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Harmonic analysis of moderately thick symmetric cross-ply laminated composite plate using FEM

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Abstract. This paper presents the vibration and harmonic analysis of orthotropic laminated composite plate. The response of plate is determined using Finite Element Method. The eight noded shell 281 elements are used to analyze the orthotropic plates and results are obtained so that the right choice can be made in applications such as aircrafts, rockets, missiles, etc. to reduce the vibration amplitudes. Initially the model response for orthotropic plate and harmonic response for isotropic plate is verified with the available literature. The results are in good agreement with the available literature. Numerical results for the natural frequency and harmonic response amplitude are presented. Effects of boundary conditions, thickness to width ratio and number of layers on natural frequency and harmonic response of the orthographic plates are also investigated. The natural frequency, mode shape and harmonic analysis of laminated composite plate has been determined using finite element package ANSYS.

Keywords: finite element method; orthotropic plate; free vibration; harmonic response

1. Introduction

Composite materials have a wide range of application, especially in weight sensitive structures like spacecraft, aircraft due to their high stiffness-to-weight ratio and high strength-to-weight ratio. Laminated plates used in these structures are often subjected to dynamic loads which create vibration. When a system operates at the system natural frequency, resonance can happen causing large deformations and even catastrophic failure in improperly constructed structures. Careful designs can minimize those unwanted vibrations. This necessitates the study of harmonic and vibrational characteristics of these plates. Laminated composite plate structures have found numerous applications in structural elements used in various fields like aerospace, military, automotive industries etc. Bending, buckling and dynamic problems of laminated plates of various shapes subjected to various combinations of boundary conditions have been the subject of many research papers.

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A detailed study of the available literature was conducted to know the present state of knowledge available in the open literature, which can assist in achieving the present goals effectively. Several methods have been used to study such types of problems. DQM is one of the newer techniques being developed to study the problems whose mathematical model is a set of differential equation(s) - linear or nonlinear, ordinary or partial. Shu (2000) presents a good study of the differential quadrature technique and its various applications in engineering problems like the ones of Navier - Stokes equation, structural analysis and chemical engineering. For e.g., Shu and Richards (1992) applied the generalized differential quadrature method to solve two-dimensional incompressible Navier-Stokes equations.

It has been proved that the boundary conditions play a defining role in such problems. Most of the researches have been carried out using the classical boundary conditions i.e., Aagaah *et al.* (2006), Andakhshideh *et al.*(2010), Civalek (2008), Kantand Swaminathan (2001), Liew *et al.* (1997), Malekzadeh (2009), Nath and Shukla (2001), Sharma *et al.* (2005) and Zhang and Yang (2009). In these classical boundary conditions, corresponding to every degree of freedom, either the corresponding force (natural boundary conditions) or the displacement (essential boundary condition) is prescribed. The boundary condition which involves some suitable relationship between a displacement component and the corresponding force is more realistic. These more realistic edge conditions are being investigated by several researchers with the help of a model being termed as `elastic edges.'

Liew *et al.* (1997) presented apparently the first known results of free vibration analysis of symmetrically laminated cross-ply rectangular plates with edges having uniform elastic restraints - translational as well as rotational. Shu and Wang (1999) applied generalized differential quadrature method for the vibration analysis of thin isotropic plates with mixed and non-uniform boundary conditions. Gorman (1997) applied superposition-Galerkin method and Zhou (2001) applied the Rayleigh-Ritz method along with static Timoshenko beam functions for obtaining the natural frequencies of isotropic Mindlin rectangular plates.

Ashour (2004) did the vibration analysis of isotropic plates having variable thickness in one direction with edges elastically restrained against both rotation and translation using the finite strip transition matrix technique. Kapuria and Achary (2005) presented the new coupled zigzag theory for the dynamic analysis of hybrid plates which is the first dynamic theory of hybrid plates and exactly satisfied the conditions on the transverse shear stresses, at the top, bottom and layer interfaces for the case of nonzero in-plane electric fields. Karami *et al.* (2006) studied the natural frequencies of moderately thick symmetric laminated plates with elastically restrained edges using the Differential Quadrature Method (DQM). Ohya *et al.* (2006) presented the natural frequencies and mode shapes of the rectangular isotropic Mindlin plates with internal columns resting on uniform elastic edge supports using the superposition method. They achieved the compatibility between the plate and the column by requiring that the column and plate rotations be equal. Using one and two dimensional Fourier series expansions for the implicit spatial Discretization. Ferreira and Fasshauer (2007) presented a study of free vibration of isotropic and composite plates based on an innovative numerical scheme, where collocation by radial basis functions and pseudospectral methods are combined to produce highly accurate results. Civalek (2008) performed for the first time the analysis of thick symmetric cross-ply laminated composite plates based on the first-order shear deformation theory using a DSC approach and found the effect of thickness parameter insignificant for deflections and stress values. Gurses *et al.* (2009) presented the discrete singular convolution method for free vibration analysis of symmetrically laminated skew plates and Civalek *et al.* (2010) used the same method for buckling analysis of rectangular Kirchhoff plates subjected to compressive loads on two-opposite edges. Li *et al.* (2009) presented an exact series solution for the transverse vibration of isotropic thin rectangular plates with general elastic boundary supports. Li and Yu (2009) developed an empirical formula based on the analytical results obtained from the Rayleigh-Ritz method for predicting natural frequencies of a thin orthotropic rectangular plate with uniformly restrained edges. Zhang and Li (2009) studied the vibration of thin isotropic rectangular plates with arbitrary non-uniform elastic edge restraints, again, using two dimensional Fourier series expansions. Hsu (2010) presented the free vibration analysis of orthotropic rectangular plates resting on nonlinear elastic foundations and having linearly elastic edge supports using DQM. Sharma *et al.* (2011) presented a simple formulation for studying the free vibration of shear-deformable antisymmetric cross-ply laminated rectangular plates having translational as well as rotational edge constraints. Asadi and Fariborz (2011) obtained the governing equations and the required boundary conditions obtained for free vibration of composite symmetric and anti-symmetric plates using a global HSDT. As already stated in a few of the references above, in the context of analysis of plates, the DQM is becoming one of the commonly used techniques to study different types of problems. Karami and Malekzadeh (2002) did the static and stability analysis of arbitrary straight-sided quadrilateral thin plates using DQM. Wang and Wang (2004) studied the free vibration of thin sector plates by a new version of differential quadrature method. Ngo-Cong *et al.* (2011) presents a new effective radial basis function (RBF) collocation technique for the free vibration analysis of laminated composite plates using the first order shear deformation theory (FSDT). Sharma and Mittal (2013) shows the application of FEM for free vibration analysis of moderately thick laminated composite plates with edges elastically restrained against translation and rotation and found that the FEM can yield convergent and accurate solutions for thin and moderately thick laminated plates with classical boundary conditions and elastic edges supports. Useche *et al.* (2012) presented the modal and harmonic analysis of orthotropic shear deformable cracked plates using a direct time-domain Boundary Element Method formulation based on the elastostatic fundamental solution of the problem. Rao and Reddy (2012) proposes a harmonic analysis to design a propeller with a metal composite material to analyze its displacements and natural frequencies using ANSYS software. From the results of harmonic analysis, Rao and Reddy shows that composite propeller is safe against resonance phenomena. Maithry and Rao (2015) investigated the dynamic response of laminated composite plates to excitations, varying arbitrarily with time using ANSYS 13.0 software and suggested the most robust fiber orientation with respect to various response parameters.

2. Methodology

2.1 Background

ANSYS mechanical APDL is used to analyse the orthotropic laminated composite plate. It consists of pre-processor (i.e., building the geometry of the model, meshing the model with a proper smart sized mesh types), Solution (i.e., applying loads on the model, generate the solution) and post processing (i.e., reviewing the result data). The analysis has been done on orthotropic plates with various combinations of different boundary conditions. Modal analysis is done to find the system vibration parameters. And then for all these combination, harmonic analysis is done to find out the frequency response of plate.

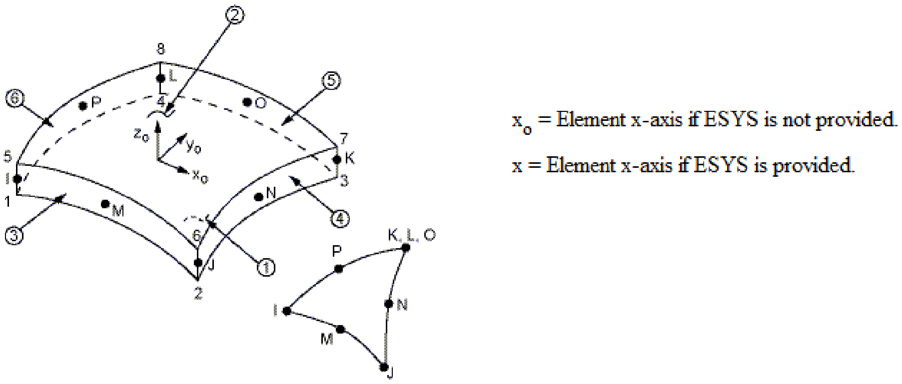


Fig. 1 SHELL281 Geometry

2.2 Modelling

For harmonic analysis, a square orthotropic plate of 1000 mm x 1000 mm of various thicknesses is considered in the study which is subjected to a force of 1 N at the node location (307.69 mm, 538.46 mm / node no. 241). The default parameters such as $E_1/E_2 = 40$, $\nu = 0.25$, $G_{12} = G_{13} = 0.6E_2, G_{23} = 0.5E_2$ and $\rho = 1$ kg/m^3 are used to provide the material properties of laminated. In order to compare with the published results of Ngo-Cong *et al.* (2011), the same shear correction factors $K_S = \pi^2/12$ and non dimensionalised natural frequencies $\varpi = \omega\left(b^2/\pi^2\right)\sqrt{\rho t/D_0}$ with $D_0 = E_2 t^3/12(1-\nu_{12}\nu_{21})$ are also employed. The element SHELL281 was chosen to mesh the model. It is suitable for analyzing thin to moderately-thick shell structures. It is an 8-node element with six degrees of freedom at each node: translations in the x, y, and z axes, and rotations about the x, y, and z-axes.
The degenerate triangular option should only be used as filler elements in mesh generation. This makes the comparison between the different orthotropic models easy.

3. Numerical results and discussion

3.1 Modal analysis

In modal analysis, firstly, the comparison study is done with available literature then non-dimensional fundamental frequency of laminated composite plates studied to determine the response of plates followed by convergence study.

3.1.1 Convergence study

In this paper the convergence study has been done for the various types of laminated plates with different boundary conditions and it is found that convergence is achieved at the mesh size of

13 × 13. Table 1 shows the convergence study of non-dimensionalised natural frequencies for clamped three ply [0°/90°/0°] square laminated for thickness ratios t/b = 0.2.

Table 2 shows the convergence study of non-dimensionalised natural frequencies for clamped three ply [0°/90°/0°] rectangular laminated plate for thickness ratios t/b = 0.2.

3.1.2 Comparison study

Table 3 shows the comparison study of non dimensionalised fundamental frequency for simply supported four-ply [0°/90°/90°/0°] square laminated symmetric plate for different thickness ratio. The results are compared with Ngo-Cong *et al.* (2011), Liew (1996) and Ferreira *et al.* (2007). It is found that the results are in the close proximity.

Table 1 Convergence study for a square cross-ply laminates [0°/90°/0°] for Clamped Boundary condition i.e., CCCC

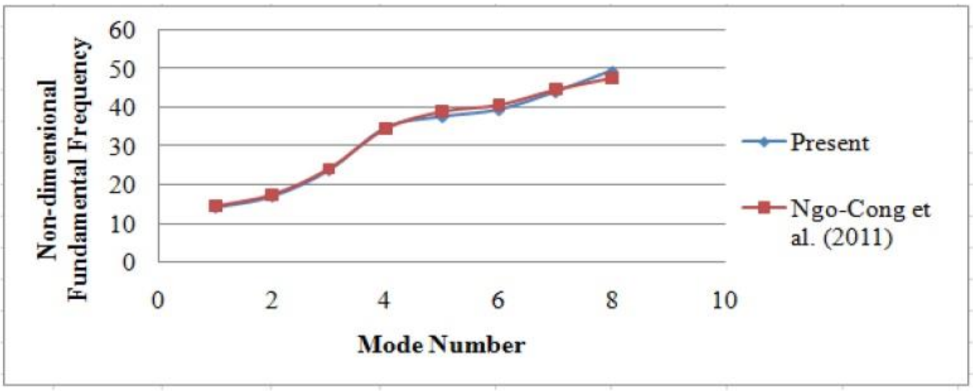
a/b	t/b	Mesh Size	$\overline{\omega} = \omega(b^2/\pi^2)\sqrt{\rho t/D_0}$									
			1	2	3	4	5	6	7	8	9	10
1	0.2	7×7	4.1488	5.6754	7.4319	7.9268	8.4120	10.0844	10.5125	11.1462	11.8309	12.2356
		9×9	4.1484	5.6720	7.4279	7.9059	8.4062	10.0642	10.4305	11.1172	11.8015	12.1593
		11×11	4.1481	5.6710	7.4265	7.8988	8.4046	10.0578	10.4031	11.1077	11.7920	12.1347
		13×13	**4.1481**	5.6707	7.4258	7.8948	8.4035	10.0541	10.3866	11.1020	11.7883	12.1246
		15×15	4.1481	5.6710	7.4265	7.8988	8.4046	10.0578	10.4031	11.1077	11.7866	12.1198
		17×17	4.1481	5.6703	7.4258	7.8941	8.4032	10.0534	10.3839	11.1013	11.7856	12.1175
		19×19	4.1481	5.6703	7.4255	7.8938	8.4032	10.0531	10.3822	11.1006	11.7849	12.1161

Table 2 Convergence study for a rectangular cross-ply laminates [0°/90°/0°] for Clamped Boundary condition i.e., CCCC

a/b	t/b	Mesh Size	$\overline{\omega} = \omega(b^2/\pi^2)\sqrt{\rho t/D_0}$									
			1	2	3	4	5	6	7	8	9	10
2	0.2	7×7	2.6690	3.9418	4.7017	5.5518	5.6507	6.8816	7.2644	7.5085	7.8546	8.4849
		9×9	2.6687	3.9394	4.6980	5.5471	5.6362	6.8667	7.2415	7.4515	7.8319	8.4291
		11×11	2.6686	3.9387	4.6966	5.5457	5.6315	6.8620	7.2340	7.4326	7.8245	8.4113
		13×13	**2.6686**	3.9384	4.6963	5.5450	5.6294	6.8600	7.2310	7.4244	7.8218	8.4042
		15×15	2.6685	3.9384	4.6960	5.5447	5.6284	6.8590	7.2297	7.4211	7.8201	8.4008
		17×17	2.6685	3.9384	4.6960	5.5447	5.6281	6.8586	7.2286	7.4190	7.8194	8.3991
		19×19	2.6685	3.9384	4.6956	5.5447	5.6277	6.8583	7.2283	7.4180	7.8191	8.3981

Table 3 Comparison of non dimensionalised fundamental frequencies for different thickness ratio for simply supported square laminated plate [0°/90°/90°/0°]

	$\varpi = \omega(b^2/\pi^2)\sqrt{\rho t/D_0}$							
t/b	0.01	0.02	0.04	0.05	0.08	0.1	0.2	0.25
Present	6.396	6.3364	6.1244	5.98	5.4723	5.11	3.6029	3.0861
Ngo-Cong *et al*. (2011)	6.6069	6.5464	6.3378	6.1882	5.6675	5.2991	3.7918	3.2806
%Error	*3.19212*	*3.20787*	*3.3671*	*3.36447*	*3.4442*	*3.56853*	*4.9818*	*5.92879*
Liew (p-Ritz) (1996)	6.606	6.549	6.338	6.193	5.677	5.311	3.807	3.295
%Error	*3.17893*	*3.2463*	*3.37015*	*3.43937*	*3.60578*	*3.7846*	*5.36118*	*6.33991*
Ferreira and Fasshauer (2007)	6.6012	6.5438	6.33	6.1844	5.6641	5.296	3.7903	3.2796
%Error	*3.10853*	*3.16941*	*3.24803*	*3.30509*	*3.38624*	*3.51208*	*4.9442*	*5.90011*



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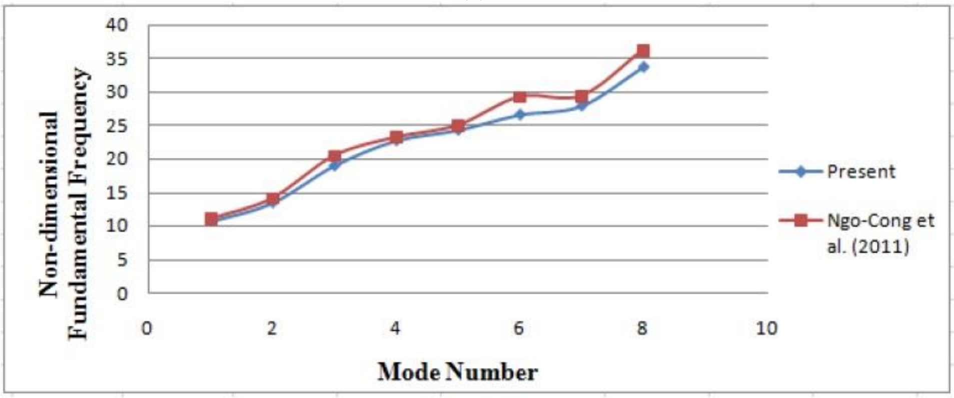

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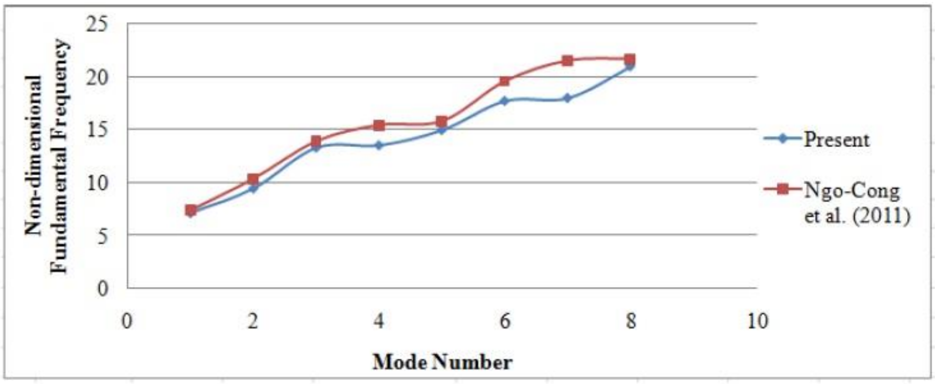

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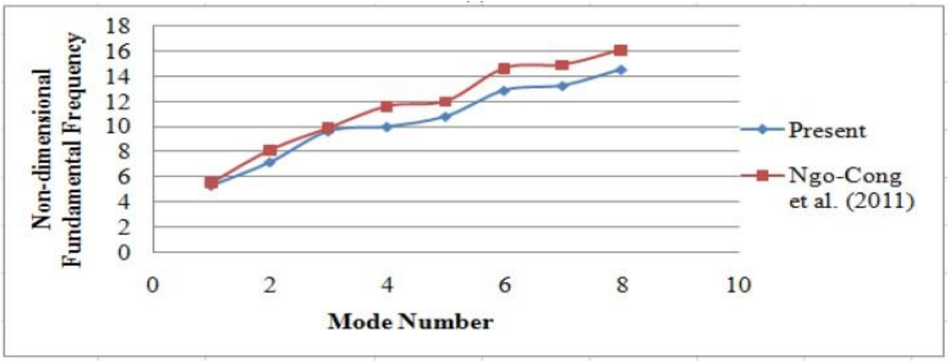

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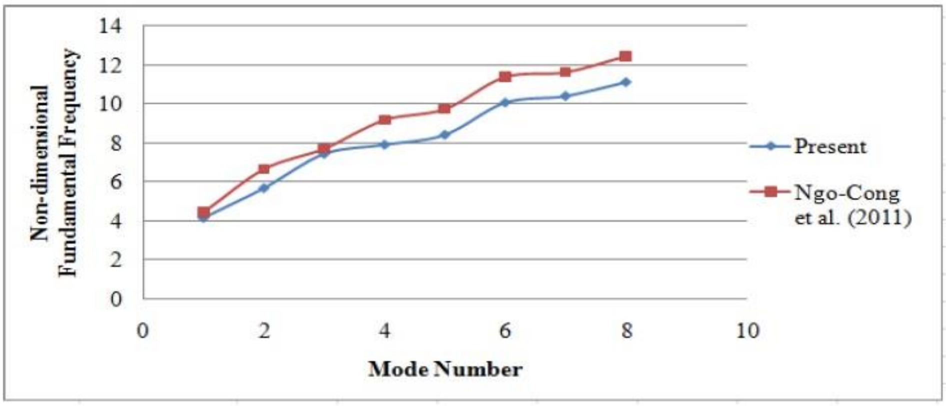
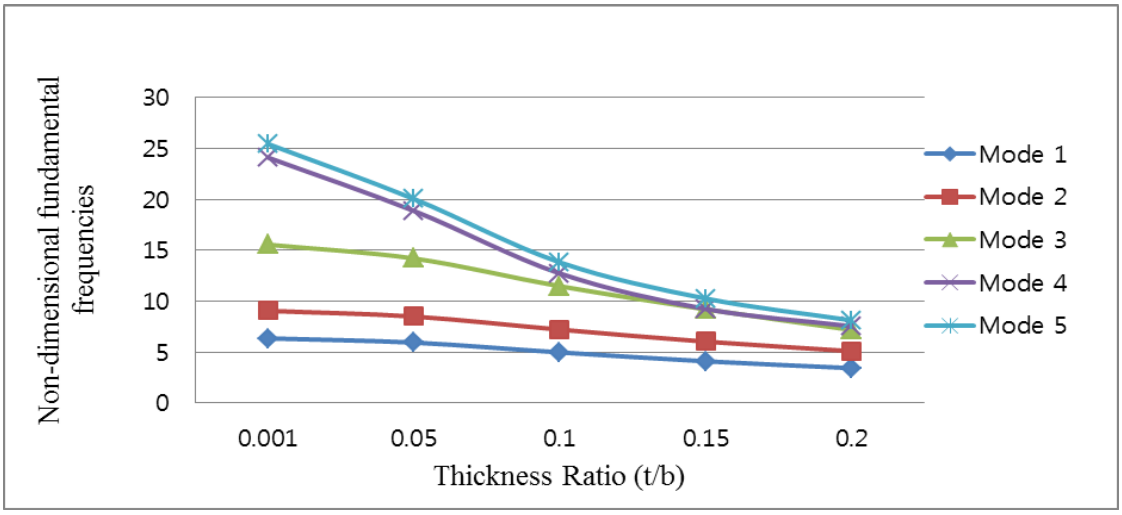


(e)

Fig. 3 (a) Comparison of First Eight non dimensionalised fundamental frequencies of square laminated plate with thickness ratio t/b = 0.001 for clamped Boundary Condition, (b) Comparison of First Eight non dimensionalised fundamental frequencies of square laminated plate with thickness ratio t/b = 0.05 for clamped Boundary Condition, (c) Comparison of First Eight non dimensionalised fundamental frequencies of square laminated plate with thickness ratio t/b = 0.1 for clamped Boundary Condition, (d) Comparison of First Eight non dimensionalised fundamental frequencies of square laminated plate with thickness ratio t/b = 0.15 for clamped Boundary Condition and (e) Comparison of First Eight non dimensionalised fundamental frequencies of square laminated plate with thickness ratio t/b = 0.2 for clamped Boundary Condition



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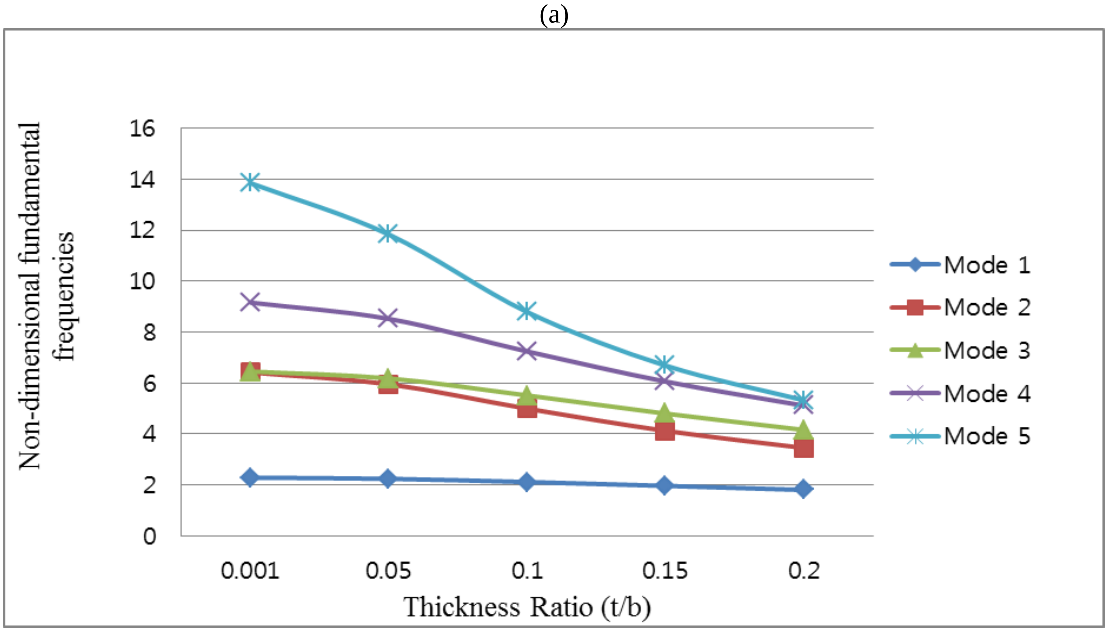


(b)

Fig. 4 (a) Variation of Non-dimensional frequencies with different thickness ratios for square laminated composite orthotropic plate with SSSS boundary condition and (b) Variation of Non-dimensional frequencies with different thickness ratios for rectangular laminated composite orthotropic plate with SSSS boundary condition

Figs. 3(a)-3(e) show the comparison of non dimensionalised fundamental frequencies for three-ply [0°/90°/0°] square laminated plates for different thickness with clamped Boundary Condition with Ngo-Cong *et al.* (2011) and it is found that the result is more accurate for the thin plate case t/b = 0.001.

The first five set of thickness ratios have been studied for three ply [0°/90°/0°] laminated composite plate for various thickness with different boundary conditions. The Block Lanczos solver is used for the vibration analysis of plates. The 13 × 13 mesh size is used. Figs. 4-6 show the variation of non-dimensional natural frequencies for the plate. It can be observed that the frequency decreases, as the plate thickness increases with the CCCC, SSSS and SCSC boundary condition.

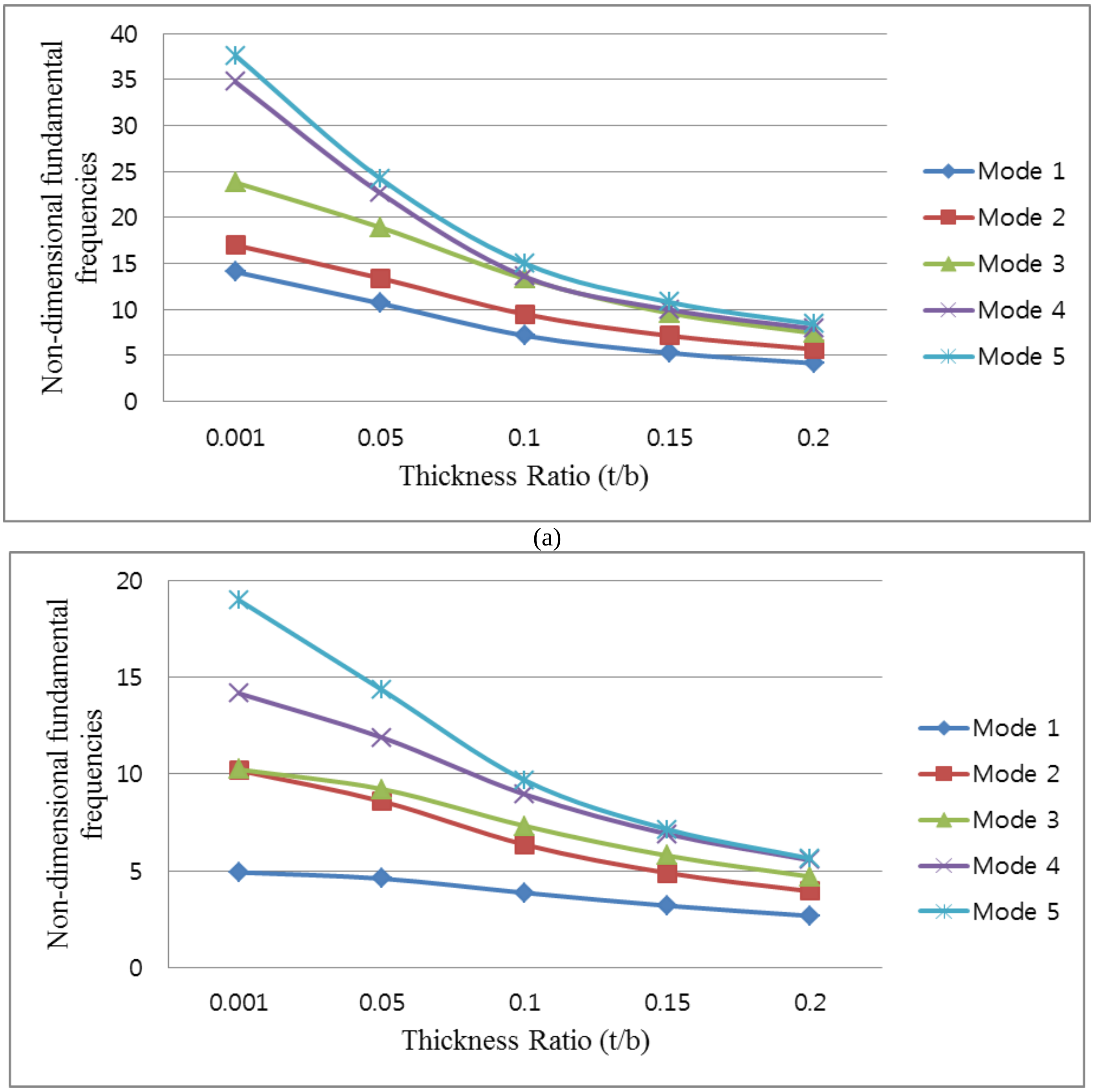
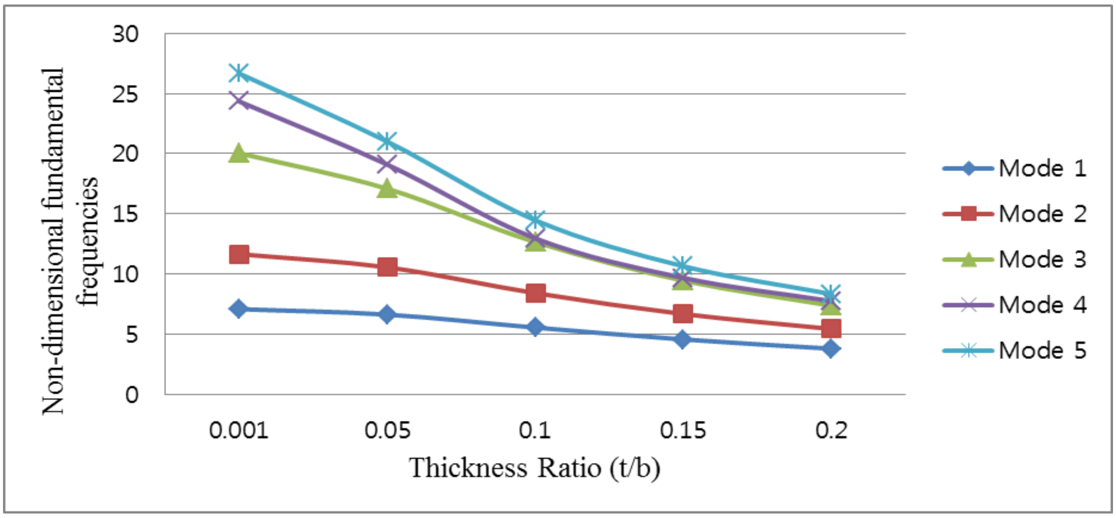
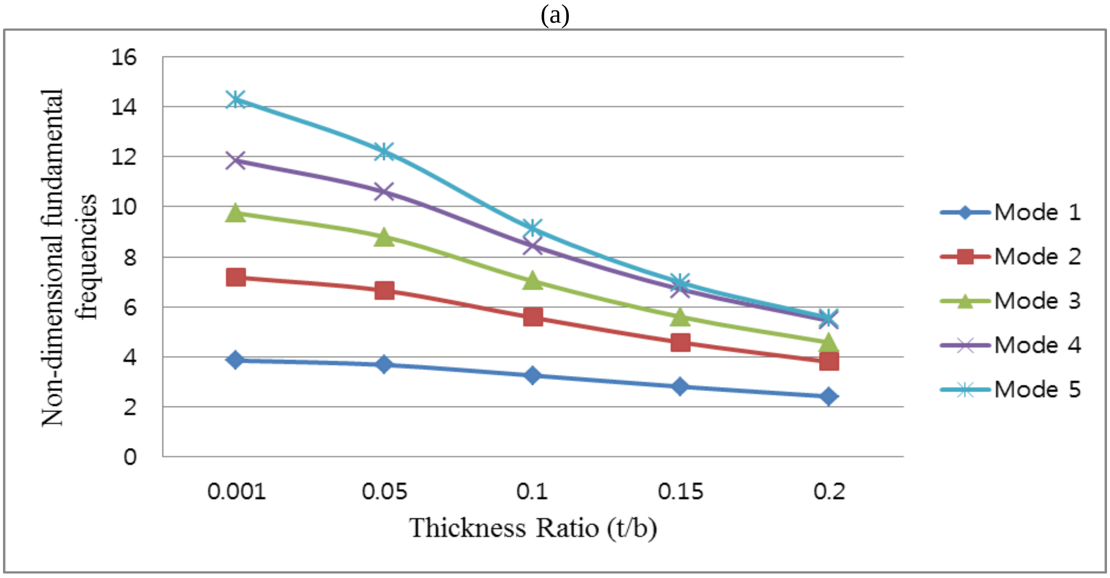


Fig. 5 (a) Variation of Non-dimensional frequencies with different thickness ratios for square laminated composite orthotropic plate with CCCC boundary condition and (b) Variation of Non-dimensional frequencies with different thickness ratios for rectangular laminated composite orthotropic plate with CCCC boundary condition



(a)



(b)

Fig. 6 (a) Variation of Non-dimensional frequencies with different thickness ratios for square laminated composite orthotropic plate with SCSC boundary condition and (b) Variation of Non-dimensional frequencies with different thickness ratios for rectangular laminated composite orthotropic plate with SCSC boundary condition

3.2 Harmonic analysis

Harmonic analysis of plate without uncertainty is solved in ANSYS, FEA software using the full method. Harmonic Analysis is done to compute the first natural frequencies for various boundary conditions with different thickness to length ratios. The results are compared for increasing layers for symmetrical laminated orthotropic plate.

3.2.1 Verification of harmonic analysis

A square plate with all edge fixed boundary condition, of dimension 304.8×304.8×2 mm of steel material studied to determine the harmonic analysis. The properties of plates material are $\rho=7.86\times10^{-9}$ tonne/mm^3, $\nu = 0.3$, E=2e5 MPa. To determine the response a force of 1 N is applied at the node location (75 mm, 152.4 mm) of the plates. The frequency range is considered from 0 to 3000 Hz.

Table 4 Comparison of the frequency response amplitude for a square steel plate for Clamped Boundary condition i.e., CCCC

Mode	***Frequency (Hz)***	***Amplitude (mm)***
Mode 1, Present	188.84	1.77e-2
Mode 1, Khan and Awari (2015)	188.06	1.8e-2
Error	0.41%	1.66%

Table 5 Effect of thickness ratio on the frequency response amplitude for symmetric laminated composite plate with various Boundary condition

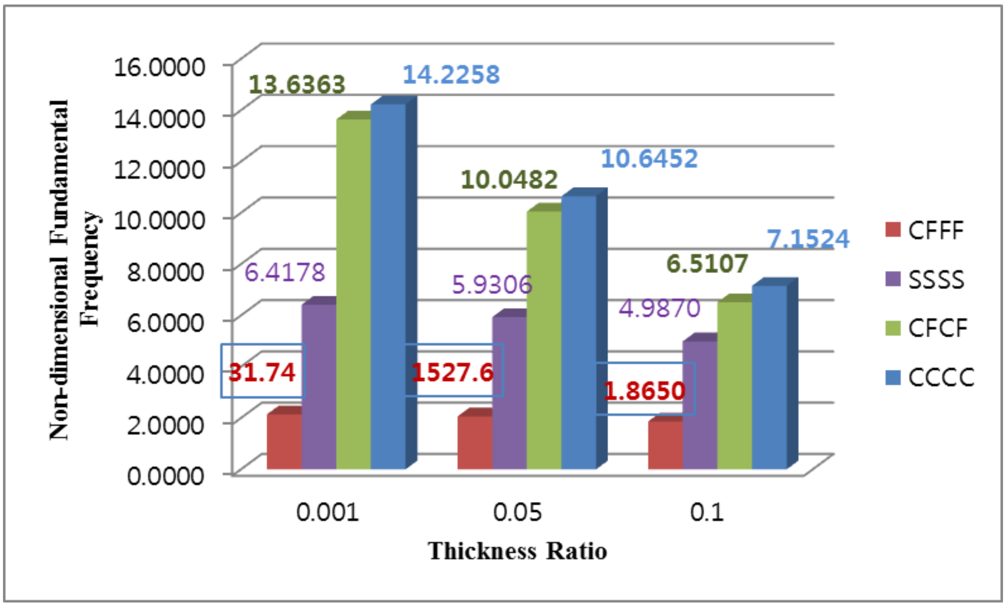
BC	**t/b**	***Three ply 0°/90°/0° plate***		***Five ply 0°/90°/0°/90°/0° plate***		***Seven ply 0°/90°/0°/90°/0°/90°/0° plate***	
		Fundamental Frequency (Hz)	Amplitude (mm)	Fundamental Frequency (Hz)	Amplitude (mm)	Fundamental Frequency (Hz)	Amplitude (mm)
CCCC	*0.001*	14.2258	124.748	14.2285	125.472	14.2298	125.529
	0.05	10.6452	0.001699	11.2521	0.0014927	11.4529	0.0014385
	0.1	7.1524	0.0004412	7.7999	0.0003532	8.0396	0.0003312
CFFF	*0.001*	2.1431	723.401	1.9486	871.4	1.8500	966.371
	0.05	2.0623	0.0066896	1.8895	0.0078427	1.7990	0.008604
	0.1	1.8650	0.0011679	1.7390	0.0013002	1.6677	0.0013961
CFCF	*0.001*	13.6363	62.6974	12.4021	74.7075	11.7748	82.0194
	0.05	10.0482	0.0009108	9.5896	0.0007880	9.2884	0.0008499
	0.1	6.5107	0.0002653	6.4425	0.0002073	6.3469	0.0002164
SSSS	*0.001*	6.4178	454.717	6.4182	454.695	6.4182	454.695
	0.05	5.9306	0.00423221	6.0273	0.0041044	6.0593	0.0040616
	0.1	4.9870	0.00074330	5.2232	0.0006795	5.3032	0.0006594

Finite element method is used to solve the problem of plate. The full method uses the full system matrices to calculate the harmonic response (no matrix reduction). During analysis, it is found that the fundamental mode for the plate is excited at the frequency 188.84 Hz, where the highest value of amplitude 1.71e-2 mm is obtained as shown in Table.

To know Frequency response function (FRF) plot, harmonic analysis is to be done by providing the different range and substeps of natural frequency. It will generate FRF plot (linear) on graph of amplitude to frequency, frequency (Hz) is taken on the x-axis and amplitude (m) on the y-axis. From this graph we come to know its resonance point. Also many details of system such as amount of displacement, by how much frequency by how much amount system excited. It can be said that the overall response of system can be known. Effect of different thickness to length ratio (t/b) on the frequency response amplitude for different sets of laminated scheme with different boundary conditions is shown in Table 5.

3.2.2 Effect of thickness ratio

Figs. 7(a) and 7(b) show the effect of thickness ratios on non-dimensional fundamental frequencies and resonance amplitude for various boundary conditions for three ply [0°/90°/0°] laminated composite Plate. It is observed as the thickness ratio increases the fundamental frequency increases but the value of resonance amplitude decreases for all boundary conditions.



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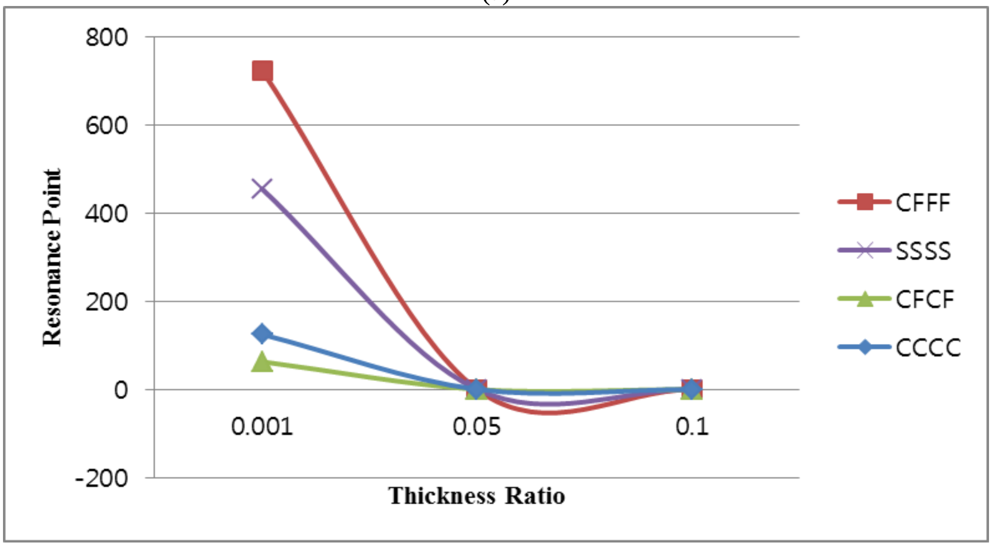
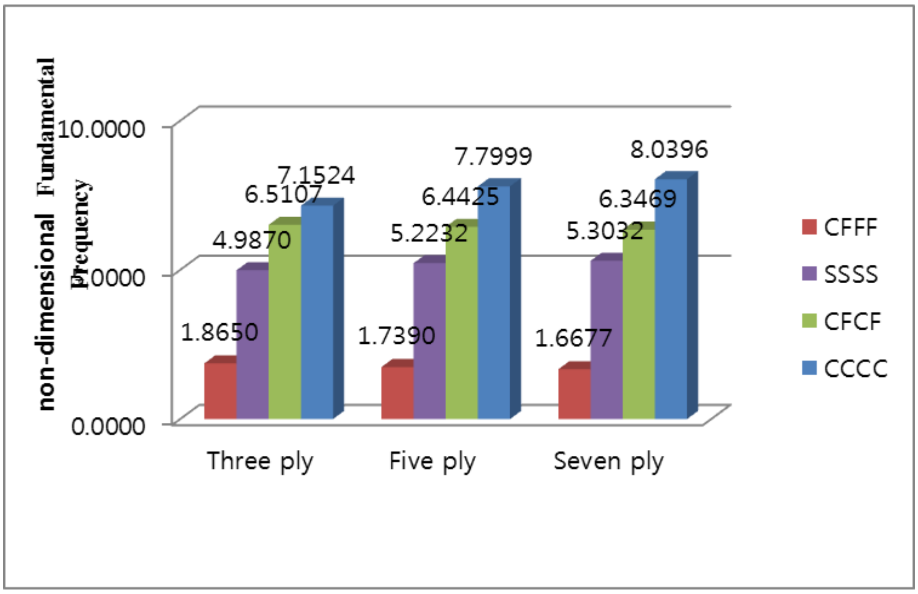


(b)

Fig. 7 (a) Effect of Thickness Ratios on non-dimensional Fundamental Frequency for various boundary conditions for three ply [0°/90°/0°] laminated composite Plate and (b) Effect of Thickness Ratios on Resonance Point for various boundary conditions three ply [0°/90°/0°] laminated composite Plate

3.2.3 Effect of number of layers

Figs. 8(a) and 8(b) show the effect of increasing layers on non-dimensional fundamental frequencies and resonance amplitude for various boundary conditions with thickness ratio 0.1. It is observed that the non-dimensional fundamental frequency increases as the number of layers increases but the variation in the non-dimensional fundamental frequency is negligible as the number of layers increases beyond the five layers. It is also observed that there is a significant variation in the resonance condition for CFFF in comparison to the other boundary conditions (i.e., SSSS, CFCF and CCCC)



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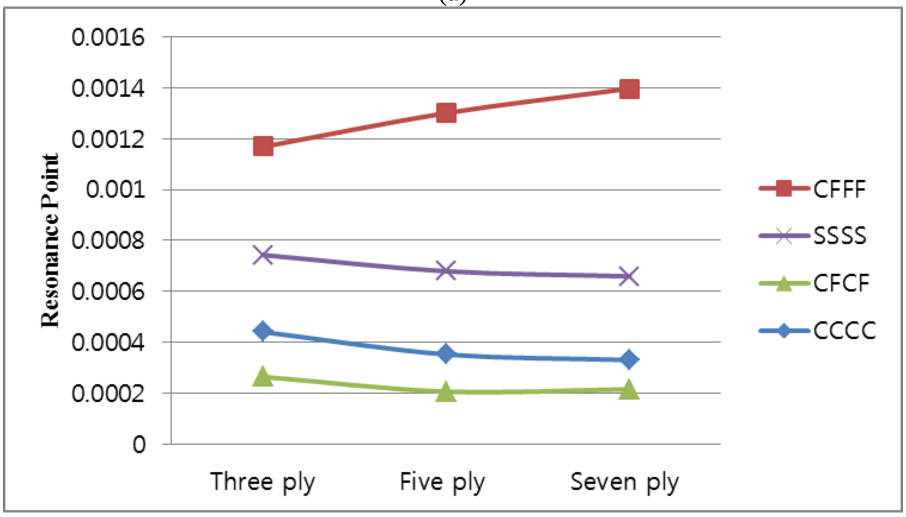
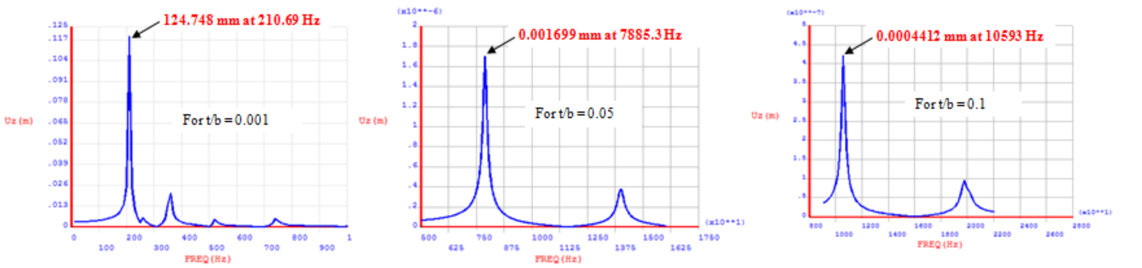


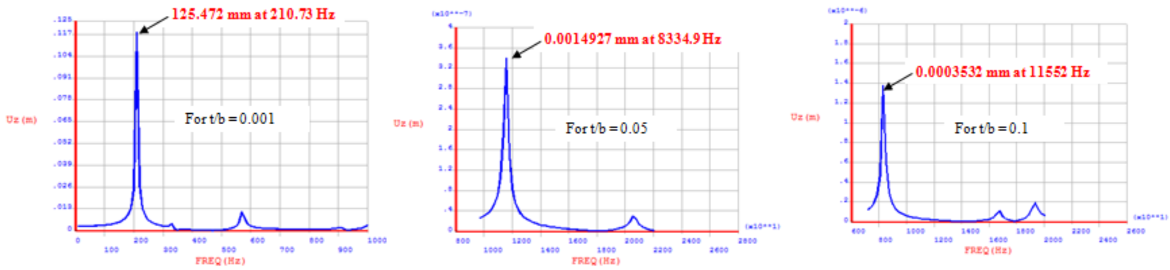
(b)

Fig. 8 (a) Effect of number of layers on non-dimensional Fundamental Frequency for various boundary conditions with thickness ratio 0.1 and (b) Effect of number of layers on Resonance Point for various boundary conditions with thickness ratio 0.1

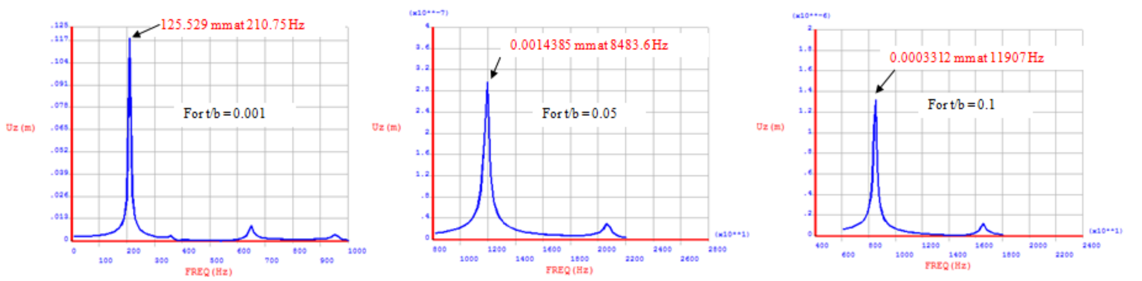
The plates with various boundary conditions are excited by external force and the response of plate is plotted on the FRF graph. The first fundamental mode is excited and single pick obtained in the graph. FRF plot of amplitude (m) to frequency (Hz) for orthotropic plate at different t/b ratio is shown on linear scale. Figs. 9-12 show the FRF plots for various thickness ratios (t/b=0.001, 0.05 & 0.1) with damping ratio (D=0.01) for orthotropic laminated composite symmetric plate at CCCC, CFFF, CFCF and SSSS boundary condition respectively with increasing layer. It shows that as increasing the value of t/b ratio, the maximum amplitude at resonance point decreases.



(a)

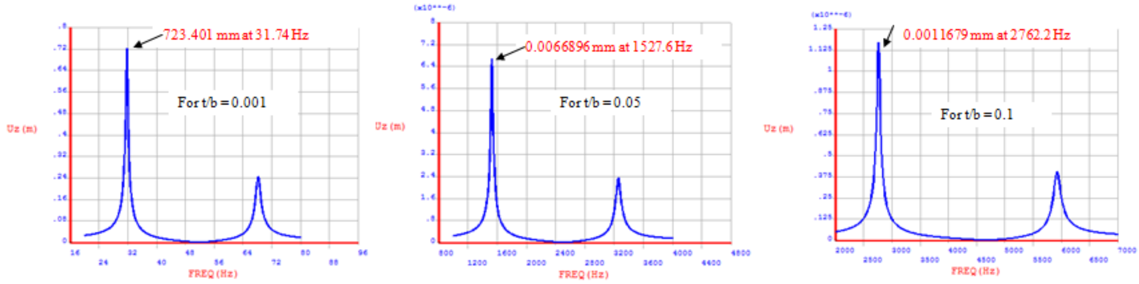


(b)

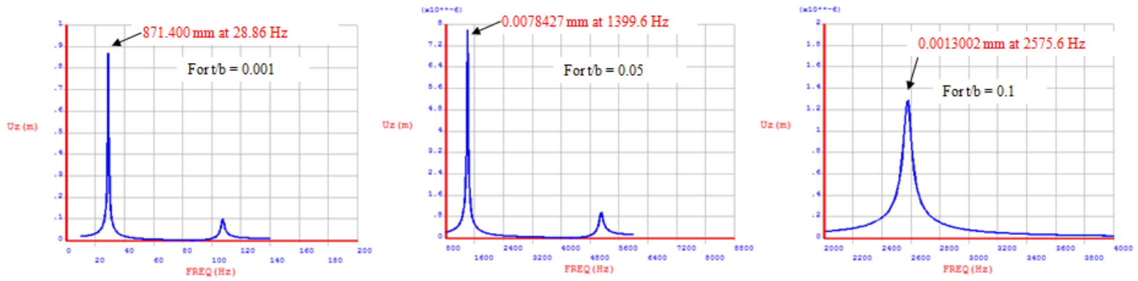


(c)

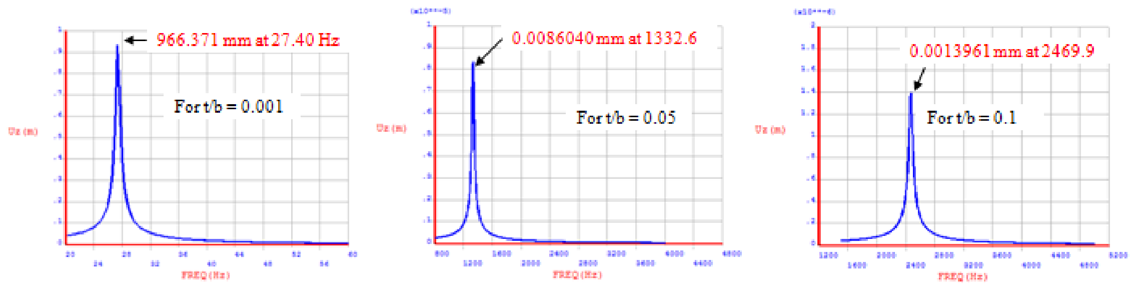
Fig. 9 (a) Harmonic response of three layered (0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at all edge fixed boundary condition i.e., CCCC, (b) Harmonic response of five layered (0°/90°/0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at all edge fixed boundary condition i.e., CCCC and (c) Harmonic response of seven layered (0°/90°/0°/90°/0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at all edge fixed boundary condition i.e., CCCC



(a)

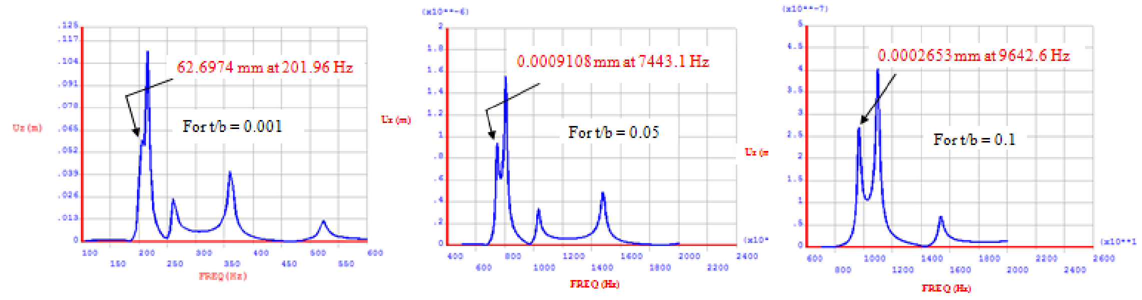


(b)

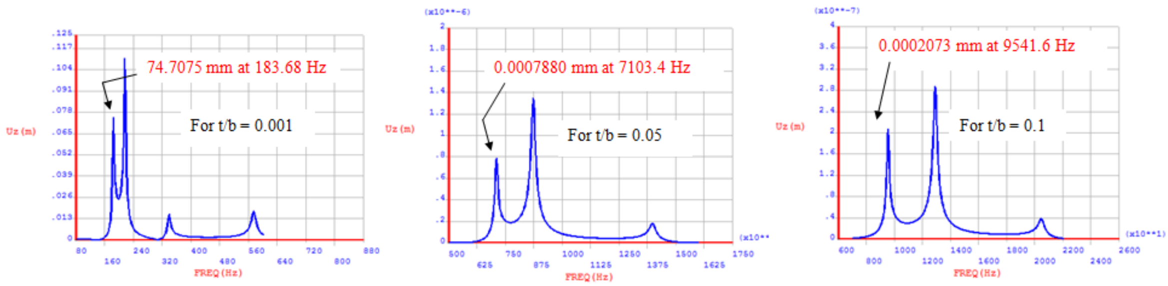


(c)

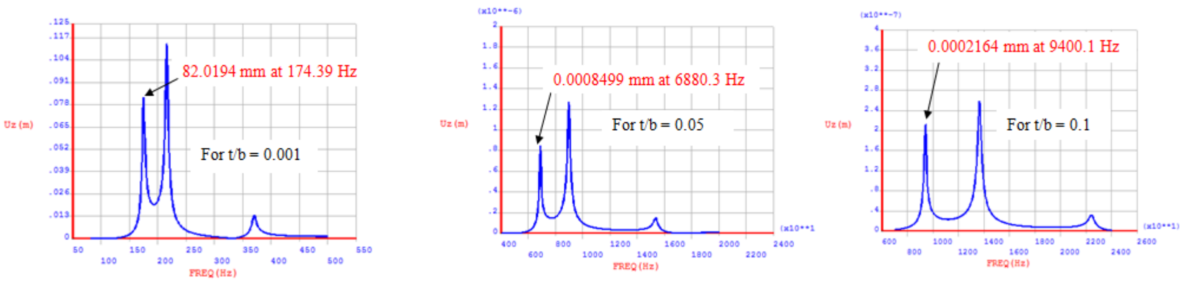
Fig. 10 (a) Harmonic response of three layered (0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at cantilever boundary condition i.e., CFFF, (b) Harmonic response of five layered (0°/90°/0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at cantilever boundary condition i.e., CFFF and (c) Harmonic response of seven layered (0°/90°/0°/90°/0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at cantilever boundary condition i.e., CFFF



(a)

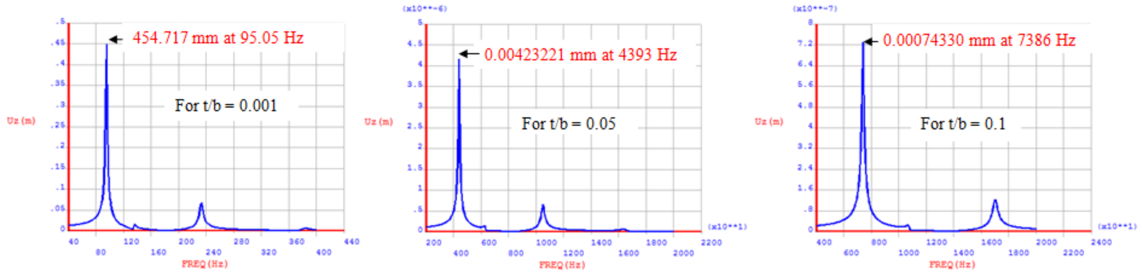


(b)

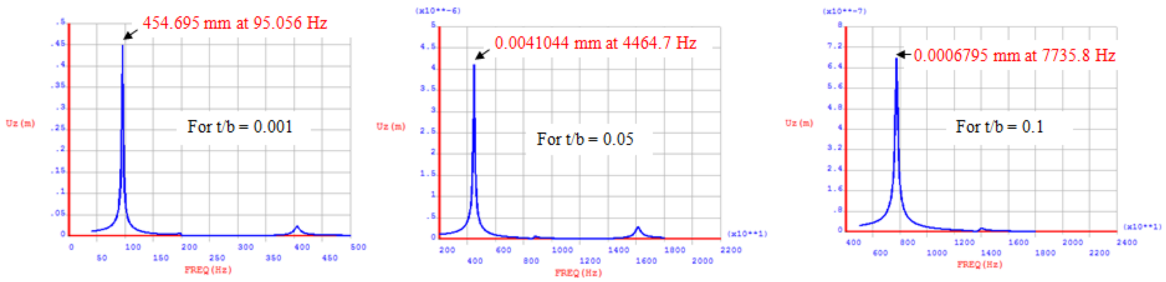


(c)

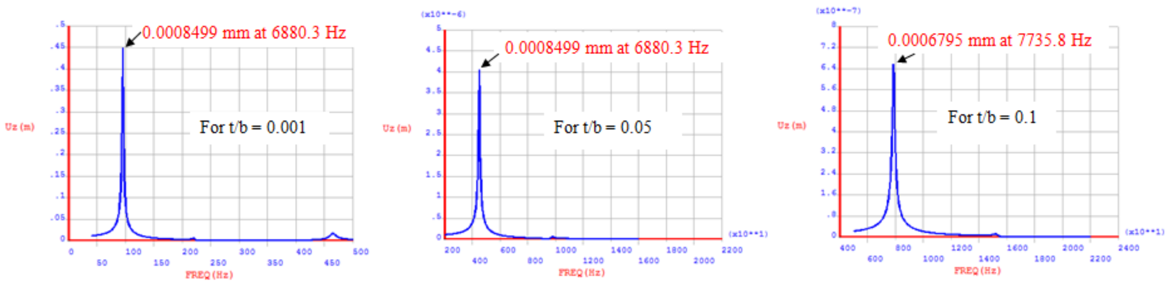
Fig. 11 (a) Harmonic response of three layered (0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at two edge clamped and two edge free boundary condition i.e., CFCF, (b) Harmonic response of five layered (0°/90°/0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at two edge clamped and two edge free boundary condition i.e., CFCF and (c) Harmonic response of seven layered (0°/90°/0°/90°/0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at at two edge clamped and two edge free boundary condition i.e., CFCF



(a)



(b)



(c)

Fig. 12 (a) Harmonic response of three layered (0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at all edge simply supported condition i.e., SSSS, (b) Harmonic response of five layered (0°/90°/0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at all edge simply supported condition i.e., SSSS and (c) Harmonic response of seven layered (0°/90°/0°/90°/0°/90°/0°) symmetric cross-ply laminated plate for different thickness ratio at all edge simply supported condition i.e., SSSS

4. Conclusions

In this study, vibration and harmonic analysis of orthotropic symmetric laminated plate is analysed. The following conclusion has been made:

- Convergence tests and comparison studies have been carried out using the Finite Element Method.
- The obtained thickness results have illustrated a good agreement with those available in the literature for different ratios, aspect ratios and different support conditions.
- It is clear from the results that the convergence is achieved at the mesh size of 13 × 13.
- The maximum variation in the non-dimensional fundamental frequency limited to 0 to 6%.
- It is observed that the non-dimensional fundamental frequency increases as the number of layers increases but the variation is negligible beyond five layers.
- It is observed that there is a significant variation in the resonance condition for CFFF in comparison to the other boundary conditions (i.e., SSSS, CFCF and CCCC)
- It is found that as the thickness ratio increases the fundamental frequency increases but the value of resonance amplitude decreases for all boundary conditions.

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