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A combined experimental and numerical method for structural response assessment applied to cable-stayed footbridges

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Abstract. This paper presents a non-destructive testing method for estimating the structural response of cable-stayed footbridges. The approach combines field measurements with a numerical static analysis of the structure. When the experimental information concerning the structure deformations is coupled with the numerical data on the structural response, it is possible to calculate the static forces and the design tension resistance in selected structural elements, and as a result, assess the condition of the entire structure. The paper discusses the method assumptions and provides an example of the use of the procedure to assess the load-carrying capacity of a real steel footbridge. The proposed method can be employed to assess cable-stayed structures including those made of other materials, e.g., concrete, timber or composites.

Keywords: footbridge; cable-stayed footbridge; static tests; numerical simulation

1. Introduction

Footbridges provide pedestrians with safe and conflict-free passage over various obstacles, for example, city streets, expressways, rivers, or sea straits. Footbridges are particularly useful in urban areas, where the pedestrian death rate is high. They are becoming increasingly popular not only in Europe but also in other urbanized regions of the world.

The structure of a pedestrian bridge built over an obstacle with a considerable width is generally the same as that of a regular bridge in terms of statics and materials used. One of the most common structures in such a case is a cable-stayed bridge, which is structurally stable and architecturally interesting. A typical cable-stayed footbridge consists of a deck structure, towers (pylons) and cables supporting the deck. The main classes of cable-stayed pedestrian bridges are: harp (Fig. 1(a)), modified fan (Fig. 1(b)) and fan (Fig 1(c)).

The history of cable-stayed bridges goes back to 1595, when a design of such a structure appeared in Fausto Veranzio's work *Machinae Novae*. Modern structures of this type were first constructed at the beginning of the 19th century. Much progress in this field, including the construction of cable-stayed pedestrian bridges, was made in the 1970s. The first cable-stayed footbridge in Poland was built at Tylmanowa in 1959 (Główczak 2003, Biliszczuk and Barcik

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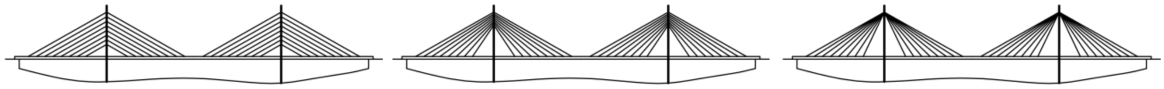


Fig. 1 The classes of cable-stayed footbridges

2006). Structures of this type are expected to have a minimum service life of 50 years. Regular in-service monitoring and inspections are required to assess their condition and determine their load-carrying capacity. Inspections and maintenance activities are particularly important in the case of structures with a long lifespan, e.g., bridges (Kossakowski 2013).

Inspections and maintenance activities are fundamental to ensure a long service life of a cable-stayed bridge. Data from the preliminary assessment of the structure condition are used to draw up target guidelines for its safe operation as well as effective and efficient maintenance. Guidelines are necessary to analyse the structure behaviour under operational conditions, indicate the key elements responsible for its safe operation, develop appropriate inspection and maintenance procedures, determine the inspection frequency and methods and recommend safety measures (Setra 2002, Post-Tensioning Institute 2001). The bridge performance should be assessed upon completion of the construction work and then regularly during service. The most important suggestions concerning the methods and range of maintenance activities are generally provided by the producer of the suspension system, the bridge designer and the institutions responsible for the monitoring and management of the bridge.

Inspections of cable-stayed bridges, including pedestrian bridges, are conducted in accordance with relevant standards and technical regulations, e.g., (VSL International 1984, Post-Tensioning Institute 2001). The basic requirements include visual inspection every two years, ultrasonic testing of the stay cables at the anchorage every four years, advanced inspection of the structure condition every six years or more frequently if necessary, and electromagnetic testing of the stay cables every ten years or more frequently if necessary (Post-Tensioning Institute 2001).

Inspections of cable-stayed bridges are part of the monitoring and control activities conducted during construction and operation. They may provide information on the structure behaviour when affected by environmental factors; they may indicate potential safety hazards or operational limitations; they may also be used to verify the design assumptions and theories.

Cable-stayed bridges are frequently inspected during service also to fully recognize the spatial performance of the structure or verify its dynamic characteristics. Insufficient amount of empirical data makes it impossible to assess the reliability of the numerical models used.

The load-carrying capacity of cable-stayed structures is more difficult to determine than that of bridges simpler in design. The calculation procedures used for cable-stayed and suspension bridges are complicated and the reason for this is the static tensile behaviour of cables. The state of knowledge in this field has advanced considerably. Currently, the research on cable-stayed bridges focuses on construction-related problems (Wang *et al.* 2004, Juozapaitis *et al.* 2013, Lozano-Galant *et al.* 2014), the development of analytical and numerical methods to optimise the bridge design process (Kiisa *et al.* 2012, Straupe and Paeglitis 2012, Chen *et al.* 2013, Recupero and Granata 2015), the optimisation of the structural systems (Janjic *et al.* 2003, Straupe and Paeglitis 2013, Vejrum and Nielsen 2014), bridge safety monitoring (Sun *et al.* 2013, Nazarian *et al.* 2016) and the application of state-of-the-art materials (Serdjuks *et al.* 2008). Problems related to the assessment of the ultimate limit state of bridges have been discussed, for instance, in the context of failure prediction based on damage mechanics e.g., (Kossakowski 2012, Kossakowski 2014, Kossakowski 2015). A separate strand of research is looking into the optimal performance of stay cables (Serdjuks *et al.* 2008, Kiisa *et al.* 2012, Juozapaitis *et al.* 2013, Recupero and Granata 2015). Further studies regarding cable-stayed bridges are vital to analyse the application of advanced materials or develop better testing and calculation methods to optimise the structural systems.

In the case of medium-span bridges, including footbridges, whose operation is not affected by severe weather conditions or seismic activity, the role of inspection is simply to control the structure condition. Inspections involve checking the functional and safety requirements, identifying the sources of disturbance as early as possible and monitoring the structure deterioration. Inspections of this type may take different forms, ranging from visual inspection to technologically advanced electromagnetic or ultrasonic testing (Fuzier and Lacroix 1995, Post-Tensioning Institute 2001).

An inspection of a cable-stayed bridge performed during service is similar to that conducted on the completion of the construction work. The procedure is dependent on the bridge structure and size. The aim is to measure the bridge deformations under known static loads applied during preliminary tests, collect data on the dynamic behaviour of the bridge (Bachmann 1995) or verify the assumptions made at the design stage. If excessive deformations, instability or other undesirable phenomena are detected, inspection results are analysed to take the right safety measures. Investigations of this type require employing advanced measurement methods, equipment as well as well-trained personnel.

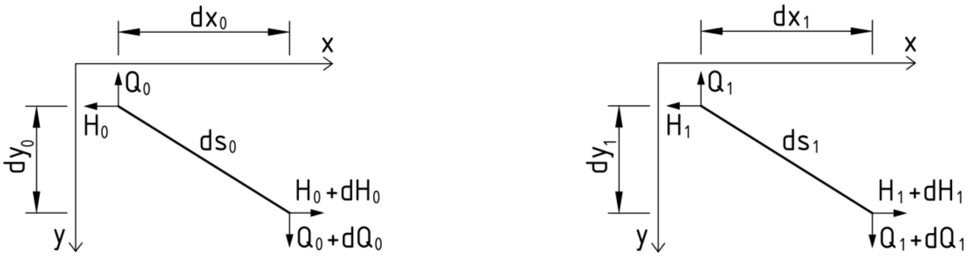
Many patterns of static loads can be used during preliminary tests to simulate the movement of pedestrians over the bridge. The aim of such tests is to compare the actual distributions of internal forces in selected elements with those assumed in the design, verify the numerical models employed and assess the quality of the construction work.

One of the most common and dangerous phenomena affecting the performance of cable-stayed bridges is vibration. The ease with which vibration can be excited is due to the application of modern materials with higher strength but smaller cross-sections and lower mass. Modern pedestrian bridges are less stable and more susceptible to dynamic loads. Dynamic tests are thus essential. These involve measuring different configurations of deflections and acceleration at selected points of the structure generated by actions to simulate the pedestrian movement.

When electromagnetic, ultrasonic or surveying (e.g., laser, induction or tensometric measurement) methods are used, advanced equipment and specially trained personnel are necessary. Meeting this requirement may often be difficult. Extensive research is being conducted to simplify the methodology for assessing the actual load-carrying capacity of cable-stayed bridges, especially small-span footbridges.

The load-carrying capacity of a cable-stayed bridge in service can be assessed on the basis of knowledge of the stress states of the structure elements, especially the stay cables. Such an analysis requires both experimental studies and numerical calculations. Actually, there are no faster or more accurate methods to assess the response of a cable-stayed bridge structure and determine the distributions of loads and stresses for all the load-carrying elements. The major problem is to analyse the behaviour of stay cables, which are non-linear elements. An iterative approach is required to significantly increase the calculation time. Classic calculation methods are not at all suitable for structures consisting of hundreds or thousands of elements.

This article discusses a combined experimental and numerical method applied to assess the load-carrying capacity of a cable-stayed pedestrian bridge. The results seem to contribute to the development of optimal methods for evaluating the performance of bridges in service. The study



(a) at the installation stage (b) under service conditions

Fig. 2 The behaviour of an elementary section of a high tension cable

consisted in measuring deformations of the stay cables and using the data to numerically model the behaviour of the whole structure. The method is well-suited to recognise the spatial performance of the structure, verify the characteristics of selected elements, determine the state of stresses in the stay cables, assess the reliability of the numerical models, evaluate the resistance of the structure and predict the safety limits under live loads. Determining the load-carrying capacity of the cable-stayed pedestrian bridge requires analysing the structure responses to different loads. It is also essential to take into account the actual factors affecting the structure performance, which were omitted at the design stage, for example, construction-related imperfections or the actual tension resistance of the stay cables. The major advantages of the method are simplified measurement and no need to use advanced testing equipment.

2. Analysis of tensile structures

A static analysis of a structure operating in the elastic region is generally not difficult. The static response and deformation of the structural elements are determined on the basis of their geometry, constraint conditions, material properties and loads. In the case of cable-stayed structures, the situation is more complicated. Stays are nonlinear elements and they operate under tension only. A static analysis of a structure with multiple stays is, thus, very complex. As the forces present in each stay are strongly related, they largely contribute to the distribution of forces in the whole bridge. A static analysis performed for cable-stayed pedestrian bridges requires that the internal forces acting on the stays should be determined correctly.

In cable-stayed structures, the principal load-carrying elements are stay cables. A stay is an element in which one of the main dimensions is many times greater than the other two and its lateral bending and torsional stiffnesses are much smaller than the longitudinal tensile stiffness. As mentioned above, only tensile forces can be applied to stays. In some cases, however, stays can be subjected to small bending or torsional moments and shear forces. The major advantage of a tensile structure is high tensile strength of the material used for the cables, which means that their cross-sections can be optimally used. As a result, tensile structures are light, economical, and architecturally and visually attractive.

The basic assumption of the theory of tensile structures is as follows: when service loads and other external forces are quasi-static in nature and do not change with time, tension elements operate within the elastic range, with Young's modulus being constant; their cross-sectional area is constant because it is not affected by deformations. In the case of high tension cables, the bending moments and lateral forces are not taken into account; however, stays can be subjected to any

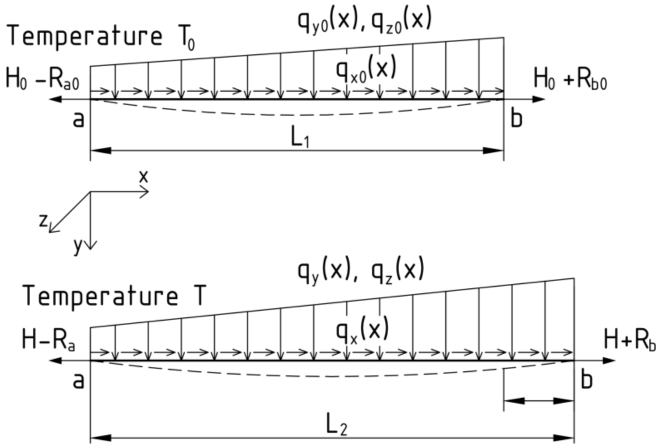


Fig. 3 The behaviour of a stay

loading, except for momentum load; large displacements u are permissible when the displacement gradients du/dx are small.

When the static behaviour of stays is analysed, high tension cables are usually taken into account. In high tension cables, the angle between the tangent and the straight line connecting the ends at any point is small. It is assumed that stays can be subjected to an arbitrarily distributed load in their plane.

The equation for a high tension cable is developed by considering an infinitely small section of a single stay. When Q_0 and H_0 act on the structure, the length of a stay is assumed to be equal to ds_0 (Fig. 2(a)). Under service conditions, however, the structure is subjected to Q_1 and H_1. Then, the length of a stay is ds_1 (Fig. 2(b)).

With assumptions made for an elementary section of a stay, it is possible to analyse a whole stay in the full scale. Fig. 3 shows diagrams of the static behaviour of a stay in the two analysed stages when subjected to loads in the x-y and z-x planes.

The elongation of the elementary section of a stay, in the function of static quantities only, can be determined assuming that there is a small slack and that the total force in a stay needs to be directed along the tangent to the stay. By integrating this quantity along the total length of a stay, we obtain the equation for a high tension cable with a small slack (1), which determines the elongation of the stay chord Δ

$$L_2 - L_1 = \Delta = \frac{Hl}{EA} - \frac{H_0 l}{EA} + \alpha_t \Delta T l + \delta - \frac{1}{2}\left[\int_0^l \frac{\left(Q_y(x)\right)^2 + \left(Q_z(x)\right)^2}{\left(H + N(x)\right)^2} dx - \int_0^l \frac{\left(Q_{y0}(x)\right)^2 + \left(Q_{z0}(x)\right)^2}{\left(H_0 + N_0(x)\right)^2} dx\right] \quad (1)$$

where: L_1-initial length of the stay (prior to the load application), L_2-final length of the stay (after the load application), a, b-beginning and end cable node, E-Young's modulus, A-cross-sectional area of a stay, EA-tensile stiffness of a stay, α_t-coefficient of thermal expansion, l-initial length of a stay, Δ-change in the distance between the supports, δ-initial, internal shortening/elongation of a stay, ΔT-change in temperature, H-tensile force, $N(x)$-function of the change in the axial force acting along the tangent to a stay, $Q_y(x)$, $Q_z(x)$-function of the change in the lateral force in relation to the y- and z-axes, respectively; the subscripts $_0$ and $_1$ refer to the initial load and service load states of a stay.

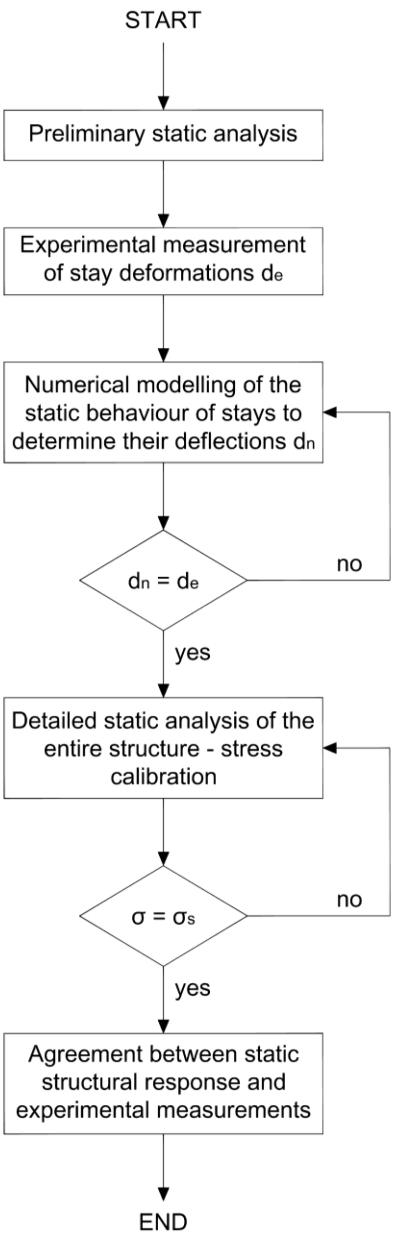


Fig. 4 The algorithm of the experimental and numerical method for assessing the structural response of cable-stayed footbridges

3. The method concept

It is relatively easy to assess the condition of simple tension systems using the theory presented above. However, in the case of structures with a complex design and/or a mixed structural system, where there are interactions between the stays and the structural members, the calculations become far more complicated. This refers also to cable-stayed bridges, including pedestrian bridges. From

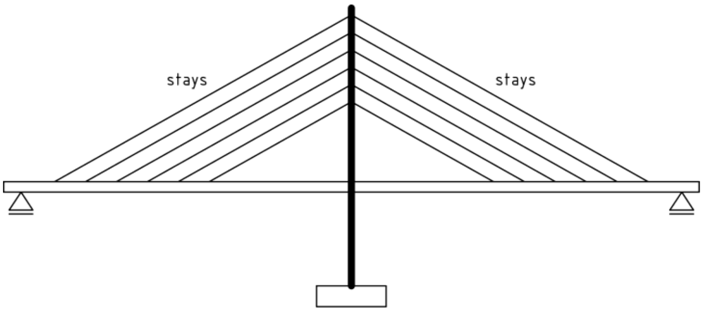


Fig. 5 A diagram of a typical cable-stayed footbridge

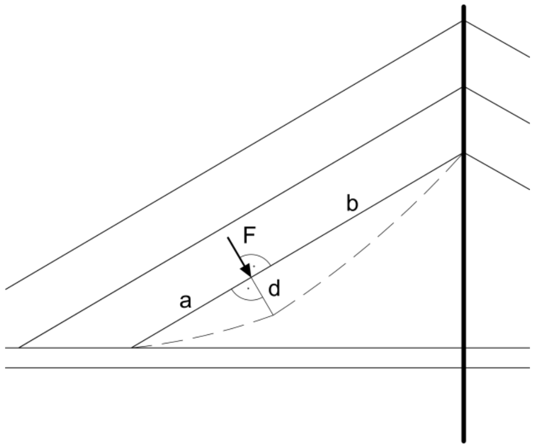


Fig. 6 Load *F* applied to a stay and the principle of measurement of the deflection *d*

a practical point of view, a static analysis of a cable-stayed footbridge should be simple to perform. The procedure should allow engineers to easily determine the internal forces when the structure is under service (live) load in order to predict its behaviour.

Details of such a procedure are provided below. The experimental and numerical approach discussed here combines field measurements with an advanced numerical FEM analysis to determine the response of the structure under live load.

The method uses the results of field measurement, i.e., the deformations of the structure, to numerically model the structural response. It is crucial that there should be agreement between the measurement data and the corresponding calculation results. Because of the problem complexity, the analysis occurs in several steps. In cable-stayed bridges, the deformations measured refer to stays. The analysis is performed at two levels. First, the behaviour of stays is modelled and it is done separately for each stay. Then, the results are used to model the behaviour of the whole structure. The general algorithm of the method is shown in Fig. 4.

The procedure begins with a preliminary static analysis of a cable-stayed structure. Since high accuracy is required, it is suggested that the calculations be performed numerically using a program based on the finite element method in order to model the nonlinear elements of the stays. The main objective of the calculations is to estimate the response of the structure and the level of the stay tension. A diagram of a typical cable-stayed footbridge is presented in Fig. 5.

The next stage involves experimental measurements of the structure deformations. The

Fig. 7 A side view of the footbridge

measurements are performed on a real structure under dead load. The main idea behind the method is to couple the experimental data of the deformations with the structural response. The deflections d of the particular stays are measured according to the diagram shown in Fig. 6. Each of the stays is loaded with a concentrated force F, applied perpendicular to the stay axis at a distance a from the point of anchorage. The value of the force F is carefully selected to allow proper deflection of the stay d in order to minimise both the measurement and calculation errors. Since the ranges of the deflections are assumed to be similar for all the stays, it is recommended that one value of the load F should be determined. After the load F is applied, the deflection d is measured perpendicular to the stay axis at the point of force application (parallel to the direction of the force).

At the next stage, the stress state is analysed separately for each stay. Then, each stay is modelled numerically. The numerical model is created taking into account the actual geometry of the structure elements. Subsequently, the value of the initial stress is selected. The modelling involves applying a load similar to the actual load used in the experiments. The deflection d is analysed for a constant load F. Each numerical model is then calibrated by iterative adjustments of the tension of the stays to obtain the best goodness of fit between experimental and numerical results of the deflection d. It is assumed that the stays are subjected to forces F and there is initial tension stress. The analysis is conducted using the theory of high tension cables, described in Section 2, on the basis of relationship (1). The stresses are calibrated iteratively up to a moment when the deflections d are equal to the values determined experimentally in order to estimate the final tension stress in the stays.

The calculations performed at the last stage of the analysis take into account the tension parameters of the stays. The static model developed at the initial stage was used to determine the characteristics of the stays, particularly, tensile stress, with the assumption that the stays are subjected to loads specified in appropriate standards. It may be necessary to slightly calibrate tensile stress if the structure is complex. This calibration is similar to that performed at an earlier stage for each stay separately. Tensile stresses present in the stays are calibrated iteratively so that there is consistency in deflections between those determined experimentally and those calculated in a static analysis. Once agreement is achieved for each stay, the results of the experimental measurements and those concerning the structural response are coupled to construct the final static model of the structure.

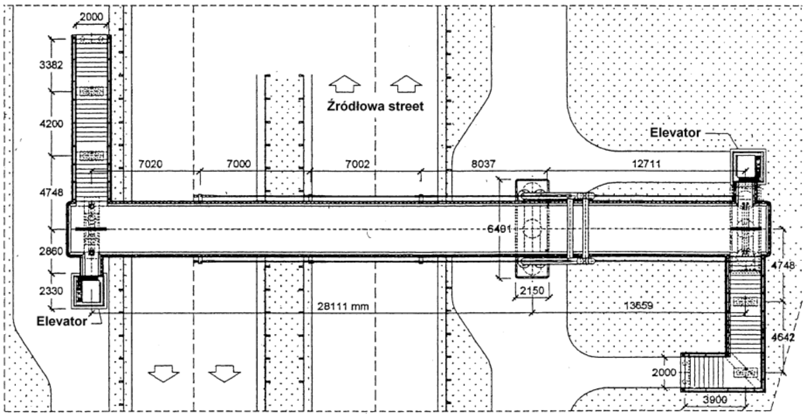


Fig. 8 The location plan of the footbridge (Biliszczuk *et al.* 2001)

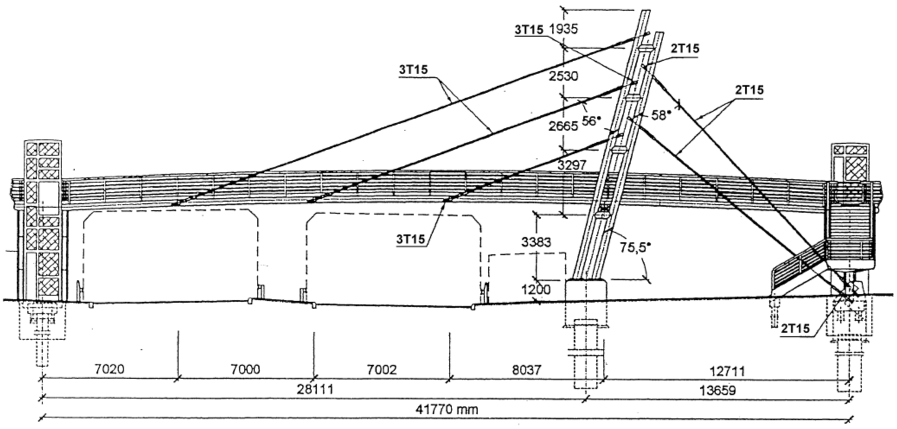


Fig. 9 A side view of the footbridge structure (Biliszczuk *et al.* 2001)

This method is suitable to analyse the statics of the particular structural elements and determine the deformations of the structure. It can also be used to study the structural response and, consequently, assess the resistance conditions of all the structural members according to the design standards.

4. An example of the method application

4.1 Description of the analysed footbridge

The cable-stayed footbridge used as an example to illustrate the application of the method is located in Kielce, Poland, over Źródłowa street, which is one of the main streets in the town. A side view of the bridge is shown in Fig. 7.

Because of the specific location in the city and the requirements concerning the height (the lowest possible), the bridge was designed to have a suspended steel span and one tower placed on the east side of Źródłowa street, as shown in Figs. 7 and 8. The useful width of the footbridge deck

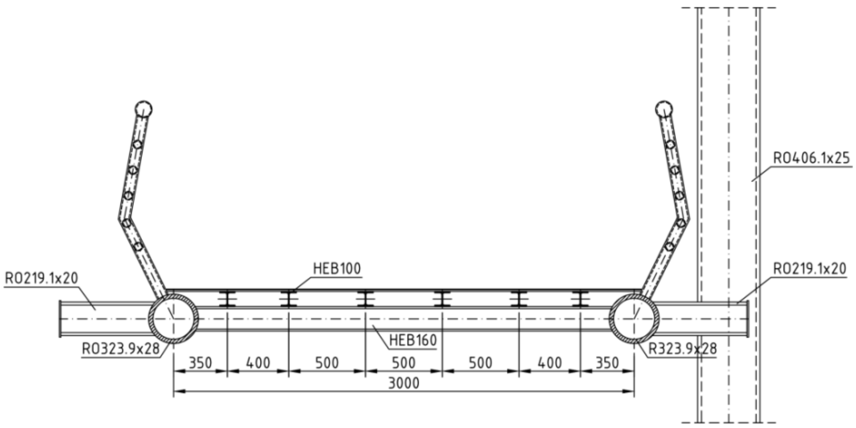


Fig. 10 A cross-sectional view of the load-carrying system of the footbridge (based on Biliszczuk *et al.* 2001)

is 3.00 m, while the total width is 3.723 m (Biliszczuk *et al*. 2001).

The pedestrian bridge was designed to have a non-symmetrical two-span steel structure attached to an inclined tower (Fig. 9). The basic dimensions of the structure are as follows (Biliszczuk *et al*. 2001):

- the length measured along the axes of the end supports - 41.77 m,
- the span length measured along the axes of supports - 29.077+12.771 m,
- the height of the tower (the steel part) - 13.805 m.

The footbridge was constructed using circular structural hollow steel sections (RO) and wide flange H steel beams (HEB). The span structure (Fig. 10) comprises two RO 323.9/20.0 round-section steel tubes, one RO 323.9/28.0 insert in the tower zone and HEB 160 sections constituting the lateral cross-bracing (in the perpendicular system). Together they form the load-carrying structure. The deck is a 10 mm thick plate. HEB 100 beams serve as longitudinal ribs. The deck is suspended on stay cables anchored at the tubular supports mounted perpendicular to the deck tubes. Each support consists of an RO 219.1/20.0 steel tube and a tube encasing the cables (Biliszczuk *et al*. 2001).

The tower elements include RO 406.4/25 m two-branch steel tubes connected with lacing tubes. The system responsible for the suspension of the deck includes tension elements in the form of steel cables with a tensile strength of 1770 MPa. The cables are made up of 7Ø 5 mm strands with a maximum breaking strength of 246 kN and a yield strength at 0.1% elongation of 212 kN. The anchorage of all the cables to the tower is passive. The cables were put into tension at the deck supports and in the foundation blocks of the tension stay cables. 3T15 cables were used for stays suspending the deck, while 2T15 cables served as high tension (rear) stays. The cables are made up of seven strands ø5 mm. The cable wires are zinc-plated and have a lubricant-filled hard polyethylene (PEHD) coating.

The tower was mounted on reinforced concrete pillars 1200 mm in diameter. The tower was attached to the base at the top of the pillars, which was 1200 mm in height.

4.2 Analysis of the footbridge response

The analysis described in this paper was conducted for the footbridge over Źródłowa street

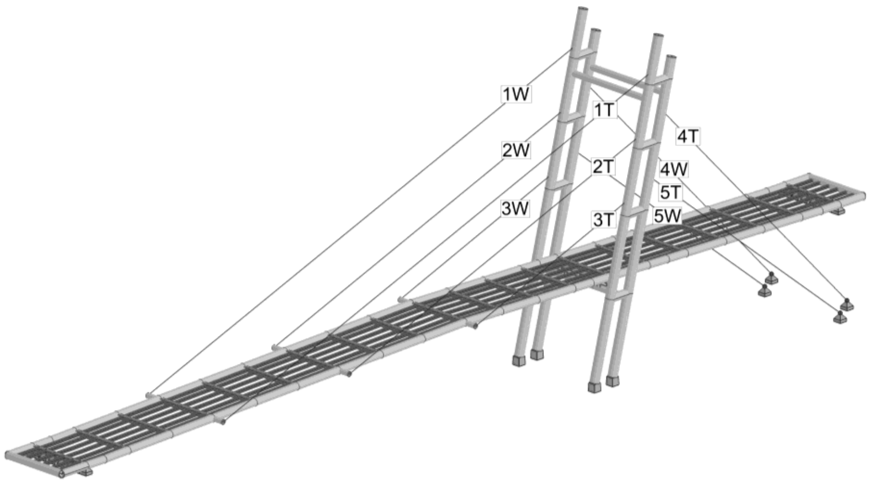


Fig. 11 Cables anchored to the deck (numbered 1W-5W) and cables anchored to the ground (numbered 1T-5T)

because some defects were found during an inspection. The owner of the bridge decided that static verification was necessary. As mentioned above, the main problem encountered during the analysis was correct determination of tensile stress in the stays. The methodology described in Section 3 was applied for this purpose. The investigation was carried out using the algorithm shown in Fig. 4, which was developed with the FEM-based program, Autodesk Robot Structural Analysis.

The first stage of the analysis involved experimental assessment of the response of the footbridge structure to a predetermined load level. It was essential to study the deformations of all the cables, i.e., those anchored to the deck (marked as W) and those anchored to the ground (marked as T), in the order in which they are numbered in Fig. 11.

It was necessary to calculate the value of the force F responsible for the stay deflections in order to minimise both the measurement and calculation errors. It was assumed that the ranges of stay deflections were similar and the level of load F was constant. The stress values were used to determine the load F applied to the structure during the experiments (see Fig. 6). The preliminary calculations were crucial to determine the range of loads that can be applied to the structure. The cable deflections d were measured while the bridge was in service. Each cable was subjected to a concentrated force F of 0.5 kN (50 kg). The measurements were taken for the structure exposed to dry air and a temperature of 12℃.

Before the main measurements were performed, the initial load was applied to check if the loading range satisfies the proposed criteria. The measurements were also preceded by calibration, which confirmed the values of F (0.5 kN). The eye-hooks for the stay cables were fitted at the distance “a” from the anchorage to the bridge deck (see Fig. 6). The force F=0.5 kN was a result of a controlled pull of the eye-hook. A dynamometer was coupled to the pull system to determine and control the load being applied. The deformations of the stay cables, i.e., their deflections, were measured with an optical system. The measurements were taken at the point of application of a constant load of 0.5 kN (see Fig. 6), which was the datum reference point. When the loads were applied to the stays, it was necessary to control the stabilisation of their deflections in order to obtain the structure response unaffected by vibration or any other dynamic effect in the static

Table 1 Properties of the steel used for the stay cables-characteristic values (according to prEN 10138-3)

Property	Symbol	Unit	Freyssinet cables	
			2T15	3T15
Load capacity	N_k	kN	492	738
Modulus of elasticity	E	GPa	195	195
Cross-sectional area	A	mm^2	278	417
Tensile strength	R_{pk}	MPa	1770	1770

Table 2 Properties of the steel used for the stay cables-design values (according to prEN 10138-3)

Property	Symbol	Unit	Freyssinet Cables	
			2T15	3T15
Load capacity	N_o	kN	320	480
Modulus of elasticity	E	GPa	195	195
Cross-sectional area	A	mm^2	278	417
Tensile strength	R_{po}	MPa	1151	1151

range. The deflections of the stays were thus measured under comparable conditions. The measurements were carried out with no pedestrians present on the deck. The total value of the deflections was calculated with reference to the “0” state, i.e., the state at which the stay cables were not subjected to any extra load. The measurements were made with an accuracy of ±1.0 mm, taking into account the scale and size of the bridge and the range of the deformations of the structure determining its response to the live load and the load applied during the experiment.

The next stage involved preliminary static calculations. The tension of the stays was determined assuming that their behaviour was similar to that of high tension cables described in Section 2. The Autodesk Robot Structural Analysis program was employed to create numerical models, according to the numbering shown in Fig. 11.

Then, each stay was modelled separately taking into account the dimensions and material properties according to the Design Project and assuming that a concentrated force, F=0.5 kN, was acting on it. The footbridge has a deck suspended by steel cables with a tensile strength of 1770 MPa. The cables are made from multiple strands of wire (7Ø5 mm) with a maximum breaking strength of 246 kN and a yield strength at 0.1% elongation of 212 kN. Their anchorage to the tower is passive. The tension acting in the cables is a result of the loads applied at the deck supports and in the foundation blocks. 3T15 cables are used for stays anchored to the deck, while 2T15 cables serve as high tension cables anchored to the ground (cf. Fig. 9). The modelling was carried out using finite elements in the form of high tension cables, the operation of which is described by relationship 1, according to the assumptions presented in Section 2. The geometrical and mechanical properties of the stay cables and the steel are provided in Tables 1 and 2.

The calculations for the steel used for the stay cables were made assuming that $N_o=0.65\cdot N_k$ and $R_{po}=0.65\cdot R_{pk}$.

The response of the load-carrying structure of the footbridge, particularly the response of the stay cables anchored to the deck, which was studied at this stage of the numerical and experimental analysis, corresponds to the state of load comprising:

- the dead load of the bridge and the load exerted by the bridge facilities,

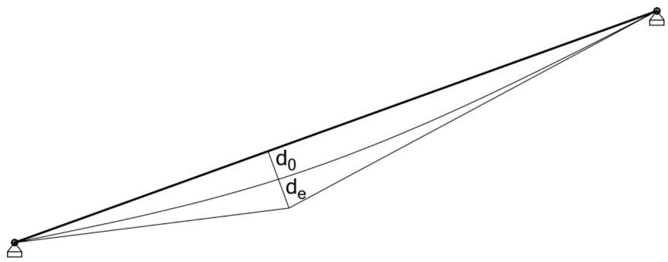


Fig. 12 Additivity of the stay cable deflections for the loads exerted by the weight of the bridge itself and the force *F*

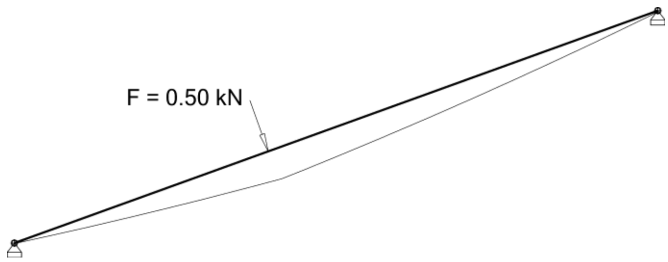


Fig. 13 The statics of stay 2W deformed under the load *F*

- the load produced by the force applied during the experiments (F=0.5 kN).

This determines the additivity of the deformations (deflections) of the stay cables, as shown schematically in Fig. 12.

The main objective of the calculations was to determine the tensile stress σ_s present in the cables, taking account of the data obtained from the measurement of the deflections d_e defined by the formula

$$d_e = d_F - d_0 \tag{2}$$

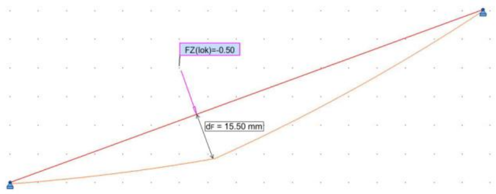
where d_e is the stay deflection estimated by measurement (resulting from the action of the force F), d_0 is the stay deflection under dead load conditions, and d_F is the stay deflection under the load exerted by the weight of the bridge and the force F.

During the first iteration, the approximate tensile stress σ_s was calculated for each stay from Eq. (1) following the assumption outlined in Section 2. Next, the iteration was applied using the optimisation criterion based on the consistency between the deflections determined by experiments d_e and those calculated numerically d_n.

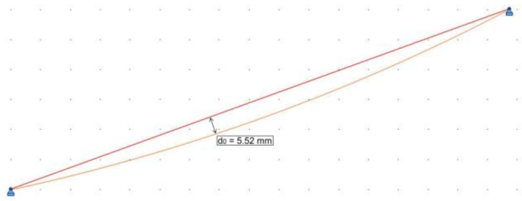
$$d_e = d_n \tag{3}$$

The detailed calculation procedure for cable 2W is presented below. Figure 13 shows the statics of stay 2W.

The displacement of the point of application of the force F (0.5 kN) measured for cable 2W was d_e=10 mm. The methodology described above was used to calibrate the pull of the cable. The calibration was performed by iteration so that condition 3 was satisfied. The numerical analysis of the response of cable 2W at the combined effect of the dead load of the bridge, the load exerted by the facilities and that exerted by a force of 0.5 kN applied during the experiments gave the value of the displacement d_F=15.50 mm; when the force F was excluded from the calculations, the value of



(a) d_F for the dead load of the bridge, the load exerted by the facilities and the load exerted by the force F



(b) d_0 for the dead load of the bridge and the load exerted by the facilities

Fig. 14 Deformations of cable 2W determined numerically

Table 3 Deflections and tensile stress in the stays

Stay No.	Coordinate mm	Deflection d_e mm	Deflection d_F mm	Deflection d_0 mm	Deflection d_n=d_F–d_0 mm	Tensile stress σ_s MPa
1W	a = 6107	25	46.42	21.47	24.95	209
1T	a = 6476	13	24.23	11.22	13.01	415
2W	a = 7019	10	15.50	5.52	9.98	506
2T	a = 7109	15	23.24	8.25	14.99	341
3W	a = 4650	10	13.13	3.24	9.89	264
3T	a = 4699	12	16.05	3.93	12.12	217
4W	b = 8305	12	14.91	2.99	11.92	581
4T	b = 8214	15	18.73	3.72	15.01	465
5W	b = 4500	23	28.78	5.88	22.9	239
5T	b = 4783	12	14.91	3.01	11.89	480

the displacement was d_0=5.52 mm. The results of the numerical calculations are shown in Figs. 14(a) and (b).

Relationship 2 was used to numerically calculate the displacement caused by the force applied during the experiment (F=0.5 kN). The value obtained through calculations (d_n=9.98 mm) was in agreement with the value measured (d_e=10 mm). As can be seen, there is a negligible difference between d_e and d_n (0.2%), resulting from high sensitivity of the response of the cable to its stress state-the pull. A slight change in the residual stresses in the cables affects their statics; hence such differences between the values of d_e and d_n. For all the stay cables, the differences were less than 1.1%, which is a very good result. It can be concluded that the results as well as the methodology presented in this paper can be used to accurately calculate the key static parameters while modelling the operation of the load-carrying structure of the footbridge and, consequently, determine the response of the structure. Thus, the stresses acting in cable 2W were calculated to be σ_s=506 MPa.

The method described above was used to determine the values of the deflections d_e and d_n for all the cables anchored to the ground. It was then possible to calculate the values of the tensile stress in each stay σ_s. The results are summarised in Table 3.

The aim of the next stage of the analysis was to assess the condition of the footbridge in the full scale. The measurement and calculation results were used to study the behaviour of the entire structure. The bridge modelling involved iterative analysis of the stays to obtain tensile stress σ_s corresponding to the basic set of the bridge load-for the dead load and the load of the facilities, as

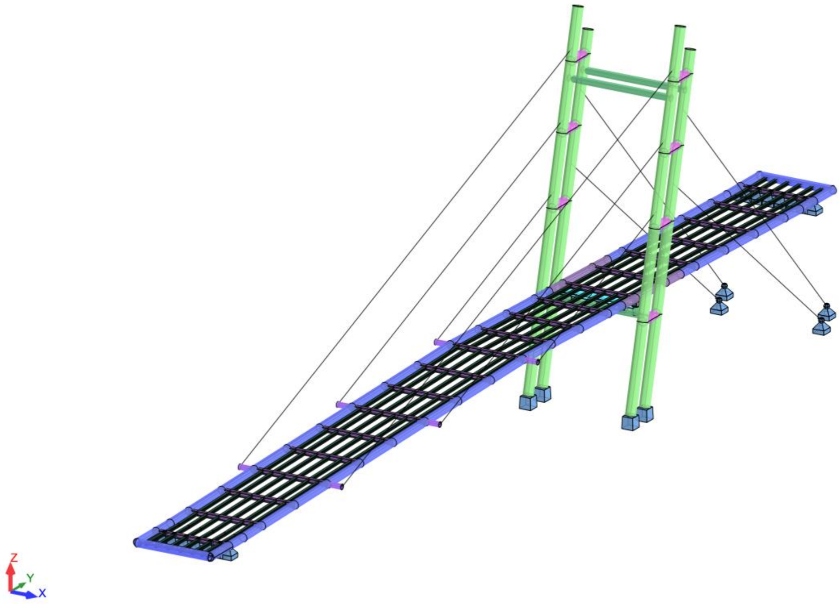


Fig. 15 A numerical model of the footbridge

Table 4 Mechanical properties of 18G2A structural steel (according to the PN-82/S-10052 standard)

Property	Symbol	Unit	Steel 18G2A
Design yield strength	R	MPa	290 (280)
Design shear strength	R_t	MPa	175 (170)
Design strength under contact stresses	R_d	MPa	350
Design strength under Hertzian contact stresses	R_{dH}	MPa	1050
Modulus of elasticity	E	GPa	205

presented in Table 3.

The numerical model was developed on the basis of the detailed design for the construction of the footbridge. The bridge geometry was created using the dimensions provided in the specifications, with the bridge length, the span lengths and the tower height being 41.77 m, 29.077 m+12.771 m and 13.805 m, respectively, as illustrated in Figs. 10 and 11. The 3D analysis was carried out using 1-D beam elements and cables. The steel structure of the footbridge was modelled using finite elements in the form of rods connected rigidly or pivotally, depending on the actual joints in the load-carrying structure. The cables anchored to the deck (marked as W) and the cables anchored to the ground (marked as T) were modelled using finite elements in the form of high tension cables, like in the previous stage of the analysis. It was assumed that pivotal joints were used at the span ends, whereas rigid restraints were applied in the tower foundations. The numerical model of the footbridge is shown in Fig. 15.

The analysis was performed for a span structure made up of two RO 323.9/20.0 round-section steel tubes, one RO 323.9/28.0 insert in the tower zone and HEB 160 sections constituting the lateral cross-bracing. The deck was considered to be a 10 mm thick plate supported on HEB 100 longitudinal beams. The deck is suspended by stay cables anchored at the tubular supports

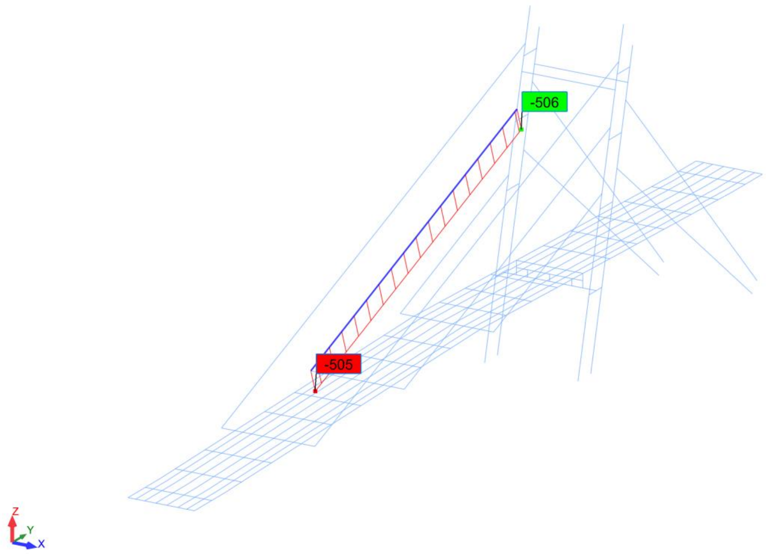


Fig. 16 Stresses in cable 2W determined through 3D analysis (σ=σ_s=506 MPa)

mounted perpendicular to the deck tubes. The modelling was done for 3T15 cables anchored to the deck and 2T15 cables anchored to the ground. Each support consists of an RO 219.1/20.0 steel tube and a tube encasing the cables. The tower comprises RO 406.4/25 m two-branch steel tubes connected with lacing tubes.

The load-carrying structure of the footbridge was made of 18G2A structural steel according to the PN-82/S-10052 standard. This material was used in the numerical analysis to define the material properties of the structural elements. The parameters of 18G2A steel used for the structure correspond to the parameters of S355 steel recommended by the PN-EN 10025-2:2007. The mechanical properties of 18G2A steel are provided in Table 4.

The values not in brackets apply to elements with a thickness of 16 mm; the values in brackets refer to elements with a thickness ranging from 16 mm to 30 mm.

The static analysis focused on calibrating the tensile stress σ_s using the data obtained in the preliminary analysis (Table 3). There was a condition that the values of the normal stress σ in the stay cables obtained at the combined effect of the dead load of the footbridge and the load exerted by the facilities should correspond to the values of σ_s determined in the previous step of the analysis

$$\sigma = \sigma_s \tag{4}$$

When the condition was satisfied, the numerical modelling was completed, and the modelled structural response was compared with that reported in a real situation.

The data concerning the performance of each stay cable and the stress σ_s occurring in it was used to study the whole numerical 3D model (Fig. 15). The analysis was conducted iteratively assuming that condition 4 had to be satisfied. For instance, at the combined effect of the dead load of the footbridge and the load exerted by the bridge facilities, the normal stress in cable 2W was σ_s=506 MPa. The pull of cable 2W was calibrated iteratively in the 3D model by defining the properties of the cable and the pull of the cable itself so that the normal stress in the cable at the

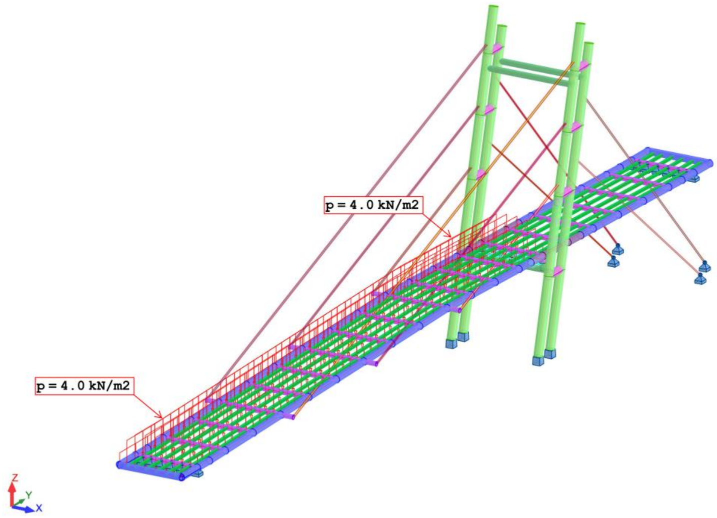


Fig. 17 An example of live load acting on the footbridge

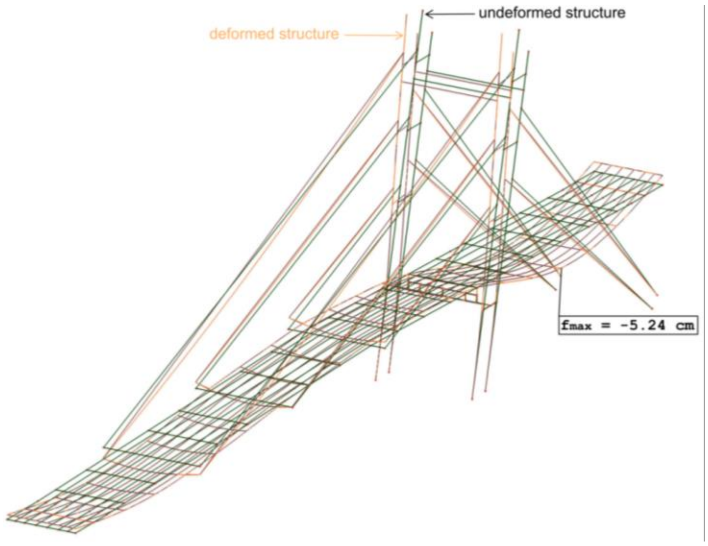


Fig. 18 A representation of the extreme deformations of the footbridge

combined effect of the dead load of the footbridge and the load exerted by the facilities was σ=506 MPa, which corresponded to the value of σ_s determined for a single cable, i.e., cable 2W (σ_s=506 MPa). The problem is illustrated in Fig. 16.

The same calibrations were performed for the other cables until condition 4 was satisfied. Once the agreement $\sigma=\sigma_s$ was achieved for all the cables analysed in the 3D model, the response of the load-carrying structure of the bridge was calculated for the combined effect of the dead load of the footbridge and the load exerted by the bridge facilities. The results were then used to determine the load-carrying capacity, usability and safe operation of the bridge structure for cases specified in the design standards.

As mentioned above, the main purpose of the analysis was to measure the responses of the

Table 5 Tensile stress and the design tension resistance in the stays

Stay No.	Tensile stress σ_s MPa	Design tension resistance condition for ULS $\sigma_{s\ max}/R_{po}$		Limit design tension resistance
1W	415.4	0.36		
1T	621.7	0.54		
2W	890.0	0.77		
2T	725.4	0.63		
3W	568.4	0.49	<	1.0
3T	521.9	0.45		
4W	875.0	0.76		
4T	759.2	0.66		
5W	472.8	0.41		
5T	714.1	0.62		

footbridge. The static strength analysis was performed on the basis of the data provided in the design standards.

The requirements of the PN-90/B-03000, PN-85/S-10030 and PN-82/S-10052 standards were used to consider two systems of loads:

(a) the primary system (P), which comprises:

- the dead load (the weight of the structure itself);
- the load exerted by the bridge facilities;
- the load exerted by the pull of the stay cables;
- the live load (the load exerted by a crowd of pedestrians).

(b) the primary and secondary systems (PD), which comprise all the loads and, additionally:

- the changes in temperature.

For the calculations it was assumed that the basic loads were: the dead load, i.e. the weight of the footbridge itself, the load exerted by the bridge facilities, the load exerted by the pull of the stay cables, and the live load, i.e., the load exerted by a crowd of pedestrians moving with an intensity of 4.0 kN/m^2. The deformations of the cables were studied by applying heating/cooling, with the temperature gradient relative to the reference temperature 20°C being -10°C and +20°C, respectively.

Three variants of the load exerted by a crowd of pedestrians were considered:

- load distributed along the whole width of the spans,
- load distributed along half of the width of the spans (northern side),
- load distributed along the southern half of the span width of the spans (southern side),

Fig. 17 illustrates one of the cases of live load applied to the footbridge.

The considerations applied to a total of 32 combinations of simple load cases, with 22 for the ultimate limit state (ULS) and 10 for the serviceability limit state (SLS). The maximum and minimum load factors, γ_{max}=1.20 and γ_{min}=0.85, respectively, were taken into account.

The analysis enabled simulation of the structure behaviour and determination of the response of the structure under service loading. First, the footbridge deformations were studied (Fig. 18). From the calculations it was clear that the maximum vertical deflection of the footbridge was $f_{\max}$=–5.24 cm; it was smaller than the admissible value, $f_{\lim}$=1.3l/300=1.3×1365.9/300=–5.92 cm. The analysis also required determining the differences in deflections between the northern and southern

Table 6 Design resistance in the footbridge structural elements

Structural element	Design resistance condition for ULS E_d/R_d		Limit resistance condition for ULS
Longitudinal beams (HEB100)	0.23		
Lateral beams (HEB160)	0.59		
Deck Tube (323.9/20.0)	0.48		
Deck Tube (323.9/28.0)	0.53	<	1.0
Horizontal braces of the tower (in the longitudinal direction)	0.80		
Horizontal braces of the tower (in the lateral direction)	0.45		

E_d-the design value of the effect of actions such as internal force, moment or a vector representing several internal forces or moments, R_d-the design value of the corresponding resistance

parts of the footbridge, with a maximum value of 1.83 cm recorded in the central part of the span where cables 1W and 1T were anchored.

Then, it was essential to determine the actual internal forces in all the structural members of the footbridge. The load-carrying capacity analysis was performed using the standard design resistance conditions defined according to the requirements of the ULS. From the results obtained for each combination of loads it was clear that, in each of the stays, the forces were not greater than the load-carrying capacity of the stays. Table 5 shows the maximum stress σ_s and the corresponding design tension resistance $\sigma_{s\ max}/R_{po}$ for each stay.

Static calculations were then performed with the aim of determining the forces acting in the structural elements of the footbridge. The load-carrying capacity was analysed and the design tension resistance was calculated on the basis of the Polish standard, PN-82/S-10052. The data are provided in Table 6.

The load-carrying capacity of the footbridge was not exceeded in the static range.

5. Conclusions

This paper has discussed a combined experimental and numerical method that can be used to assess structural responses of cable-stayed footbridges and other structures similar in type and size. The following are the conclusions drawn from the study:

1. The proposed methodology provides an opportunity, first, to easily measure deformations of a structure under actual service conditions, then, to model the behaviour of the stays and the entire load-carrying system of a footbridge and, finally, to determine the structural responses.

2. The experimental data obtained in the study may be used to verify the load-carrying capacity of the cable-stayed footbridge and determine the deformations of the structural elements at the most unfavourable combination of loads according to the standards. The verification is easy and accurate. It was possible to determine the internal forces acting in all the main load-carrying footbridge elements, including the stays. The highest value of design resistance condition was reported for the horizontal braces of the tower (in the longitudinal direction); it amounted to 80%

of the limit design resistance. The maximum value of tension resistance condition for the stay defined by the normal stresses constituted 77% of the permissible tensile stress. The load-carrying capacity of the structure was not exceeded in the range of static loads.

3. The deformations of the footbridge numerically calculated from the deflections of the particular elements also satisfied the standard requirements; the ratio of the maximum deflections of the most deformed element to the permissible deflection was 89%. This suggests that the analysed structure meets the requirements of the serviceability limit state.

4. The combined experimental and numerical method for assessing the structural response is relatively easy to perform. It can be used to analyse the structural safety of cable-stayed footbridges and make decisions on their further service or maintenance procedures.

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