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Electron spin relaxation control in single electron QDs

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Abstract. So far, all reviews and control approaches of spin relaxation have been done on lateral single electron quantum dots. In such structures, many efforts have been done, in order to eliminate spin-lattice relaxation, to obtain equal Rashba and linear Dresselhaus parameters. But, ratio of these parameters can be adjustable up to 0.7 in a material like GaAs under high- electric field magnitudes. In this article we have proposed a single electron QD structure, where confinements in all of three directions are considered to be almost identical. In this case the effect of cubic Dresselhaus interaction will have a significant amount, which undermines the linear effect of Dresselhaus while it was destructive in lateral QDs. Then it enhances the ratio of the Rashba and Dresselhaus parameters in the proposed structure as much as required and decreases the spin states up and down mixing and the deviation angle from the net spin-down As a result to the least possible value.

Keywords: spin relaxation; spin orbit interaction; Rashba spin-orbit interaction; Dresselhaus spin-orbit interaction; lateral quantum dot

1. Introduction

A promising candidate for realization of a single qubit by a confined electron in a semiconductor quantum dot, that whose spin states $|\downarrow\rangle$ and $|\uparrow\rangle$ represent the logical qubit states (Loss and DiVincenzo 1998). The electron in the quantum dot is not isolated from environment and there exists a coupling between them, which limits the lifetime of the stored information as a result. Then, by controlling the spin-qubit interaction, one can increase the lifetime of stored information till the quantum gates are capable of performing the information processing tasks.

Spin-orbit interaction is the main source of the spin flips for the 3D and 2D electron states in the GaAs-type crystal without an inversion center. Besides, in such a polar-type crystal one finds a strong coupling of electrons to the bosonic environment via the piezoelectric interaction with acoustic phonons. The combination of these two mechanisms provides an effective spin-lattice relaxation (Khaetskii and Nazarov 2001).

In this article we have studied spin-orbit interaction (SOI) as a limiting factor of the lifetime of quantum information.

This paper attempts to discuss size affecting on the electron spin relaxation for a single-electron GaAs quantum dot that the confinement for the electron in the *x* and *y* and *z* directions is almost identical.

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2. Content

In a magnetic Field, B, the spin states of the electron are split by the Zeeman energy $\Delta = |g| \mu_B B$, providing a two level quantum system that can be used as a qubit (Loss and DiVincenzo 1998).

At first we have considered a single electron in a isotropic parabolic confinement potential with form of $(m^*/2)\ \omega_0^2(x^2+y^2)$ in the presence of a perpendicular magnetic field to the surface of the QD, which has been investigated by Fock and Darwin (Darwin 1930, Fock 1928). The Hamiltonian of this system is

$$H_0 = H_1 + H_z \tag{1}$$

$$H_z = p_z^2 / 2m^* + eEz \tag{2}$$

$$H_1 = \frac{(p + \frac{eA}{c})^2}{2m^*} + \frac{1}{2} m\omega_0^2 (x^2 + y^2) + \frac{1}{2} g_0 \mu_B \sigma_z B \tag{3}$$

Where, $p = (p_x, p_y, 0)$ and $A = B/2(-y, x, 0)$ are the electron momentum and the magnetic potential vector respectively. e is the electron charge, c is the speed of light, m^* is electron effective mass and ω_0 is frequency of the confining parabolic potential, g_0 is the bulk g factor, μ_B is the Bohr magneton and σ_z is the diagonal Pauli matrices.

The interaction Hamiltonian, H_1, will be diagonal if we write it as a function of Fock-Darwin number operators $n_\pm = a_\pm^+ a_\pm$ (Jacak *et al.* 1998)

$$H_1 = \hbar\omega_+ (n_+ + 1/2) + \hbar\omega_- (n_- + 1/2) + \frac{1}{2} g_0 \mu_B \sigma_z B \tag{4}$$

$$a_\pm^+ = \frac{1}{2l}(x \pm i\, y) - \frac{l}{2}(\partial_x \pm i\, \partial_y) \tag{5}$$

$$a_\pm = \frac{1}{2l}(x \mp i\, y) + \frac{l}{2}(\partial_x \mp i\, \partial_y) \tag{6}$$

Where

$$\omega_\pm = \Omega \pm \omega_c / 2 \quad , \quad \Omega = \sqrt{\omega_0^2 + \omega_c^2 / 4} \tag{7}$$

Here, ω_c is cyclotron frequency and $l = \sqrt{\hbar / m^* \Omega}$ will be the Fock-Darwin radius.
One can easily obtain eigenvalues of the H_1 as

$$E_{n_+, n_-, \sigma_z} = (n_+ + 1/2)\, \hbar\omega_+ + (n_- + 1/2)\, \hbar\omega_- - \frac{1}{2} \hbar\omega_z \sigma_z \tag{8}$$

Where, $n_+, n_- = 0, 1, 2, \ldots$ and $\omega_z = |g| \mu_B B / \hbar$ is the Zeeman frequency.

The last term of the system Hamiltonian, H_z, Eq. (1) suggests confinement of a quantum well in z growth direction with ground state (de Sousa and Das Sarma 2003)

$$\psi_{0z}(z) = 1.461\kappa^{1/2} Ai(\kappa z + \zeta_1) \tag{9}$$

$$\kappa = (2m^* eE/\hbar^2)^{1/3} \tag{10}$$

Where *Ai* is Airy function and ζ_1=-2.3381 is the first zero of the *Ai*, than in Wentzel-Kramers-Brillouin (WKB) approximation, is given as (Abu-Safe 2003)

$$\psi_{0z} = \sqrt{\frac{2}{L_1}} \sin\frac{\pi z}{L_1} \tag{11}$$

In this approximation, the width of the well is given by an effective width as

$$L_1 = \frac{2}{(0.7637)^2 \kappa} \tag{12}$$

And the ground-state energy is E_{0z}=- $\zeta_1 eE/\kappa$.

In the spin flip process, firstly spin-orbit interaction causes an orbital spins-up and spins-down to mix with different spins of other orbitals, then the acoustic phonons causes the transition of the newly produced eigenfunctions among the Zeeman sublevels.

First of all, in order to calculate transition rate of spin relaxation, we obtain the corrected eigenfunctions considering the spin-orbit interaction as a perturbation. Afterward using the Fermi Golden Rule we obtain the spin relaxation rate by phonon between Zeeman sublevels of the ground state (de Sousa and Das Sarma 2003)

$$\Gamma_{ep} = \frac{2\pi}{\hbar}\sum_q \left|\left\langle 00\downarrow\left|V_{ep}(q)\right|00\uparrow\right\rangle\right|^2 \delta(\Delta - \hbar\omega_q) \quad = \frac{v}{(2\pi)^2\hbar}\int d^3q \left|\left\langle 00\downarrow\left|V_{ep}(q)\right|00\uparrow\right\rangle\right|^2 \delta(\Delta - \hbar\omega_q) \tag{13}$$

Where, *v* is volume of the system and ω_q is the phonon frequency.

2.1 Spin orbit interaction

The SOI is taken in to account by adding the Dresselhaus (Dresselhaus 1955) and Rashba (Bychkov and Rashba 1984) terms for conduction band electrons.

A kind of spin-orbit interaction called Bychkov- Rashba will appear because of structure inversion asymmetry in solid state system described by the following Hamiltonian

$$H_{BR} = \alpha_{BR}(\sigma_x K_y - \sigma_y K_x) \tag{14}$$

Where, α_{BR} is the coefficient of Bychkov-Rashba interaction and is taken to be proportional to the average effective electric field

$$\alpha_{BR} = \alpha_0\, e\langle E\rangle \tag{15}$$

$$E = (-1/e)\frac{d}{dz}V(z) \tag{16}$$

That α_0 depends on band structure parameters (de Andrada e Silva *et al.* 1994, 1997).

Another kind of spin-orbit interaction is because of bulk inversion asymmetry, called Dresselhaus which is divided in to two types of linear Dresselhaus and cubic Dresselhaus described by the following Hamiltonians

$$H_D = \gamma_c \left\langle K_z^2 \right\rangle (-\sigma_x K_x + \sigma_y K_y) \tag{17}$$

$$H_{D2} = (\gamma_c / 2)(\sigma_x K_x K_y^2 - \sigma_y K_y K_x^2) + H.C. \tag{18}$$

In which γ_c is the determined by the band structure parameters.

Here we express Hamiltonians of the Rashba and linear Dresselhaus together, also Hamiltonian of the cubic Dresselhaus based on the Fock-Darwin operators as follows (de Sousa and Das Sarma 2003)

$$H_{D1} + H_R = V\sigma_+ + V^+\sigma_- \tag{19}$$

$$V = -\alpha_- a_-^+ + \alpha_+ a_+ + i\beta_- a_- - i\beta_+ a_+^+ \tag{20}$$

Where, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$

And spin-orbit energy scale defined as

$$\alpha_\pm = \alpha_0 eE\zeta_\pm \tag{21}$$

$$\beta_\pm = \left\langle K_z^2 \right\rangle \gamma_c \zeta_\pm \tag{22}$$

$$\zeta_\pm = \frac{1}{\ell} \pm \frac{eB\ell}{2\hbar c} \tag{23}$$

$$\begin{aligned} H_{D2} = -i\sigma_+ [&\lambda_1 a_-^{+2} a_+ + \lambda_2 a_+^{+2} a_- + \lambda_4 a_- n_- - \lambda_3 a_+^+ (n_+ + 1) + \frac{\lambda_2}{3} a_- (2n_+ + 1) \\ &- \frac{\lambda_1}{3} a_+^+ (2n_- + 1) + \frac{\lambda_2}{3} a_+^{+2} a_-^+ - \frac{\lambda_1}{3} a_+ a_-^2 - \lambda_4 a_-^{+3} + \lambda_3 a_+^3] + H.c. \end{aligned} \tag{24}$$

Where $\lambda_1 = 3/4\ \gamma_c \zeta_+ \zeta_-^2$, $\lambda_2 = 3/4\ \gamma_c \zeta_- \zeta_+^2$, $\lambda_3 = 1/4\ \gamma_c\ \zeta_+^3$, $\lambda_4 = 1/4\ \gamma_c\ \zeta_-^3$. Since lengths of the spin-orbit interaction (Bulaev and Loss 2005) are $L_{BR} = \hbar^2/2\ \mathrm{m}^*\ \alpha_{BR}$, $L_D = \hbar^2/2\ \mathrm{m}^*\ \gamma_c \langle Kz^2 \rangle$, $L_{D3} = \hbar^2 l_0^2 / 2\gamma_c \mathrm{m}^*$ ($\approx 2\mu$m). Which are so much larger than the size of quantum dot, considering the spin-orbit interaction as a small perturbation (Bulaev and Loss 2005), we will obtain the two lowest electron eigenstates in first-order perturbation theory.

$$\begin{aligned} \psi_+ = \left|00\downarrow\right\rangle + \frac{\alpha_-}{\hbar\omega_-}\left|01\uparrow\right\rangle + i\frac{\beta_+}{\hbar\omega_+}\left|10\uparrow\right\rangle - i\Big(&\frac{(\lambda_3 + \lambda_1/3)}{\hbar\omega_+}\left|10\uparrow\right\rangle \\ &- \frac{\lambda_2/3}{2\hbar\omega_+ + \hbar\omega_-}\left|21\uparrow\right\rangle + \frac{\lambda_4}{3\hbar\omega_-}\left|03\uparrow\right\rangle\Big) \end{aligned} \tag{25}$$

$$\psi_{-}=\left|00\uparrow\right\rangle-\frac{\alpha_{+}}{\hbar\omega_{+}}\left|10\downarrow\right\rangle+i\frac{\beta_{-}}{\hbar\omega_{-}}\left|01\downarrow\right\rangle-i(\frac{(\lambda_4+\lambda_2/3)}{\hbar\omega_{-}}\left|01\downarrow\right\rangle$$
$$-\frac{\lambda_1/3}{\hbar\omega_{+}+2\hbar\omega_{-}}\left|12\downarrow\right\rangle+\frac{\lambda_3}{3\hbar\omega_{+}}\left|30\downarrow\right\rangle) \tag{26}$$

And we get $E_0=E_0^{(0)}$.

Where in Fig. 1, S_{-}, S_{+} shows the eigenspinors of the two eigenfunctions of Ψ-, Ψ_{+} and as shown in this figure, θ is deviation of the S_{+} from the net spin-down. Thus, electron spin in the solid state contrary to what we might think won't have complete spin-up and spin-down polarizations and true polarizations depend on the intensity of the spin-orbit interaction.

It would be easily shown that expected values of the Pauli matrices on the spinor S_{+} will be

$$S_{+z}\approx-\frac{\hbar}{2}\frac{(\frac{\alpha_{-}}{\hbar\omega_{-}})^2+(\frac{\beta_{+}}{\hbar\omega_{+}})^2(1-F)^2-1}{1+(\frac{\alpha_{-}}{\hbar\omega_{-}})^2+(\frac{\beta_{+}}{\hbar\omega_{+}})^2(1-F)^2} \tag{27}$$

$$S_{+x}\approx\frac{\hbar}{2}\frac{(2\alpha_{-}/\hbar\omega_{-})}{1+(\frac{\alpha_{-}}{\hbar\omega_{-}})^2+(\frac{\beta_{+}}{\hbar\omega_{+}})^2(1-F)^2} \tag{28}$$

$$S_{+y}\approx\frac{\hbar}{2}\frac{(2\beta_{+}(1-F)/\hbar\omega_{+})}{1+(\frac{\alpha_{-}}{\hbar\omega_{-}})^2+(\frac{\beta_{+}}{\hbar\omega_{+}})^2(1-F)^2} \tag{29}$$

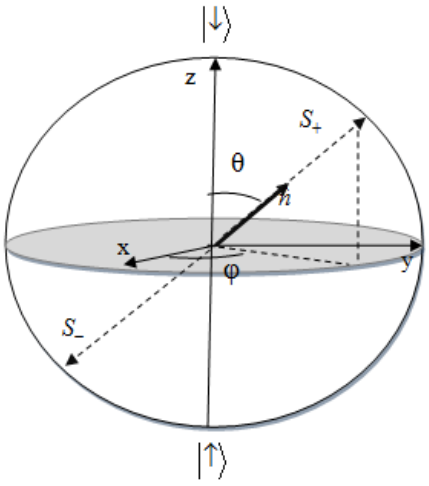

Fig. 1 The spin polarization in two Zeeman energy levels on Bloch sphere

And, also expected values of the spinor S_- are as follows

$$S_{-z} \approx \frac{\hbar}{2} \frac{(\frac{\alpha_+}{\hbar\omega_+})^2 + (\frac{\beta_-}{\hbar\omega_-})^2(1-F)^2 - 1}{1+(\frac{\alpha_+}{\hbar\omega_+})^2 + (\frac{\beta_-}{\hbar\omega_-})^2(1-F)^2} \tag{30}$$

$$S_{-x} \approx -\frac{\hbar}{2} \frac{(2\alpha_+ / \hbar\omega_+)}{1+(\frac{\alpha_+}{\hbar\omega+})^2 + (\frac{\beta_-}{\hbar\omega_-})^2(1-F)^2} \tag{31}$$

$$S_{-y} \approx -\frac{\hbar}{2} \frac{(2\beta_-(1-F)/\hbar\omega_-)}{1+(\frac{\alpha_+}{\hbar\omega_+})^2 + (\frac{\beta_-}{\hbar\omega_-})^2(1-F)^2} \tag{32}$$

Where, $F= (\lambda_3+\lambda_1/3)/\beta_+$
As seen, in above, the spins in high and low Zeeman energy levels are anti-parallel.

Comparing above values to the expected values of the Pauli matrices on an arbitrary vector *n* in the polar coordinates with an amplitude of one on Bloch sphere

$$\theta = \cos^{-1}\left(\frac{1-(\frac{\alpha_-}{\hbar\omega_-})^2 - (\frac{\beta_+}{\hbar\omega_+})^2(1-F)^2}{1+(\frac{\alpha_-}{\hbar\omega_-})^2 + (\frac{\beta_+}{\hbar\omega_+})^2(1-F)^2}\right) \tag{33}$$

Hence, by adjusting values of the $\alpha_\pm$ and $\beta_\pm$ (1-*F*) in relation to each other, one can decrease the spins-up and down mixing to the least possible value.

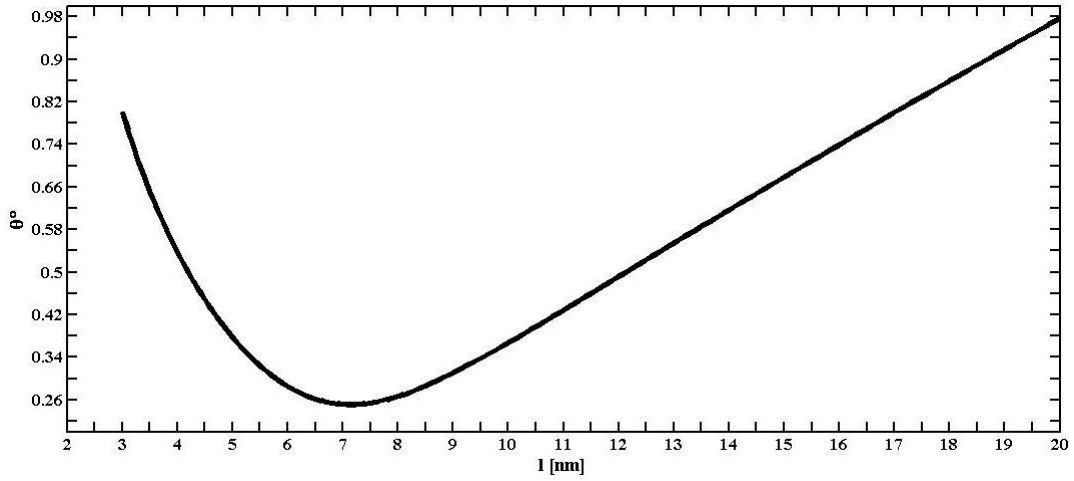

Fig. 2 Deviation from the net spin-down as a function of the Fock-Darwin radius

As shown in above diagram the minimum value of Θ for the discussed GaAs quantum dot will happen for l=7 nm.

The used parameters for the Galium Arsenide are as follows:

m =0.067 m_e (where m_e is the free electron mass) g=-0.44 and γ_c=27.5 ev.A$^{\circ 3}$.

The linear Dresselhaus coefficient $\gamma_c<K_z^2>$ = 4.5 mev.A$^{\circ}$ for the confinement width of 11 nm in the ground state of a quantum well is triangular and α_{BR} equals 3.3 mev.A$^{\circ}$ (Miller *et al*. 2003).

3. Conclusions

In this article, we have studied a quantum dot, in which confinement in all of three directions is nearly equal. Then we have figured out spin deviation angle from its initial direction. We found out that the spin deviation angle has a direct relationship to defined spin orbit energy scales in Eqs. (21), (22), (23).

By reducing length of the confinement in x and y directions, we have increased effect of the cubic Dresselhaus, which caused reduction of the linear Dresselhaus. Also, Reduction of the Fock-Darwin radius caused reduction of the linear Dresselhaus effect and resulted in adjusting the coefficients $\alpha_{\pm}$ and $\beta_{\pm}$ (1-F) and getting this ratio near one, thus we prevented spin relaxation among Zeeman sublevels.

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