Advances in Nano Research, *Vol. 3, No. 4 (2015) 193-206*
DOI: http://dx.doi.org/10.12989/anr.2015.3.4.193

Critical buckling load of chiral double-walled carbon nanotube using non-local theory elasticity

Awda Chemi[1], Houari Heireche[1], Mohamed Zidour[*2,3], Kaddour Rakrak[1,4] and Abdelmoumen Anis Bousahla[1]

[1]*Laboratoire de Modélisation et Simulation Multi-échelle, Université de Sidi Bel Abbés, Algerie*
[2]*Laboratoire des Matériaux et Hydrologie, Université de Sidi Bel Abbés, BP 89 Cité Ben M'hidi, 22000 Sidi Bel Abbés, Algerie*
[3]*Université Ibn Khaldoun, BP 78 Zaaroura, 14000 Tiaret, Algerie*
[4]*Département de Physique, Faculté des Sciences, Université Hassiba benbouali, Chlef, Algerie*

(Received November 26, 2015, Revised December 23, 2015, Accepted December 28, 2015)

Abstract. The present paper investigate the elastic buckling of chiral double-walled carbon nanotubes (DWCNTs) under axial compression. Using the non-local elasticity theory, Timoshenko beam model has been implemented. According to the governing equations of non-local theory, the analytical solution is derived and the solution for non-local critical buckling loads is obtained. The numerical results show the influence of non-local small-scale coefficient, the vibrational mode number, the chirality of carbon nanotube and aspect ratio of the (DWCNTs) on non-local critical buckling loads of the (DWCNTs). The results indicate the dependence of non-local critical buckling loads on the chirality of single-walled carbon nanotube with increase the non-local small-scale coefficient, the vibrational mode number and aspect ratio of length to diameter.

Keywords: double-walled carbon nanotubes; chirality; buckling; small-scale; non-local elasticity

1. Introduction

The single-walled carbon nanotube (SWNT) and multi-walled carbon nanotube (MWNT) were first discovered by Iijima (1991), Iijima and Ichihashi (1993). Recent studies indicated that carbon nanotubes (CNTs) have a mechanical and thermal properties (Dresselhaus and Avouris 2001, Zidour *et al*. 2015), possess superior electronic, others studies have showed that they have good properties Recently CNT can be used in nanocomposite structures (carbon nanotube-reinforced composite (CNTRC)) (Tagrara *et al*. 2015, Aydogdu 2014), can be used for nanoelectronics and nanodevices (Dai *et al*. 1996, Baghdadi *et al*. 2015, Besseghier *et al*. 2015).

Due the limited to systems computation of molecular dynamics (MD) simulations and the difficulties encountered in experimental methods to predict the responses of nanostructures under different loading conditions, the continuum mechanics methods are often used to investigate the behaviour of carbon nanotubes (CNTs) (Bouazza *et al*. 2015, Zidour *et al*. 2014). Recently, the

∗Corresponding author, Ph.D., E-mail: zidour.mohamed@yahoo.fr

http://www.techno-press.com/?journal=anr&subpage=7 ISSN: 2287-237X (Print), 2287-2388 (Online)

continuum mechanics approach has been widely used to study the responses of micro and nanostructures, such as the static and dynamic (Belabed *et al*. 2013, Bourada *et al*. 2015, Ait Yahia *et al*. 2015, Benzair *et al*. 2008, Mahi *et al*. 2015, Hebali *et al*. 2014), the buckling and thermo-mechanical analysis of (CNTs) (Ait Amar Meziane *et al*. 2014, Amara *et al*. 2010, Hamidi *et al*. 2015, Zidi *et al*. 2014, Bouderba *et al*. 2013, Tounsi *et al*. 2013). More recently, Yakobson *et al*. (1996) utilize a continuum shell model to predict the buckling of a single-walled carbon nanotube and their results are compared with molecular dynamics simulations. Murmu and Adhikari (2010) have analyzed the longitudinal vibration of double nanorod systems using the non-local elasticity.

Continuum elastic-beam models have been widely used to study the critical buckling loads in (CNTs). For example, Semmah *et al*. (2014) presented the effect of the chirality on critical Buckling temperature of zigzag single-walled carbon nanotubes using the nonlocal continuum theory. Their results indicated that the critical buckling temperature can be overestimated by the local beam model if the small-scale effect is overlooked for long nanotubes. In addition, significant dependence of the critical buckling temperatures on the chirality of zigzag carbon nanotube is confirmed. Mohammadimehr *et al*. (2011) investigated the Buckling analysis of double-walled carbon nanotubes embedded in an elastic medium under axial compression using non-local Timoshenko beam theory. A molecular dynamics simulation is also utilized by Odegard *et al*. (2002), Zhang *et al*. (2007) to investigate the buckling behavior of nanotubes.

The Young's moduli used in this study of three types of single-walled carbon nanotubes (SWCNTs), armchair, zigzag and chiral tubules, are calculated by Bao *et al*. (2004) based on molecular dynamics (MD) simulation. They numeric results are in good agreement with the existing experimental ones (Liu *et al*. 2001, Tombler *et al*. 2000). This approach represents the dynamics of atoms or molecules of the materials by a discrete solution of Newton's classical equations of motion. The inter-atomic forces required for the equations of motions are obtained on the basis of interaction energy functions. Cornwell and Wille (1997) used the (MD) with the Tersoff-Brenner potential (1990) to obtain the Young's modulus of (SWCNTs) about 0.8 TPa. Jin *et al*. (2003) used (MD) and force-constant approach and reported the Young's modulus of (SWCNTs) to be about 1236±7 GPa.

The present study is concerned with the use of the non-local Timoshenko beam model to analyse the non-local critical buckling loads of double-walled carbon nanotubes (DWCNTs). The solution for critical loads is obtained. Influence of the chirality of carbon nanotube, aspect ratio of the (SWCNTs), non-local small-scale coefficient and the vibrational mode number, are studied and discussed.

2. Single-walled carbon nanotube (SWCNT)

A single-walled carbon nanotube (SWCNT) is theoretically assumed to be made by rolling a graphene sheet (Fig. 1). In terms of the chiral vector ($\vec{C}_h$), the fundamental structure of carbon nanotubes can be classified into three categories as zigzag, armchair and chiral shown in (Fig. 1).

The chiral vector can be expressed in terms of base vectors ($\vec{a}_1$) and ($\vec{a}_2$)

$$\vec{C}_h = m\vec{a}_1 + n\vec{a}_2 \tag{1}$$

where the integer pair (*n*, *m*) are the indices of translation.

According to different values of integers (*n*, *m*), (SWCNTs) can be classified into zigzag ((*n or*

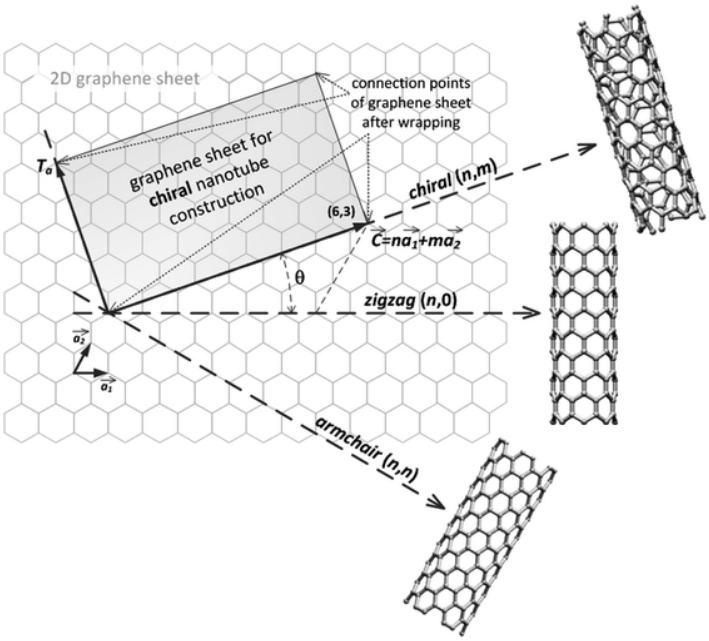

Fig. 1 Schematic the chiral vector of carbon nanotube

m)=0), armchair (*n*=*m*) and chiral (*n*≠*m*) (Fig. 1).

The diameter of (SWCNTs) can be expressed in terms of integers (*n*, *m*) (Tokio 1995)

$$d = a\sqrt{3(n^2 + m^2 + nm)}/\pi \tag{2}$$

where (*a*) is the length of the carbon-carbon bond which is (1.42 ***A***°).

3. Nonlocal Timoshenko beam models of (SWCNTs)

The nonlocal continuum elasticity theory assumed that the stress at a reference point is considered to be a functional of the strain field at every point in the body (Eringen 1983). The nonlocal elasticity theory is applied in various types of nanostructures (nano FGM structures, nanotube..) such as the static (Zemri *et al*. 2015, Aissani *et al* 2015), the buckling (Larbi Chaht *et al*. 2015), free vibration (Belkorissat *et al*. 2015), wave propagation (Heireche *et al*. 2008) and thermo-mechanical analysis of (CNTs) (Tounsi *et al*. 2013).The local or classical theory of elasticity is obtained when the effects of strains at points other than *x* are neglected. For homogeneous and isotropic elastic solids, the constitutive equation of non-local elasticity can be given by Eringen. Non-local stress tensor (*t*) at point (*x*′) is defined by

$$\begin{aligned} &\sigma_{ij,j} = 0 \\ &\sigma_{ij}(x) = \int K(|x - x'|,\ \tau) C_{ijkl}\varepsilon_{kl}(x')dV(x'), \quad \forall x \in V \\ &\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \end{aligned} \tag{3}$$

Where (C_{ijkl}) is the classical, macroscopic stress tensor at point x', σ_{ij} and ε_{ij} are stress and strain tensors respectively. $K(|x-x'|, \tau)$ is the kernel function and ($\tau=e0a/l$) is a material constant that depends on internal and external characteristic length (such as the lattice spacing and wavelength), where ($e0$) is a constant appropriate to each material, a is an internal characteristic length, e.g., length of (C-C) bond, lattice parameter, granular distance, and (l) is an external characteristic length.

Non-local constitutive relations for present nano beams can be approximated to a one-dimensional form as

$$\left(1-e0a^2\frac{\partial^2}{\partial x^2}\right)\sigma_x = E\left(z\frac{\partial \psi}{\partial x}\right) \tag{4}$$

$$\left(1-e0a^2\frac{\partial^2}{\partial x^2}\right)\tau_{xz} = G\left(\psi-\frac{\partial w}{\partial x}\right) \tag{5}$$

where E and G are the Young's and shear modulus, respectively, ψ is the rotation angle of cross-section of the beam and w is the transverse displacement. Thus, the scale coefficient ($e0a$) in the modelling will lead to small-scale effect on the response of structures at nano size. In addition, $e0$ is a constant appropriate to each material, and a is an internal characteristic length of the material (e.g., length of C-C bond, lattice spacing, granular distance).

The shear force and the bending moment can be defined by

$$M = \int_A z\sigma_x dA \quad , \quad T = \int_A \tau_{xy} dA \tag{6}$$

The force equilibrium equations in vertical direction and the moment on the one-dimensional structure can be easily provided From the free body diagram of an infinitesimal element of a beam structure subjected to an axial loading P.

$$\frac{dT}{dx} = -q(x) \quad \text{and} \quad \frac{dM}{dx} = T - P\frac{dw}{dx} \tag{7}$$

where M and V are the resultant bending moment and the resultant shear force, respectively, P is the axial compression.

Based on the Eqs. (4), (5), (6) and (7), the shear force T and the bending moment M for the non-local model can be expressed as

$$T = \beta AG\left(\psi-\frac{dw}{dx}\right) - e0a^2P\frac{dq(x)}{dx} \tag{8}$$

$$M = EI\frac{d\psi}{dx} + e0a^2\left(\frac{dT}{dx} - P\frac{d^2w}{dx^2}\right) \tag{9}$$

where A is the cross-section area of the beam, ($I = \int_A z^2 dA$) is the moment of inertia, and β a correction factor depending on the shape of the cross-section of the considered beam.

Substituting Eqs. (8) and (9) into Eq. (7) and eliminating ψ yield the following differential equation of a non-local Timoshenko beam theory.

$$EI\frac{d^4w}{dx^4}+\left(1-e0a^2\frac{d^2}{dx^2}\right)\left(\left(1-\frac{EI}{\beta AG}\frac{d^2}{dx^2}\right)q(x)+P\frac{d^2w}{dx^2}\right)=0 \tag{10}$$

4. Nonlocal Timoshenko beam models of (DWCNTs)

The above equation is the equilibrium equation of a Timoshenko beam considering the non-local effects.

The double-walled carbon nanotubes are distinguished from traditional elastic beam by their hollow two-layer structures and associated with van der Waals interaction forces.

Assuming that the inner and outer tubes have the same effective material constants and layer thickness, the Eq. (10) can be used to each of the inner and outer tubes of the double-walled carbon nanotubes.

$$EI_1\frac{d^4w_1}{dx^4}+\left(1-e0a^2\frac{d^2}{dx^2}\right)\left(\left(1-\frac{EI_1}{\beta A_1G}\frac{d^2}{dx^2}\right)q_{12}(x)+P\frac{d^2w_1}{dx^2}\right)=0 \tag{11a}$$

$$EI_2\frac{d^4w_2}{dx^4}+\left(1-e0a^2\frac{d^2}{dx^2}\right)\left(\left(1-\frac{EI_2}{\beta A_2G}\frac{d^2}{dx^2}\right)q_{21}(x)+P\frac{d^2w_2}{dx^2}\right)=0 \tag{11b}$$

where subscripts 1 and 2 are used to denote the quantities associated with the inner and outer tubes, respectively, $\boldsymbol{q}_{12}$ and $\boldsymbol{q}_{21}$ denote the van der Waals pressure per unit axial length.

The deflection of two tubes is coupled through the van der Waals force (Reulet *et al*. 2000). Based on the Lennard-Jones model the van der Waals interaction potential as a function of the interlayer spacing between two adjacent tubes. The interlayer interaction potential between two adjacent tubes can be approximated by the potential obtained for two flat graphite monolayers, denoted by $g(\Delta)$, where Δ is the interlayer spacing (Girifalco and Lad 1956, Girifalco and Chem 1991). The van der Waals pressure should be a linear function of the difference of the deflections of the two adjacent layers at the point as follows

$$q_{12}=tc(w_2-w_1)\quad,\quad q_{21}=-\frac{R_1}{R_2}tc(w_2-w_1) \tag{12}$$

where R_1 and R_2 are the radius of the inner and the outer tube, respectively. c is the intertube interaction coefficient per unit length between two tubes, which can be estimated by (Sudak 2003).

$$c=\frac{320\,(2R_1)\,erg/cm^2}{0.16\,d^2}\quad(d=0.142\,nm) \tag{13}$$

Let us assume the buckling modes as

$$w_1=A\sin\left(\frac{m\pi}{L}x\right)\qquad\text{and}\qquad w_2=B\sin\left(\frac{m\pi}{L}x\right) \tag{14}$$

The above equations satisfy the simply supported boundary conditions which are

$$w_i = \frac{d^2 w_i}{dx^2} = 0 \qquad \text{at} \qquad x = 0,\ L \quad (i = 1, 2) \tag{15}$$

Replacing Eq. (14) into Eq. (11), one can easily obtain the homogeneous system

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = 0 \tag{16}$$

By setting the determinant of coefficients equal to zero, the non-trivial solution for the homogeneous system (19) can be obtained.

Solving equation yields the buckling pressure of the DWCNT in which the effects of different parameters are shown.

$$P = \frac{1}{2}\left(-\alpha_n \pm \sqrt{(\alpha_n)^2 - 4\beta_n} \right) \tag{17}$$

where α_n and β_n in Eq. (17) are defined as

$$\alpha_n = \frac{ct}{\lambda^2}\left(\left(1 + \frac{EI_2}{\beta A_2 G}\lambda^2 \right)\frac{d_1}{d_2} + 1 + \frac{EI_1}{\beta A_1 G}\lambda^2 \right) - \frac{(EI_1 + EI_2)\lambda^2}{\left(1 + \mathrm{e0a}^2\lambda^2\right)}$$

$$\beta_n = \frac{E^2 I_1 I_2 \lambda^4}{\left(1 + \mathrm{e0a}^2\lambda^2\right)^2} - \frac{(EI_1 + EI_2)}{\left(1 + \mathrm{e0a}^2\lambda^2\right)} ct\left(\left(1 + \frac{EI_2}{\beta A_2 G}\lambda^2 \right)\frac{d_1}{d_2} + 1 + \frac{EI_1}{\beta A_1 G}\lambda^2 \right) + \left(1 + \frac{EI_2}{\beta A_2 G}\lambda^2 \right)\left(1 + \frac{EI_1}{\beta A_1 G}\lambda^2 \right)\frac{ct^2}{\lambda^4}$$

If neither the rotary inertial nor the shear deformation is taken into account, the Solving equation yields the buckling pressure of the DWCNT using nonlocal elastic Euler beam as follows:

$$\alpha_n = \frac{ct}{\lambda^2}\left(\frac{d_1}{d_2} + 1 \right) - \frac{(EI_1 + EI_2)\lambda^4}{\left(1 + \mathrm{e0a}^2\lambda^2\right)}$$

$$\beta_n = \frac{E^2 I_1 I_2 \lambda^4}{\left(1 + \mathrm{e0a}^2\lambda^2\right)^2} - \frac{(EI_1 + EI_2)}{\left(1 + \mathrm{e0a}^2\lambda^2\right)}\lambda^4 ct\left(\frac{d_1}{d_2} + 1 \right)$$

5. Results and discussions

The Young's moduli used in this study of three types of single-walled carbon nanotubes (SWCNTs), armchair, zigzag and chiral tubules, are calculated by Bao *et al*. (2004) based on molecular dynamics (MD) simulation. They numeric results are in good agreement with the existing experimental ones (Liu *et al*. 2001, Tombler *et al*. 2000).

Tu and Ou-Yang **(**2002) indicated that the relation between Young's modulus of multi-walled carbon nanotubes (MWCNTs) and the layer number N' can be expressed as

$$E_{MWNT} = \frac{N'}{N' - 1 + t/h}\frac{t}{h}E_{SWNT} \tag{18}$$

where E_{MWNT}, E_{SWNT}, t, $N'$and h are Young's modulus of multi-walled nanotubes, Young's modulus of single-walled nanotubes, effective wall thickness of single-walled nanotubes, number of layers and layer distance. In the case of single-walled carbon nanotubes, N'=1 and E_{MWNT}=E_{SWNT}.

Based on the formulations obtained above with the nonlocal Timoshenko beam models, the critical buckling loads of double-walled carbon nanotubes (DWCNT's) are discussed here. To investigate the critical buckling loads of (DWCNTs), the results including the aspect ratio of the (DWCNTs), the vibrational mode number and effect of nonlocal small-scale coefficient. In addition, to explore the effect of chirality, the critical buckling loads of different chiral of (DWCNTs) are compared. The parameters used in calculations of (DWCNT) are given as follows: the effective thickness of (CNTs) taken to be 0.285 nm, the mass density

ρ=2.3 g/cm^3, layer distance h=0.34 nm and poisson ratio υ=0.19. (Naceri *et al*. 2011, Boumia *et al*. 2014).

The Young's modulus of (SWCNTs) and (DWCNTs) employed in this study (Table 1), are calculated by Bai *et al.* (2004), Tu and Ou-Yang **(**2002) respectively. The results show the decreasing of Young's modulus (DWCNTs) for some chirality nanotube. The reason for this phenomenon is attributed to the weak van der Waals forces between the inner and outer tube.

Table 1 lists the values of Young's modulus of single and double carbon nanotube for different chirality's

(n,m)	Young's modulus (SWNT) (GPa) Bao Wen Xing *et al*. (2004)	Young's modulus (DWNT) (GPa) Tu and Ou-Yang (2002)
Armchair		
(8,8) (12,12)	934.960	852.684
(10,10) (15,15)	935.470	853.149
(12,12) (17,17)	935.462	853.141
(14,14) (19,19)	935.454	853.134
(16,16) (21,21)	939.515	856.838
(18,18) (23,23)	934.727	852.471
(20,20) (25,25)	935.048	852.764
Zigzag		
(14,0) (23,0)	939.032	856.397
(17,0) (26,0)	938.553	855.960
(21,0) (30,0)	936.936	854.486
(24,0) (33,0)	934.201	851.991
(28,0) (37,0)	932.626	850.555
(31,0) (40,0)	932.598	850.529
(35,0) (44,0)	933.061	850.952
Chiral		
(12,6) (18,10)	927.671	846.036
(14,6) (20,10)	921.616	840.514
(16,8) (22,12)	928.013	846.348
(18,9) (24,13)	927.113	845.527
(20,12) (26,16)	904.353	824.770
(24,11) (30,15)	910.605	830.472
(30,8) (36,12)	908.792	828.818

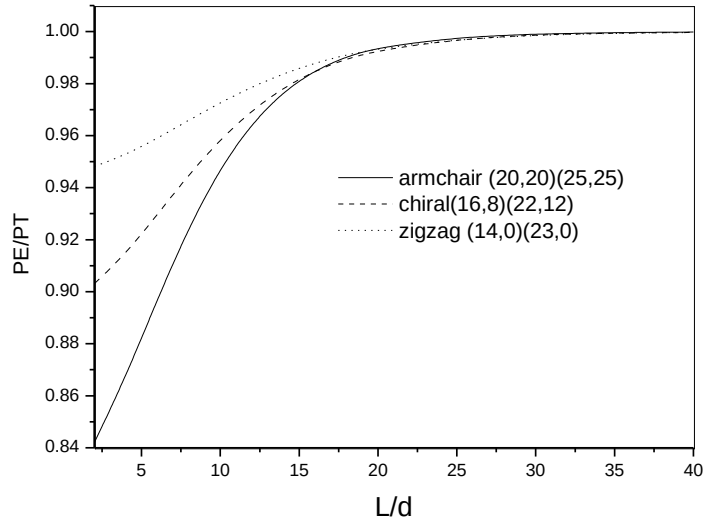

Fig. 2 The values of ratios (PE/PT) of carbon nanotube, with respect ratio using the nonlocal Timoshenko beam model (PT) and nonlocal Euler beam model (PE); The scale coefficient is (*e*0*a*=2 nm)

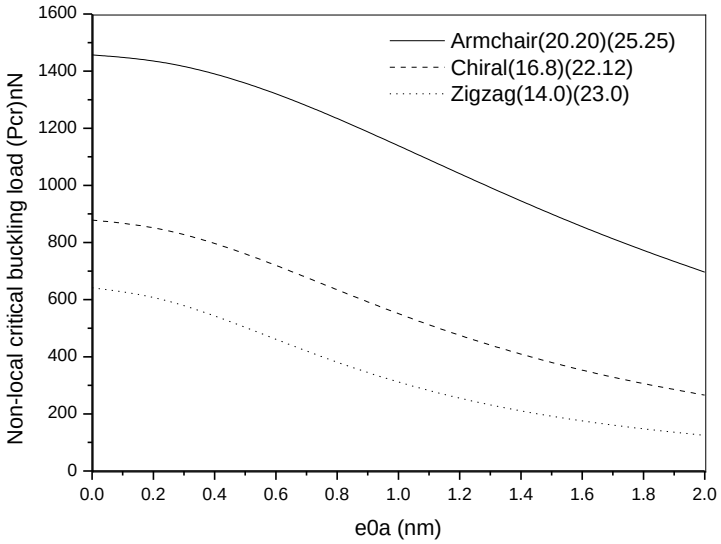

Fig. 3 Relation between the critical buckling load and the scale coefficients (*e*0*a*) of double carbon nanotube; The value of (*L/d*) is 10

To analyse the deference between the nonlocal Timoshenko (NTB) and nonlocal Euler (NEB) beam model in the present study, the (Fig. 2) depicts The critical buckling loads ratios (PE/PT) of three types of double-walled carbon nanotubes (DWCNTs), armchair, zigzag and chiral tubules,with respect to length-to-diameter ratio. It can be concluded that when (*L/d*>20) the difference between the results predicted by (NTB) and (NEB) is negligible. This is due to the fact that the shear effect is negligible for long nanotubes.

In the present study, the (Fig. 3) illustrate the dependence of the non-local critical buckling load

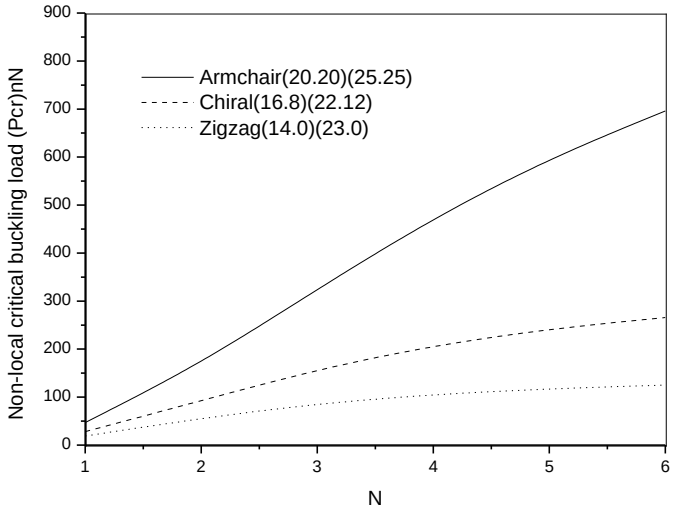

Fig. 4 Relation between the non-local critical buckling load and the mode number (*N*) of double carbon nanotube; The value of (*L/d*) is 10

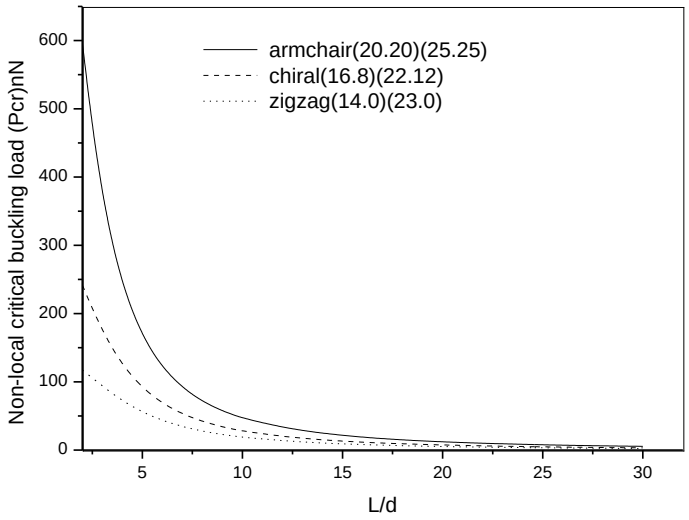

Fig. 5 Effect of aspect ratio and chirality of double carbon nanotube on the Non-local critical buckling load in fundamental mode and scale coefficient (*e*0*a*=2 nm)

on the chirality of double-walled carbon nanotubes (DWCNTs), armchair, zigzag and chiral for different values of small-scale coefficient. The ratio of the length to the diameter, *L/d*, is 10. It is clearly seen from (Fig. 3) that the ranges of the non-local critical buckling loads for these chirality of double-walled carbon nanotubes (DWCNTs) are quite different, the range is the smallest for zigzag (14,0) (23,0), but the range is the largest for armchair (20,20) (25,25). The reason for this difference perhaps is attributed to the increasing or decreasing of carbon nanotube diameter. In additional, when the small scale effects is not considered (*e*0*a*=0), the local Timoshenko beam model give a higher values for the critical buckling load. Therefore, it is clear that the critical loads

Table 2 lists the values of non-local critical buckling load for different armchair chirality's, mode number (N) and aspect ratios (L/d) of carbon nanotube, when the value of scale coefficients ($e0a$) is 2 nm

Armchair	L/d=5		L/d=10	
	N=1	N=6	N=1	N=6
(8,8) (12,12)	55.1875	136.5681	18.7654	122.2091
(10,10) (15,15)	72.5996	211.1002	23.2310	183.0479
(12,12) (17,17)	91.0246	308.8093	27.8344	258.5388
(14,14) (19,19)	110.1593	432.1884	32.5434	348.4674
(16,16) (21,21)	130.3348	585.5184	37.4919	453.9389
(18,18) (23,23)	149.6086	762.7478	42.1415	568.0853
(20,20) (25,25)	169.8218	973.1429	47.0423	696.1516

Table 3 lists the values of non-local critical buckling load for different Chiral chirality's, mode number (N) and aspect ratios (L/d) of carbon nanotube, when the value of scale coefficients ($e0a$) is 2 nm

Chiral	L/d=5		L/d=10	
	N=1	N=6	N=1	N=6
(12,6) (18,10)	64.6583	175.9445	21.1724	154.6661
(14,6) (20,10)	73.8794	219.4681	23.4770	189.4703
(16,8) (22,12)	92.3700	318.7122	28.1228	265.7540
(18,9) (24,13)	106.7662	411.7080	31.6601	333.5611
(20,12) (26,16)	127.0945	578.7193	36.4763	447.0672
(24,11) (30,15)	144.7845	735.0185	40.8128	548.4167
(30,8) (36,12)	165.4185	951.8376	45.8015	680.1950

Table 4 lists the values of non-local critical buckling load for different Zigzag chirality's, mode number (N) and aspect ratios (L/d) of carbon nanotube, when the value of scale coefficients ($e0a$) is 2 nm

Zigzag	L/d=5		L/d=10	
	N=1	N=6	N=1	N=6
(14,0) (23,0)	56.1207	139.8065	19.0286	124.9510
(17,0) (26,0)	71.1717	203.9082	22.8868	177.3549
(21,0) (30,0)	92.3398	316.1741	28.1687	264.1026
(24,0) (33,0)	108.6232	421.9061	32.1594	341.1178
(28,0) (37,0)	131.0270	595.3372	37.6157	460.0075
(31,0) (40,0)	148.2492	751.2628	41.7978	560.6713
(35,0) (44,0)	171.5586	994.7399	47.4497	708.6199

decrease for increasing values of small-scale coefficient. This variation demonstrate the precision of the nonlocal theory.

The effect of mode number (N) on the non-local critical buckling loads for various chirality of double-carbon nanotube is demonstrated in (Fig. 4) with aspect ratio (L/d=10) and small-scale coefficient ($e0a$=2 nm). In this figure, it is observed that as the mode number increases, the critical

buckling load increases and the difference becomes obvious between the three types of double-walled carbon nanotubes (DWCNTs), armchair, zigzag and chiral. This significance in higher modes is attributed to the diameter and the influence of small wavelength. For smaller wavelengths, interactions between atoms are increasing and these loads to an increase in the nonlocal effects.

The (Fig. 5) show the effect of aspect ratio (*L/d*) on the non-local critical buckling loads for various chirality of double-walled carbon nanotubes (DWCNTs), armchair, zigzag and chiral with small-scale coefficient ($e0a$=2 nm) and fundamental mode number. In these figure, we present the effect of long and diameter of double-walled carbon nanotubes (DWCNTs) on the non-local critical buckling loads. The critical buckling load gets reduced as one transit from the armchair (20,20) to the chiral (16,8) and then, zigzag (14,0) chirality, when the diameter of nanotube is decreasing. This reduction in the critical buckling load is most pronounced when the nanotube is short. However, it is observed, that the non-local critical buckling loads is more affected by the long of nanotube. The reason for this phenomenon is that a carbon nanotube with larger long or smaller diameter has a larger curvature, which results in a more significant distortion of (C-C) bonds and low critical loads.

The variation of non-local critical buckling loads of double-walled carbon nanotubes (DWCNTs) armchair, chiral and zigzag chirality for the first and the sixth modes with different length-to-diameter ratios based on the non-local Timoshenko beam model are listed in (Table 2). The effects of chirality, mode number and aspect ratio presented in (Figs. 3-5) are detailed in this table for various chirality nanotube. The results show the dependence of the different chirality's of carbon nanotube, Aspect Ratio and, vibrational mode number on the non-local critical buckling loads.

6. Conclusions

This paper studies the Influence of non-local small-scale coefficient, the vibrational mode number, the aspect ratio and the chirality of double-carbon nanotube (DWCNTs) on the nonlocal critical buckling loads using non-local Timoshenko beam theory. The theoretical formulations include the different parameters, the governing equations and the boundary conditions for the (DWCNTs) are solved and the non-local critical buckling loads are obtained.

According to the study, the results showed the dependence of the nonlocal critical buckling loads on the different parameters (chirality of carbon nanotube, small-scale coefficients, Aspect Ratio and mode number). However, it is observed that as the mode number or diameter of (DWCNTs) increases, the nonlocal critical buckling loads increases. The reason of this increases is attributed to the influence of small wavelength when the interactions between atoms are increasing and the large diameter.

In addition, the critical loads also are affected by the increasing or decreasing of small-scale coefficients and long of (DWCNTs). This affection in the critical buckling load is most pronounced in higher values of small-scale coefficients and for short nanotube. The reason for this phenomenon is that a carbon nanotube with higher long has a larger curvature, which results in a more significant distortion of (C-C) bonds and low critical loads.

The presente study may be is helpful in the use of (DWCNTs), such as nanocomposites nanodevices, nanoelectronics and mechanical sensors.

Acknowledgments

This research was supported by the Algerian national agency for development of university research (ANDRU) and university of sidi bel abbes (UDL SBA) in Algeria.

References

Aissani, K., Bachir Bouiadjra, M., Ahouel, M. and Tounsi, A. (2015), "A new nonlocal hyperbolic shear deformation theory for nanobeams embedded in an elastic medium", *Struct. Eng. Mech.*, **55**(4), 743-762.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech.*, `**53**(6), 1143-1165.

Amara, K., Tounsi, A., Mechab, I. and Adda Bedia, E. (2010), "Nonlocal elasticity effect on column buckling of multiwalled carbon nanotubes under temperature field", *Appl. Math. Model.*, **34**, 3933-3942.

Aydogdu, M. (2014), "On the vibration of aligned carbon nanotube reinforced composite beams", *Adv. Nano Res.*, **2**(4), 199-210.

Baghdadi, H., Tounsi, A., Zidour, M. and Benzair, A. (2015), "Thermal effect on vibration characteristics of armchair and zigzag single-walled carbon nanotubes using nonlocal parabolic beam theory", *Full. Nanotub. Carb. Nanostr.*, **23**, 266-272.

Bao, W.X., Zhu, Ch.Ch. and Cui, W.Zh. (2004), "Simulation of Young's modulus of single-walled carbon nanotubes by molecular dynamics", *Physica B*, **352**, 156-163.

Batra, R.C. and Sears, A. (2007), "Continuum models of multi-walled carbon nanotubes", *Int. J. Solid. Stuct.*, **44**, 7577.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos. Part B*, **60**, 274-283.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct.*, **18**(4), 1063-1081.

Besseghier, A., Heireche, H., Bousahla, A.A., Tounsi, A. and Benzair, A. (2015), "Nonlinear vibration properties of a zigzag single-walled carbon nanotube embedded in a polymer matrix", *Adv. Nano Res.*, **3**(1), 29-37.

Benzair, A., Tounsi, A., Besseghier, A., Heireche, H., Moulay, N., and Boumia, L. (2008), "The thermal effect on vibration of single-walled carbon nanotubes using nonlocal Timoshenko beam theory", *J. Phys. D.*, **41**, 225404.

Bouazza, M., Amara, K., Zidour, M., Tounsi, A. and Adda Bedia El A. (2015), "Postbuckling analysis of nanobeams using trigonometric Shear deformation theory", *Appl. Sci. Report.*, **10**(2), 112-121.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct.*, **14**(1), 85-104.

Boumia, L., Zidour, M., Benzair, A. and Tounsi, A. (2014), "Timoshenko beam model for vibration analysis of chiral single-walled carbon nanotubes", *Physica E*, **59**, 186-191.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct.*, **18**(2), 409-423.

Brenner, D.W. (1990), *Phys. Rev. B*, **42**, 9458.

Cornwell, C.F. and Wille, L.T. (1997), "Elastic properties of single-walled carbon nanotubes in compression", *Solid. State. Commun.*, **101**, 555.

Dai, H., Hafner, J.H., Rinzler, A.G., Colbert, D.T. and Smalley R.E. (1996), "Nanotubes as nanoprobes in scanning probe microscopy", *Nature*, **384**, 147-50.

Dresselhaus, M.S. and Avouris, P. (2001), "Carbon nanotubes: synthesis, structure, properties and application", *Top. Appl. Phys.*, **80**, 1-11.

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**, 4703-4710.

Girifalco, L.A. and Lad, R.A., Chem, (1956), "Energy of cohesion, compressibility, and the potential energy functions of the graphite system", *J. Phys.*, **25**, 693.

Girifalco, L.A. (1991), "Interaction potential for carbon (C60) molecules", *J. Phys.*, **95**, 5370.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct.*, **18**(1), 235-253.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *ASCE J. Eng. Mech.*, **140**, 374-383.

Heireche, H., Tounsi, A., Benzair, A., Maachou, M. and Adda Bedia, E.A. (2008) "Sound wave propagation in single-walled carbon nanotubes using nonlocal elasticity", *Physica E*, **40**, 2791-2799.

Iijima, S. (1991), "Helical microtubules of graphitic carbon", *Nature*, **354**, 56-8.

Iijima, S. and Ichihashi, T. (1993), "Single-shell carbon nanotubes of 1 nm diameter", *Nature*, **363**, 603.

Jin, Y. and Yuan, F. G. (2003), "Simulation of elastic properties of single-walled carbon nanotubes", *Compos. Sci. Technol*, **63**, 1507.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct.*, **18**(2), 425-442.

Liu, J.Z., Zheng, Q. S. and Jiang, Q. (2001), "Effect of a rippling mode on resonances of carbon nanotubes", *Phys. Rev. Lett*, **86**, 4843.

Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**, 2489-2508.

Mohammadimehr, M., Saidi, A. R., Ghorbanpour Arani, A., Arefmanesh, A. and Han, Q. (2011), "Buckling analysis of double-walled carbon nanotubes embedded in an elastic medium under axial compression using non-local Timoshenko beam theory", *Proc. IMech E Vol. 225 Part C: J. Mechanical Engineering Science*.

Murmu, T. and Adhikari, S. (2010), "Thermal effects on the stability of embedded carbon nanotubes", *Physica E*, **43**, 415-422.

Naceri, M., Zidour, M., Semmah, A., Houari, S.A., Benzair, A. and Tounsi, A. (2011), "Sound wave propagation in armchair single walled carbon nanotubes under thermal environment", *J. Appl. Phys.*, **110**, 124322.

Odegard, G.M., Gates, Nicholson, T.S.L.M., and Wise, K.E. (2002), "Equivalent-continuum modeling of nano-structured materials", *Compos. Sci. Technol*, **62**, 1869-1880.

Reulet, B. *et al.* (2000), "Acoustoelectric effects in carbon nanotubes", *Phys. Rev. Lett.*, **85**, 2829.

Sears, C. and Batra, A.R.C. (2006), "Buckling of carbon nanotubes under axial compression", *Phys. Rev. B.*, **73**, 085410.

Semmah, A., Tounsi, A., Zidour, M., Heireche H. and Naceri M. (2014), "Effect of the chirality on critical buckling temperature of zigzag single-walled carbon nanotubes using the nonlocal continuum theory", *Full. Nanotub. Carb. Nanostr.*, **23**, 518-522.

Sudak, L.J. (2003), "Column buckling of multiwalled carbon nanotubes using nonlocal continuum mechanics", *J. Appl. Phys.*, **94**, 7281.

Tagrara, S.H., Benachour, A., Bouiadjra, M.B. and Tounsi, A. (2015), "On bending, buckling and vibration responses of functionally graded carbon nanotube-reinforced composite beams", *Steel Compos. Struct.*, **19**(5), 1259-1277.

Tokio, Y. (1995), "Recent development of carbon nanotube", *Synth. Met.*, **70**, 1511-8.
Tombler, T.W., Zhou, C.W., Alexseyev, L. *et al.* (2000), "Reversible nanotube electro-mechanical characteristics under local probe manipulation", *Nature*, **405**, 769.
Tounsi, A, Benguediab, S., Adda Bedia, E.A., Semmah, A. and Zidour, M. (2013), "Nonlocal effects on thermal buckling properties of double-walled carbon nanotubes", *Adv. Nano Res.*, **1**(1), 1-11.
Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Tech.*, **24**, 209-220.
Tu, Z.C. and Ou-Yang, Z.C. (2002), "Single-walled and multiwalled carbon nanotubes viewed as elastic tubes with the effective Young's modulus dependent on layer number", *Phys. Rev. B*, **65**, 233407.
Yakobson, B.I., Brabec, C.J. and J. Bernholc, (1996), "Nanomechanics of carbon tubes: instabilities beyond linear response", *Phys. Rev. Lett.*, **76**, 2511-2514.
Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech.*, **54**(4), 693-710.
Zhang, H.W., Wang, L. and Wang, J. B. (2007), "Computer simulation of buckling behaviour of double-walled carbon nanotubes with abnormal interlayer distances", *Comput. Mater. Sci.*, **39**, 664.
Zidi, M., Tounsi, A., Houari, M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Tech.*, **34**, 24-34.
Zidour, M., Daouadji, T.H., Benrahou, K.H., Tounsi, A., Adda Bedia, El A. and Hadji, L. (2014), "Buckling analysis of chiral single-walled carbon nanotubes by using the nonlocal timoshenko beam theory", *Mech. Compos. Mater.*, **50**(1), 95-104.
Zidour, M., Hadji, L., Bouazza, M., Tounsi, A. and Adda Bedia, El A. (2015), "The mechanical properties of Zigzag carbon nanotube using the energy-equivalent model", *J. Chem. Mater. Res.*, **3**, 9-14.

CC