobeam for various nonlocal parameter, magnetic potentials and electric voltages (*L/h*=25)

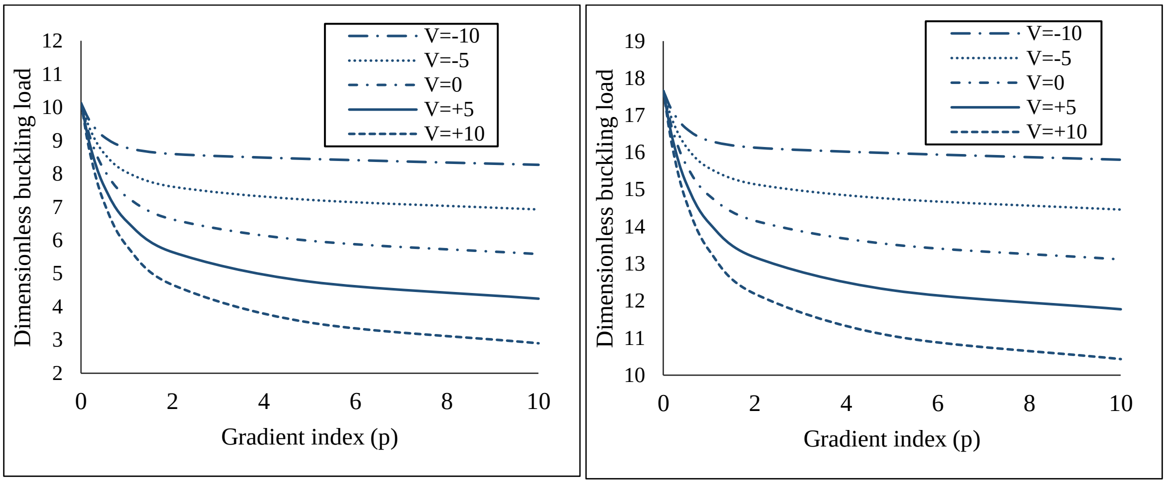
(K_w, K_p)	μ		Ω=-0.05			Ω=0			Ω=+0.05		
			p=0.2	*p*=1	*p*=5	*p*=0.2	*p*=1	*p*=5	*p*=0.2	*p*=1	*p*=5
(0,0)	0	V=-5	6.28461	7.15540	8.57532	9.45495	9.05760	9.20939	12.6253	10.9598	9.84346
		V=0	5.80384	5.71309	6.17148	8.97418	7.61530	6.80554	12.1445	9.51750	7.43961
		V=+5	5.32307	4.27078	3.76763	8.49342	6.17299	4.40170	11.6638	8.07520	5.03577
	1	V=-5	5.47846	6.47131	7.96398	8.64880	8.37352	8.59805	11.8191	10.2757	9.23212
		V=0	4.99769	5.02901	5.56013	8.16803	6.93121	6.19420	11.3384	8.83342	6.82827
		V=+5	4.51692	3.58670	3.15629	7.68726	5.48891	3.79036	10.8576	7.39111	4.42442
	2	V=-5	4.80520	5.9000	7.45342	7.97554	7.80221	8.08749	11.1459	9.70442	8.72155
		V=0	4.32443	4.45770	5.04957	7.49478	6.35990	5.68364	10.6651	8.26211	6.31771
		V=+5	3.84366	3.01539	2.64572	7.01401	4.91760	3.27979	10.1844	6.81980	3.91386
(25,5)	0	V=-5	13.8176	14.6884	16.1084	16.988	16.5906	16.7424	20.1583	18.4928	17.3765
		V=0	13.3369	13.2461	13.7045	16.5072	15.1483	14.3386	19.6776	17.0505	14.9726
		V=+5	12.8561	11.8038	11.3007	16.0264	13.7060	11.9347	19.1968	15.6082	12.5688
	1	V=-5	13.0115	14.0043	15.4970	16.1818	15.9066	16.1311	19.3522	17.8088	16.7651
		V=0	12.5307	12.5620	13.0932	15.7011	14.4642	13.7272	18.8714	16.3665	14.3613
		V=+5	12.0499	11.1197	10.6893	15.2203	13.0219	11.3234	18.3906	14.9241	11.9575
	2	V=-5	12.3382	13.4330	14.9864	15.5086	15.3352	15.6205	18.6789	17.2374	16.2546
		V=0	11.8575	11.9907	12.5826	15.0278	13.8929	13.2167	18.1982	15.7951	13.8507
		V=+5	11.3767	10.5484	10.1788	14.5470	12.4506	10.8128	17.7174	14.3528	11.4469



(a) K_w=K_p=0 (b) K_w=25, K_p=5

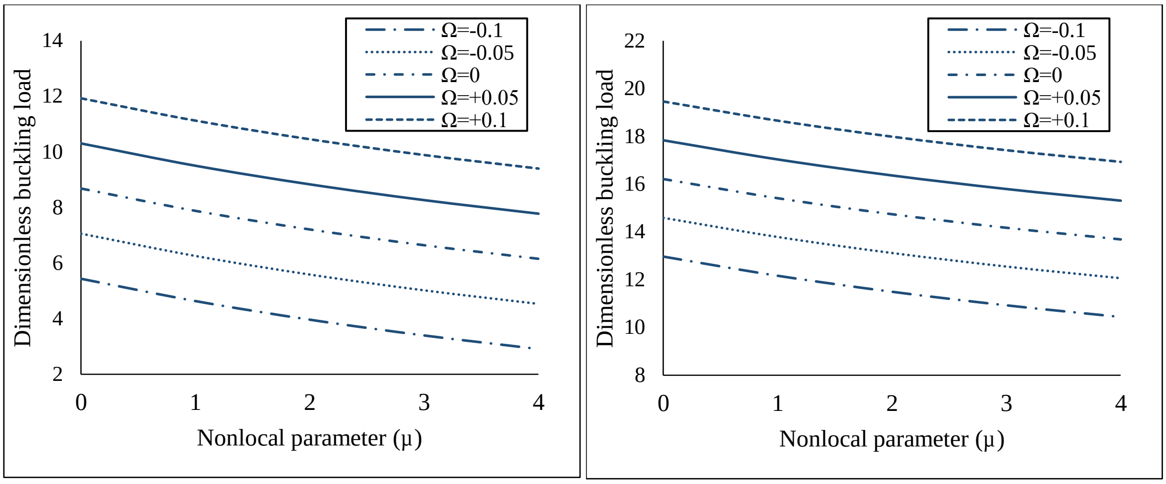
Fig. 2 Effect of external magnetic potential on the dimensionless buckling load of the S-S FG nanobeam with respect to gradient index (*L/h*=20, *V*=+5, μ=2)

reduction in buckling load is more significant with respect to the positive magnetic potentials and external electric voltages. Moreover, it is observed that influence of larger values of gradient index



(a) K_w=K_p=0 (b) K_w=25, K_p=5

Fig. 3 Effect of external electric voltage on the dimensionless buckling load of the S-S FG nanobeam with respect to gradient index (L/h=20, Ω =+0.05, μ=2)



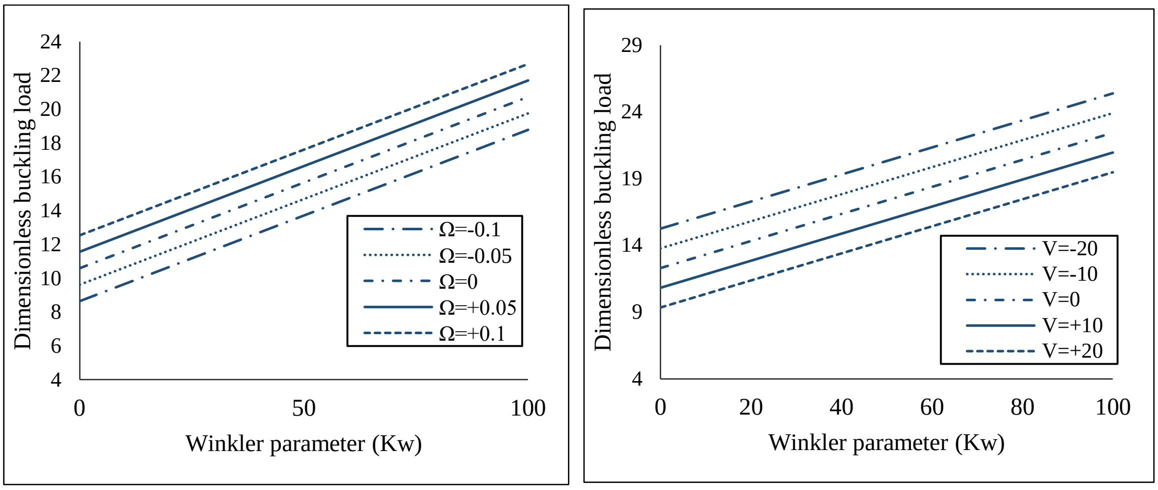
(a) K_w=K_p=0 (b) K_w=25, K_p=5

Fig. 4 Effect of nonlocal parameter and magnetic field on the dimensionless buckling load of the S-S FG nanobeam (L/h=20, V=+5, μ=0.2)

on the magnetic potential is less than lower gradient indexes, whereas this trend is reverse for electric voltage and the impact of higher gradient indexes on electric voltage is more sensible.

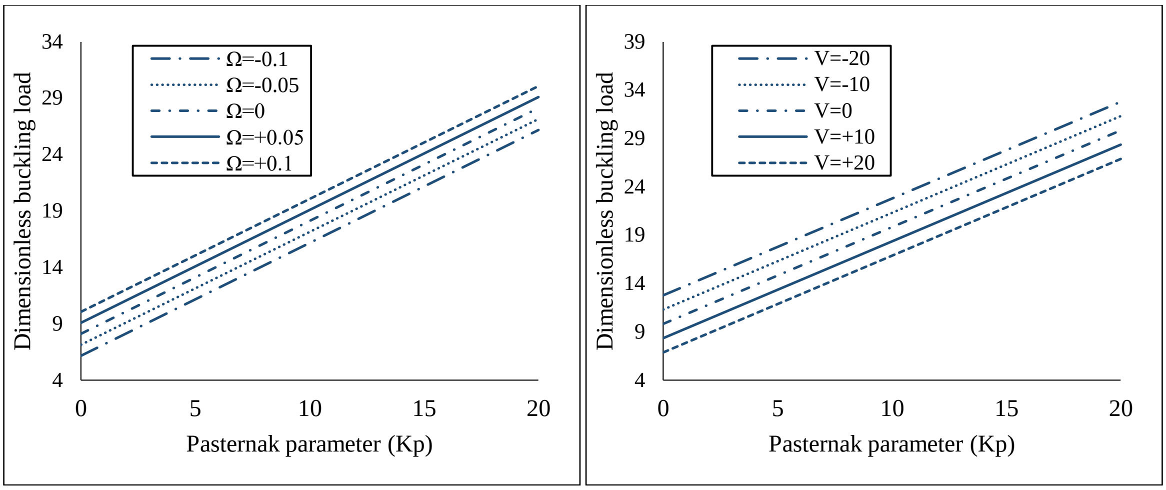
The effect of nonlocal parameter on the first non-dimensional buckling load of the S-S MEE-FG nanobeams is depicted in Fig. 4 (L/h=20, V=+5, μ=0.2). It is apparently seen that nonlocal parameter has a softening influence on the beam structure and reduces the buckling loads. So, nonlocal beam model produces smaller buckling loads compared to local beam model. Also, it is observed that nonlocality is independent of magnetic field.

The variations of the dimensionless buckling load of MEE-FG nanobeams versus the Winkler and Pasternak parameters for various magnetic potentials and electric voltages at L/h=20, p=0.2 and μ=2 are presented in Figs. 5 and 6, respectively. One can find that, with the increase of



(a) (b)

Fig. 5 Effect of external magnetic potential and electric voltage on the dimensionless buckling load of the S-S FG nanobeam with respect to Winkler parameter; (a) V=+5, (b) Ω=+0.05 (L/h=20, μ =2, p=0.2, K_p=5)

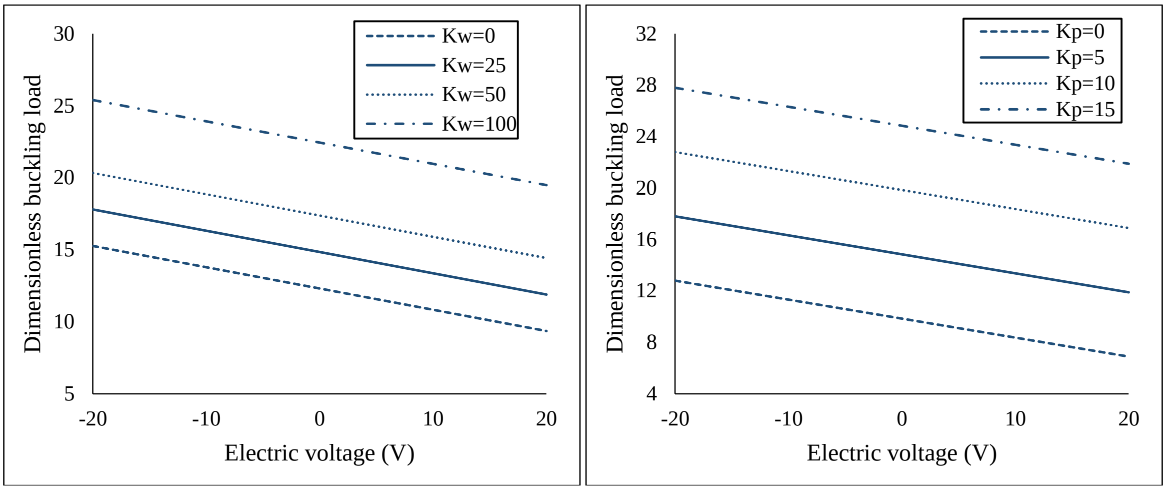


(a) (b)

Fig. 6 Effect of external magnetic potential and electric voltage on the dimensionless buckling load of the S-S FG nanobeam with respect to Pasternak parameter; (a) V=+5, (b) Ω=+0.05 (L/h=20, μ =2, p=0.2, K_p=25)

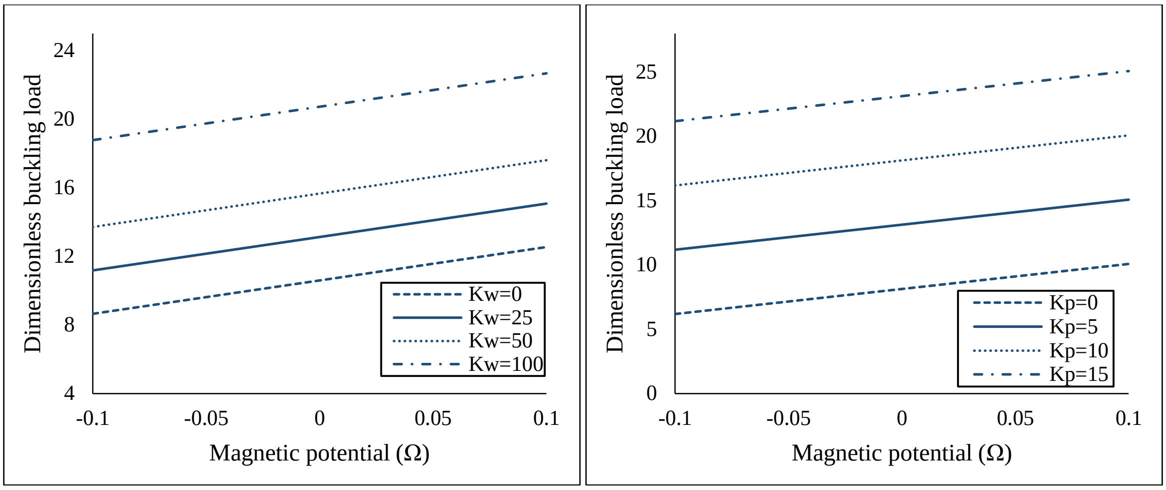
Winkler and Pasternak parameters for any sign and magnitude of magnetic potential and electric voltage, the non-dimensional buckling load increases, due to the enhancement in stiffens of the MME-FG nanobeam structure. Moreover, it is clearly seen that the effect of Pasternak elastic parameter on the non-dimensional buckling load is more than Winkler parameter. Therefore, the shear layer or Pasternak parameter of elastic foundation plays an important role on the mechanical responses of FG structure and should be considered in their analysis.

Figs. 7-8 demonstrate the variations the dimensionless buckling load of nonlocal FG beams made of magneto-electro-elastic materials with respect to external electric voltage and magnetic



(a) K_p=5 (b) K_w=25

Fig. 7 Effect of elastic foundation on the dimensionless buckling load of the S-S FG nanobeam with respect to electric voltage (L/h=20, Ω=+0.05, μ=2, p=1)

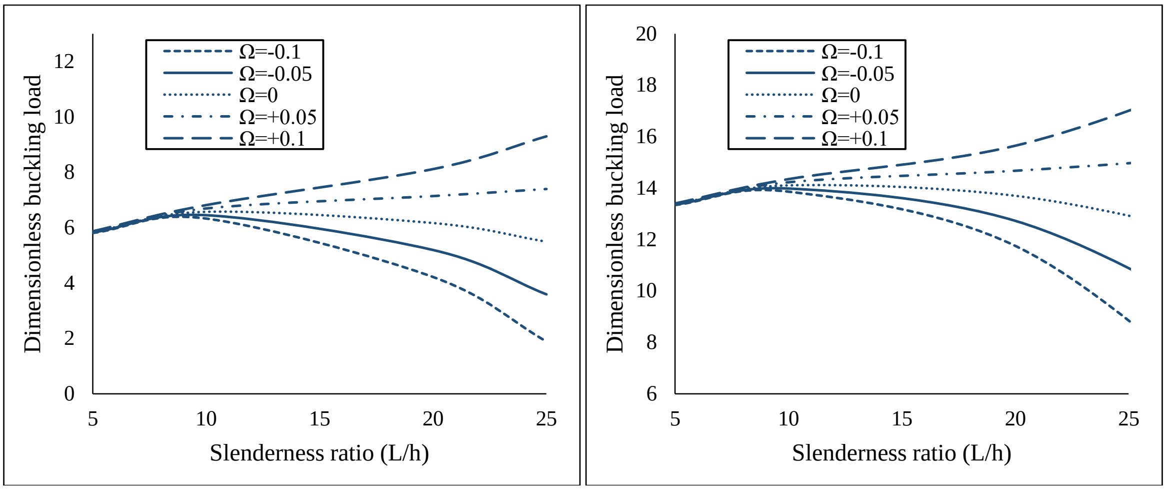


(a) K_p=5 (b) K_w=25

Fig. 8 Effect of elastic foundation on the dimensionless buckling load of the S-S FG nanobeam with respect to magnetic potential (L/h=20, V=+5, μ=2, p=1)

potential, respectively at L/h=20 for various Winkler and Pasternak parameters. It is evident that external electric voltage has a decreasing influence on the buckling loads of MEE-FG nanobeams when it changes from negative values to positive one, whereas by varying the magnetic potential values from negative values to positive one, the non-dimensional buckling load rise. As a general consequence, it must be mentioned that the impact of magnetic field on the buckling behavior of FG nanobeams is more than electric field.

Finally, Fig. 9 depicts the variations of the non-dimensional buckling load of MEE-FG nanobeam with respect to slenderness ratio for different magnetic potentials at power-law index



(a) K_w=K_p=0 (b) K_w=25, K_p=5

Fig. 9 Effect of slenderness ratio on the dimensionless buckling load of the S-S FG nanobeam for various magnetic potentials (V=+5, μ=1, p=1)

p=1 and nonlocal parameter μ=1 (nm)^2. It is shown that slenderness ratio has a significant effect on the stability of MEE-FG nanobeams. Hence, the higher values of slenderness ratio have more influence on the dimensionless buckling load. Also, it is observable that positive values of magnetic potential show an increasing influence on buckling loads of FG nanobeams, whereas the negative ones have a reducing impact. This is due to the reason that compressive and tensile in-plane forces are generated in the nanobeam when positive and negative magnetic potentials are applied, respectively.

5. Conclusions

This paper presents a nonlocal higher-order beam model for buckling analysis of magneto-electro-elastic FG nanobeams resting on two-parameter elastic foundation including linear springs and a shear layer. Governing equations obtained using Hamilton's principle as well as nonlocal elasticity theory which captures the small size influences are solved applying Navier solution method. Magneto-electro-elastic properties of the FG nanobeams are supposed to be varied continuously through the thickness direction according to power-law model. A detailed parametric study is conducted to study the influences of the magnetic potential, electric voltage, elastic foundation, nonlocal parameter, material composition and slenderness ratio on the buckling responses of the MEE-FG nanobeams. It is deduced that nonlocality and power-law exponent yields in reduction on both rigidity of the nanobeam structure and buckling loads. But with an increment in value of Winkler or Pasternak parameters the rigidity of the MEE-FG nanobeam and buckling load growth. Also, it is observed that the magnitude and sign of magnetic potential and electric voltage have a notable influence on the buckling loads of MEE-FG nanobeams.

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